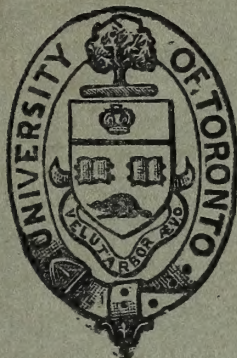


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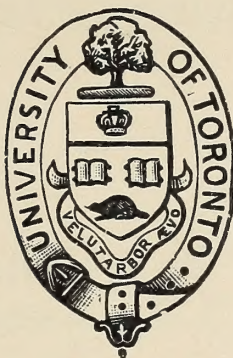


BULLETIN NO. 5

1925

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THE ADJUSTMENT AND COMPUTATION OF THE GULL LAKE TRIANGULATION

By L. B. STEWART, *Professor of Surveying and Geodesy*

On the establishment of the University of Toronto survey camp at the head of Gull Lake, in the Township of Lutterworth, Ontario, the advantage to be derived from the establishment of triangulation stations near the camp, and their connection with some primary station of the Geodetic Survey of Canada, was at once recognized. The staff in Surveying therefore resolved to carry out a survey with that object at the earliest opportunity.

During a conversation with the Director of the Geodetic Survey in the fall of 1921 he suggested that a good plan would be to lay out a chain of secondary triangulation connecting the primary stations Anson and Somerville, the former of which is situated about five miles west of the camp, and the latter almost due south of it. Points near the camp could then be included in the triangulation scheme, and the lengths of the lines could be made to depend upon that of the line joining those two stations as given by the primary triangulation, thus obviating the necessity for measuring a base.

In the summer of 1923 the opportunity occurred for putting this into effect, the reconnaissance being begun by the writer early in June of that year. On visiting Somerville station, which is situated near the south boundary of the township bearing that name, it was found that Anson station, about twenty-one miles distant, was invisible, owing, as was afterwards found, to the growth of trees on intervening high ground since the angles at those stations were observed. It was therefore decided to dispense with the direct use of that line.

The secondary stations, Coboconk, Dongola, Laxton, Miner's Bay, and Gull Lake, were then established in the positions and giving the figure shown in Fig. 1. The last-named station, being within a quarter of a mile of the survey camp, is conveniently placed to serve for connecting the surveys made in the neighbourhood of the camp with the primary triangulation network of the Dominion.

Two of the stations, Laxton and Dongola, required observing towers 78 and 40 feet in height, respectively, in order that the sight lines to the surrounding stations might clear the neighbouring trees. These were built by a tower-building party of the Geodetic Survey.

The angles were observed with a theodolite having a 7-inch circle read by two opposite verniers to 10". The method of observation was

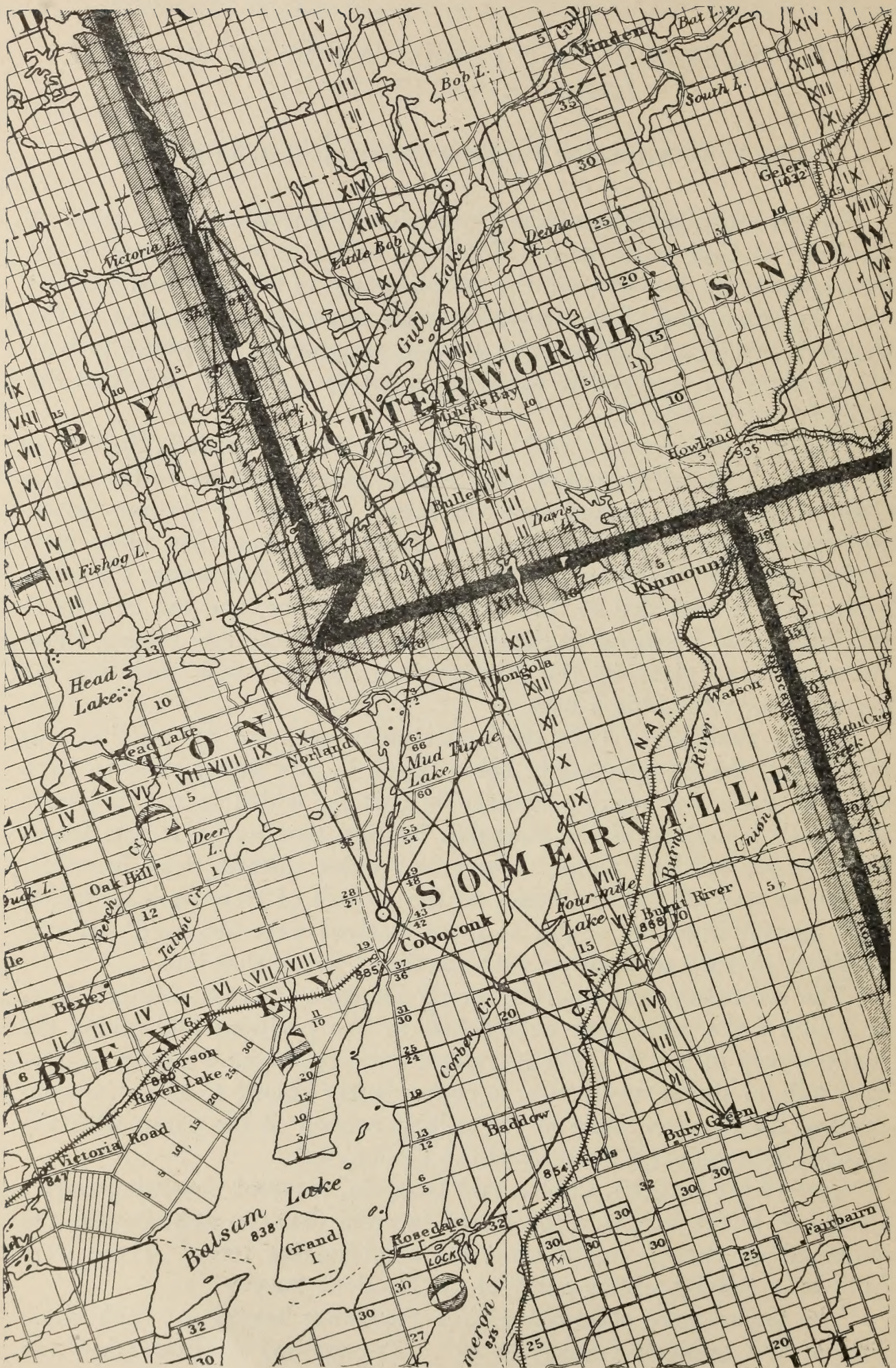


FIG. 1

by repetitions, twelve repetitions of each angle being made six of the angle itself with clamp right, and six of the explement of the angle with clamp left. The angles observed are shown in Fig. 2, numbered 1 to 27, their values being as follows:

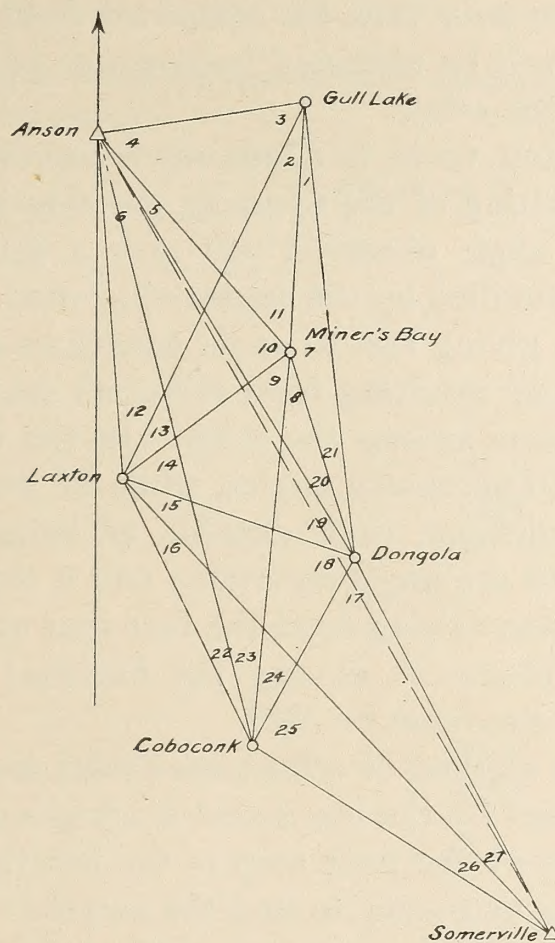


Fig. 2—Diagram of Triangulation

FIG. 2

Angle	Obs'd value			Angle	Obs'd value		
	°	'	"		°	'	"
1	8	25	46.0	15	27	24	30.1
2	22	36	20.1	16	17	41	54.8
3	55	55	00.0	17	57	04	38.1
4	56	56	11.8	18	80	11	05.4
5	10	49	40.0	19	40	47	49.6
6	26	20	40.7	20	15	12	19.2
7	161	29	25.8	21	10	04	48.0
8	21	38	01.8	22	12	07	14.8
9	46	41	09.1	23	20	24	24.1
10	85	38	55.5	24	22	10	44.9
11	44	32	25.8	25	93	04	23.5
12	29	58	28.6	26	14	31	14.1
13	27	12	19.7	27	15	19	46.3
14	55	40	44.9				

The ultimate object of a trigonometric survey is to determine the geodetic latitudes and longitudes of the stations, and the azimuths of

the lines joining them. The first step towards this end is the adjustment of the angles. This means the determination of the most probable system of corrections to the angles, which will at the same time give corrected values that will exactly fulfil all possible geometrical conditions. The length of any line may then be computed from that of any other, and the same result will be obtained irrespective of the route followed in going from one to the other.

In any triangulation there is a minimum number of angles which will permit of the plotting of the figure or the solution of the triangles, and each additional angle observed will give a geometrical condition that must be exactly fulfilled by the angles concerned. These conditions are also of two sorts, giving rise to angle equations and side equations, respectively, the former resulting from relations among the angles, and the latter from relations among the sides. In the triangulation under discussion there are 10 necessary angles, while 27 were observed; there are therefore 17 conditions, each capable of being expressed by an equation. Of these 10 are angle equations and 6 side equations; there is also a station equation arising from the fact that at one of the stations all the angles were observed, closing the horizon; the sum of their adjusted values must therefore be 360° .

The theory of the method of adjustment used cannot be given here, but it will be illustrated by the numerical work given below.

After the adjustment the next step is the solution of the triangles. In this instance it was necessary to find the lengths of the lines in terms of that of the line joining the primary stations Anson and Somerville, and as that line had not been observed it was necessary to resort to an indirect process. A length was assumed for the line between Dongola and Somerville, the triangles solved, and thence a provisional length found for the line between Anson and Somerville. The ratio of the correct to the provisional length of that line, being the same for all the lines used in determining the latter, served for finding the corrected lengths of those lines.

The final operation is the computation of the geodetic positions and azimuths, and the method used below in making these computations is essentially the same as that of the Geodetic Survey of Canada.

The numerical work involved in carrying out the operations outlined above follow.

The condition equations used in the adjustment of the triangulation are as follows:

Ten angle equations given by the triangles AGM, AML, AGL, LMD, ADL, GDM, MLC, MDC, LSC, and LDS;

Six side equations given by the quadrilaterals AGML, AMDL, GDLM, MDCL, ADCL, and LDSC; and

One station equation given by the angles about station M.

NOTE.—The stations are denoted by their initial letters.

FORMATION OF THE CONDITION EQUATIONS.
SPHERICAL EXCESSES OF TRIANGLES—

A preliminary solution of the triangles is made, assuming $AG = 5$ miles.

Formula for spherical excess—

$$e = \frac{a b \sin C}{2 \rho_m \rho_n \sin 1''} = m . ab \sin C$$

$$m = \frac{1}{2 \rho_m \rho_n \sin 1''}$$

		Log	
Triangle AGM —	a	0.69897	
	b	0.77622	
$C = G = 78^\circ 31' 20''$	$\sin C$	9.99123	
	5280^2	7.44526	
	m	<u>10.37204</u>	
	e	<u>1.28372</u>	$e = 0''.192$
Triangle AML —	a	0.70090	
	b	0.84418	
$C = M = 85^\circ 38' 55''$	$\sin C$	9.99875	
	5280^2	7.44526	
	m	<u>10.37204</u>	
	e	<u>1.36113</u>	$e = 0''.230$
Triangle AGL —	a	0.69897	
	b	0.91846	
$C = A = 94^\circ 06' 33''$	$\sin C$	9.99888	
	5280^2	7.44526	
	m	<u>10.37204</u>	
	e	<u>1.43361</u>	$e = 0''.271$
Triangle LMD —	a	0.70090	
	b	0.69924	
$C = M = 68^\circ 19' 11''$	$\sin C$	9.96814	
	5280^2	7.44526	
	m	<u>10.37205</u>	
	e	<u>1.18559</u>	$e = 0''.153$
Triangle ADL —	a	0.91846	
	b	0.75046	
$C = L = 112^\circ 51' 34''$	$\sin C$	9.96448	
	5280^2	7.44526	
	m	<u>10.37204</u>	
	e	<u>1.45070</u>	$e = 0''.282$

Triangle <i>GDM</i> —	<i>a</i>	0.77622	
	<i>b</i>	0.69924	
$C = M = 161^{\circ} 29' 26''$	$\sin C$	9.50169	
	5280^2	7.44526	
	<i>m</i>	<u>10.37204</u>	
	<i>e</i>	<u>2.79445</u>	$e = 0''.062$
Triangle <i>MLC</i> —	<i>a</i>	0.70090	
	<i>b</i>	0.83226	
$C = L = 100^{\circ} 47' 10''$	$\sin C$	9.99226	
	5280^2	7.44526	
	<i>m</i>	<u>10.37205</u>	
	<i>e</i>	<u>1.34273</u>	$e = 0''.220$
Triangle <i>MDC</i> —	<i>a</i>	0.69924	
	<i>b</i>	0.68891	
$C = D = 136^{\circ} 11' 14''$	$\sin C$	9.84029	
	5280^2	7.44526	
	<i>m</i>	<u>10.37205</u>	
	<i>e</i>	<u>1.04580</u>	$e = 0''.111$
Triangle <i>LSC</i> —	<i>a</i>	0.83226	
	<i>b</i>	0.91593	
$C = C = 147^{\circ} 46' 47''$	$\sin C$	9.72687	
	5280^2	7.44526	
	<i>m</i>	<u>10.37205</u>	
	<i>e</i>	<u>1.29237</u>	$e = 0''.196$
Triangle <i>LDS</i> —	<i>a</i>	0.75046	
	<i>b</i>	0.99134	
$C = D = 137^{\circ} 15' 43''$	$\sin C$	9.83165	
	5280^2	7.44526	
	<i>m</i>	<u>10.37205</u>	
	<i>e</i>	<u>1.39076</u>	$e = 0''.246$
Triangle <i>LDC</i> —	<i>a</i>	0.75046	
	<i>b</i>	0.83226	
$C = L = 45^{\circ} 06' 25''$	$\sin C$	9.85030	
	5280^2	7.44526	
	<i>m</i>	<u>10.37205</u>	
	<i>e</i>	<u>1.25033</u>	$e = 0''.178$

ANGLE EQUATIONS.

The corrections to the angles are denoted by the corresponding numbers bracketed.

Triangle *AGM*—
° ' "

2 = 22 36 20.1	(2) + (3) + (4) + (11) - 2.5 = 0
3 = 55 55 00.0	
4 = 56 56 11.8	
11 = 44 32 25.8	
179 59 57.7	
180 00 00.2	
- 2.5	

Triangle *AML*—
° ' "

5 = 10 49 40.0	(5) + (6) + (10) + (12) + (13) + 4.3 = 0
6 = 26 20 40.7	
10 = 85 38 55.5	
12 = 29 58 28.6	
13 = 27 12 19.7	
180 00 04.5	
180 00 00.2	
+ 4.3	

Triangle *AGL*—
° ' "

3 = 55 55 00.0	(3) + (4) + (5) + (6) + (12) + 0.8 = 0
4 = 56 56 11.8	
5 = 10 49 40.0	
6 = 26 20 40.7	
12 = 29 58 28.6	
180 00 01.1	
180 00 00.3	
+ 0.8	

Triangle *AMD*—
° ' "

8 = 21 38 01.8	(8) + (9) + (14) + (19) + (20) + 4.4 = 0
9 = 46 41 09.1	
14 = 55 40 44.9	
19 = 40 47 49.6	
20 = 15 12 19.2	
180 00 04.6	
180 00 00.2	
+ 4.4	

Triangle *ADL*—
 ° ' "

6 = 26 20 40.7	(6) + (12) + (13) + (14) + (19) + 3.2 = 0
12 = 29 58 28.6	
13 = 27 12 19.7	
14 = 55 40 44.9	
19 = 40 47 49.6	
180 00 03.5	
180 00 00.3	
+ 3.2	

Triangle *GDM*—
 ° ' "

1 = 8 25 46.0	(1) + (7) + (21) - 0.3 = 0
7 = 161 29 25.8	
21 = 10 04 48.0	
179 59 59.8	
180 00 00.1	
- 0.3	

Triangle *MLC*—
 ° ' "

9 = 46 41 09.1	(9) + (14) + (15) + (16) + (22) + (23) - 2.4 = 0
14 = 55 40 44.9	
15 = 27 24 30.1	
16 = 17 41 54.8	
22 = 12 07 14.8	
23 = 20 24 24.1	
179 59 57.8	
180 00 00.2	
- 2.4	

Triangle *MDC*—
 ° ' "

8 = 21 38 01.8	(8) + (18) + (19) + (20) + (24) + 0.8 = 0
18 = 80 11 05.4	
19 = 40 47 49.6	
20 = 15 12 19.2	
24 = 22 10 44.9	
180 00 00.9	
180 00 00.1	
+ 0.8	

Triangle *LSC*—
° ' "

16=17 41 54.8
22=12 07 14.8
23=20 24 24.1
24=22 10 44.9
25=93 04 23.5
26=14 31 14.1

(16)+(22)+(23)+(24)+(25)+(26)−4.0=0

179 59 56.2
180 00 00.2

−4.0

Triangle *LDS*—
° ' "

15=27 24 30.1
17=57 04 38.1
18=80 11 05.4
27=15 19 46.3

(15)+(17)+(18)+(27)−0.3=0

179 59 59.9
180 00 00.2

−0.3

SIDE EQUATIONS

Quadrilateral *AGML*. Pole at intersection of diagonals.

sin 4. sin 2. sin 10. sin 12

sin (5+6).sin 3. sin 11. sin 13

=1

Log. sin	<i>D.</i> for 1''	Log. sin	<i>D.</i> for 1''
4.....9.923279,0	1.37	5+6...9.781191,9	2.77
2.....9.584766,7	5.06	3.....9.918147,5	1.42
10.....9.998746,4	0.16	11.....9.845974,0	2.14
12.....9.698636,5	3.65	13.....9.660090,0	4.10
9.205428,6		9.205403,4	
9.205403,4			
+25.2			

5.06 (2)−1.42 (3)+1.37 (4)−2.77 (5)−2.77 (6)+0.16 (10)−2.14 (11)+3.65 (12)
−4.10 (13)+25.2=0.

Quadrilateral *AMDL*. Pole at *M*.

sin 20. sin (5+6), sin 14

sin 5. sin (12+13) sin (19+20)

=1

Log. sin	D. for 1''	Log. sin	D. for 1
20.....9.418763,5	7.74	5.....9.273828,6	11.01
5+6.....9.781191,9	2.77	12+13.....9.924474,8	1.36
14.....9.916923,8	1.43	19+20.....9.918586,7	1.42
9.116879,2		9.116890,1	
9.116890,1			
-10.9			

$$-8.24\ (5)+2.77\ (6)-1.36\ (12)-1.36\ (13)+1.43\ (14)-1.42\ (19)+6.32\ (20)-10.9=0.$$

Quadrilateral *GDL**M*. Pole at *M*.

$$\frac{\sin 1 \cdot \sin (19+20) \cdot \sin 13}{\sin 2 \cdot \sin 21 \cdot \sin 14} = 1$$

Log. sin	D. for 1''	Log. sin	D. for 1''
1....9.166108,5	14.21	2.....9.584766,7	5.06
19+20....9.918586,7	1.42	21.....9.243095,3	11.85
13....9.660090,0	4.10	14.....9.916923,8	1.43
8.744785,2		8.744785,8	
8.744785,8			
-0.6			

$$14.21\ (1)-5.06\ (2)+4.10\ (13)-1.43\ (14)+1.42\ (19)+1.42\ (20)-11.85\ (21)-0.6=0$$

Quadrilateral *MDCL*. Pole at intersection of diagonals.

$$\frac{\sin 8 \cdot \sin 18 \cdot \sin (22+23) \cdot \sin 14}{\sin 9 \cdot \sin (19+20) \sin 24 \cdot \sin (15+16)} = 1$$

Log. sin	D. for 1''	Log. sin	D. for 1''
8....9.566642,0	5.31	9....9.861894,8	1.99
18....9.993596,1	0.36	19+20....9.918586,7	1.42
22+23....9.730543,2	3.30	24....9.576921,2	5.16
14....9.916923,8	1.43	15+16....9.850293,9	2.10
9.207705,1		9.207696,6	
9.207696,6			
+8.5			

$$5.31\ (8)-1.99\ (9)+1.43\ (14)-2.10\ (15)-2.10\ (16)+0.36\ (18)-1.42\ (19)-1.42\ (20)+3.30\ (22)+3.30\ (23)-5.16\ (24)+8.5=0.$$

Quadrilateral *ADCL*. Pole at *A*.

$$\frac{\sin 19 \cdot \sin (12+ \dots 16) \cdot \sin (23+24)}{\sin (12+13+14) \cdot \sin 22 \cdot \sin (18+19)} = 1$$

	Log. sin	<i>D.</i> for 1''		Log. sin	<i>D.</i> for 1''
19.....	9.815167,4	2.44	12+13+14..	9.964477,5	-0.89
12+ . . 16..	9.574210,2	-5.20	22.....	9.322163,7	9.80
23+24.....	9.830392,3	2.29	18+19..	9.933147,8	-1.26
	9.219769,9			9.219789,0	
	9.219789,0				
	-19.1				
-4.31 (12)-4.31 (13)-4.31 (14)-5.20 (15)-5.20 (16)+1.26 (18)+3.70 (19)					
-9.80 (22)+2.29 (23)+2.29 (24)-19.1=0.					

Quadrilateral *LDSC*. Pole at *L*.

$$\frac{\sin 18. \sin (22+ . . 25). \sin 27}{\sin (22+23+24). \sin 26. \sin (17+18)} = 1$$

	Log. sin	<i>D.</i> for 1''		Log. sin	<i>D.</i> for 1''
18.....	9.993596,1	0.36	22+ . . 24....	9.911798,9	1.49
22+ . . 25..	9.726869,3	-3.35	26.....	9.399202,4	8.13
27.....	9.422212,4	7.68	17+18.....	9.831643,2	-2.28
	9.142677,8			9.142644,5	
	9.142644,5				
	+33.3				
2.28 (17)+2.64 (18)-4.84 (22)-4.84 (23)-4.84 (24)-3.35 (25)-8.13 (26)					
+7.68 (27)+33.3=0.					

Station equation.

$$\begin{array}{l} \text{o. ' ''} \\ 7 = 161 \ 29 \ 25.8 \\ 8 = 21 \ 38 \ 01.8 \\ 9 = 46 \ 41 \ 09.1 \\ 10 = 85 \ 38 \ 55.5 \\ 11 = 44 \ 32 \ 25.8 \\ \hline 359 \ 59 \ 58.0 \\ 360 \ 00 \ 00.0 \\ \hline -2.0 \\ \hline \end{array} \quad (7) + (8) + (9) + (10) + (11) - 2.0 = 0.$$

The solution of the normal equations gives the following values for the correlates

$$\begin{aligned}
 k_1 &= -0.1746 \\
 k_2 &= -2.5426 \\
 k_3 &= +1.2382 \\
 k_4 &= -4.0162 \\
 k_5 &= +1.4574 \\
 k_6 &= -0.5124 \\
 k_7 &= +2.0517 \\
 k_8 &= +1.1178 \\
 k_9 &= -1.2179 \\
 k_{10} &= +0.1910 \\
 k_{11} &= +1.8747 \\
 k_{12} &= -0.4297 \\
 k_{13} &= +0.2808 \\
 k_{14} &= -0.0159 \\
 k_{15} &= -0.0397 \\
 k_{16} &= +0.1958 \\
 k_{17} &= -0.2327
 \end{aligned}$$

These values, substituted in the correlate equations, give the following values for the corrections, and thence the corrected values of the angles:

Corrections	Corrected angles
	° ' "
(1) = -0.7383	1 = 8 25 45.26
(2) = -2.2684	2 = 22 36 17.83
(3) = +1.6738	3 = 55 55 01.67
(4) = +0.4748	4 = 56 56 12.27
(5) = -2.4279	5 = 10 9 37.57
(6) = +2.1211	6 = 26 20 42.82
(7) = +1.3623	7 = 161 29 27.16
(8) = -1.2345	8 = 21 38 00.57
(9) = -0.0108	9 = 46 41 09.09
(10) = -0.7367	10 = 85 38 54.76
(11) = +2.6197	11 = 44 32 28.42
(12) = -2.6412	12 = 29 58 25.96
(13) = -0.6144	13 = 27 12 19.09
(14) = -0.9836	14 = 55 40 43.92
(15) = +1.3079	15 = 27 24 31.41
(16) = -0.1010	16 = 17 41 54.70
(17) = -0.3396	17 = 57 04 37.76
(18) = +0.9269	18 = 80 11 06.33
(19) = -1.0814	19 = 40 47 48.52
(20) = -1.0899	20 = 15 12 18.11
(21) = -0.3240	21 = 10 04 47.68
(22) = -0.0897	22 = 12 07 14.71
(23) = +2.2775	23 = 20 24 26.38
(24) = +1.6795	24 = 22 10 46.58
(25) = -0.4384	25 = 93 04 23.06
(26) = +0.6740	26 = 14 31 14.77
(27) = -1.5961	27 = 15 19 44.70
Mean =	1.1790

[illegible]

	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}	k_{16}	k_{17}
(1) =						+1								+14.01			
(2) =	+1											+5.06		-5.06			
(3) =	+1											-1.42					
(4) =	+1											+1.37					
(5) =			+1									-2.77		-8.24			
(6) =			+1			+1						-2.77		+2.77			
(7) =							+1				+1						
(8) =					+1			+1			+1					+5.31	
(9) =					+1			+1			+1					-1.99	
(10) =		+1									+1	+0.16					
(11) =	+1										+1	-2.14					
(12) =		+1	+1			+1						+3.65		-1.36			-4.31
(13) =		+1				+1						-4.10		-1.36	+4.10		-4.31
(14) =					+1	+1							+1.43	-1.43		+1.43	-4.31
(15) =							+1			+1					-2.10	-5.20	
(16) =							+1								-2.10	-5.20	
(17) =									+1		+1						+2.28
(18) =								+1			+1						+2.64
(19) =				+1	+1			+1							+0.36	+1.26	
(20) =				+1				+1					-1.42	+1.42	-1.42	+3.70	
(21) =						+1							+6.32	+1.42			
(22) =									+1					-11.85			
(23) =							+1								+3.30	-9.80	-4.84
(24) =															+3.30	+2.29	-4.84
(25) =								+1							-5.16	+2.29	-4.84
(26) =																	-3.35
(27) =																	-8.13
										+1							+7.68

The values of the corrections given by these equations are now substituted in the condition equations, giving the normal equations, as below.



SOLUTION OF TRIANGLES.

Log. of assumed length of line $DS=4.1979876$.Triangle DSC —

Sph. angles ° ' "	Pl. angles "	Log.
$D=57\ 04\ 37.76$	37.66	$DS\ldots\ldots 4.1979876$
$S=29\ 50\ 59.47$	59.38	$\sin C\ldots\ldots 9.9993750$
$C=93\ 04\ 23.06$	22.96	$ 4.1986126$
		$\sin D\ldots\ldots 9.9239705$
180 00 00.29		$\sin S\ldots\ldots 9.6969924$
		$ 4.1225831$
		$CS\ldots\ldots 4.1225831$
		$CD\ldots\ldots 3.8956050$

Triangle LDC —

° ' "	"	
$L=45\ 06\ 26.11$	26.08	$CD\ldots\ldots 3.8956050$
$D=80\ 11\ 06.33$	06.29	$\sin L\ldots\ldots 9.8502964$
$C=54\ 42\ 27.67$	27.63	$ 4.0453086$
		$\sin D\ldots\ldots 9.9935965$
180 00 00.11		$\sin C\ldots\ldots 9.9118046$
		$ 4.0389051$
		$LC\ldots\ldots 4.0389051$
		$LD\ldots\ldots 3.9571132$

Triangle MDL —

° ' "	"	
$M=68\ 19\ 09.66$	09.59	$LD\ldots\ldots 3.9571132$
$D=56\ 00\ 06.63$	06.56	$\sin M\ldots\ldots 9.9681360$
$L=55\ 40\ 43.92$	43.85	$ 3.9889772$
		$\sin D\ldots\ldots 9.9185835$
180 00 00.21		$\sin L\ldots\ldots 9.9169223$
		$ 3.9075607$
		$LM\ldots\ldots 3.9075607$
		$MD\ldots\ldots 3.9058995$

Triangle AML —

° ' "	"	
$A=37\ 10\ 20.39$	20.33	$LM\ldots\ldots 3.9075607$
$M=85\ 38\ 54.76$	54.69	$\sin A\ldots\ldots 9.7811909$
$L=57\ 10\ 45.05$	44.98	$ 4.1263698$
		$\sin M\ldots\ldots 9.9987463$
180 00 00.20		$\sin L\ldots\ldots 9.9244703$
		$ 4.1251161$
		$AL\ldots\ldots 4.1251161$
		$AM\ldots\ldots 4.0508401$

Computation of LS —

$$\text{Formulæ—}\cos \theta = \frac{\sqrt{ab}}{a+b} 2 \cos \frac{1}{2} C \quad c = (a+b) \sin \theta.$$

Triangle LDS —

	Log
$LD = 9059.6875$	3.9571132
$DS = 15775.6618$	4.1979876
$LD.DS$	8.1551008
$\sqrt{LD.DS}$	4.0775504
2	0.3010300
$\cos \frac{1}{2} LDS$	9.5615440
$LD + DS = 24835.3493$	4.3950703
	3.9401244
$\cos \theta (\theta = 69^\circ 27' 50''.46)$	9.5450541
$\sin \theta$	9.9714856
$LD + DS$	4.3950703
LS	4.3665559

Triangle LCS —

$LC = 10937.1738$	4.0389051
$CS = 13261.2080$	4.1225831
$LC.CS$	8.1614882
$\sqrt{LC.CS}$	4.0807441
2	0.3010300
$\cos \frac{1}{2} LCS$	9.4432256
$LC + CS = 24198.3818$	4.3837863
	3.8249997
$\cos \theta (\theta = 73^\circ 58' 00''.66)$	9.4412134
$\sin \theta$	9.9827695
$LC + CS$	4.3837863
LS	4.3665558

Computation of AD —Triangle AMD —

	Log.
$AM = 11241.9096$	4.0508401
$MD = 8051.9204$	3.9058995
$AM.MD$	7.9567396
$\sqrt{AM.MD}$	3.9783698
2	0.3010300
$\cos \frac{1}{2} AMD$	9.3526149
$AM + MD = 19293.8300$	4.2854185
	3.6320147
$\cos \theta (\theta = 77^\circ 09' 58''.18)$	9.3465962
$\sin \theta$	9.9890128
$AM + MD$	4.2854185
AD	4.2744313

Triangle ALD —

$AL = 13338.7791$	4.1251161
$LD = 9059.6875$	3.9571132
$AL.LD$	8.0822293
$\sqrt{AL.LD}$	4.0411146
2	0.3010300
$\cos \frac{1}{2} ALD$	9.7427014
$AL + LD = 22398.4666$	4.3502183
	4.0848460
$\cos \theta (\theta = 57^\circ 07' 35''.71)$	9.7346277
$\sin \theta$	9.9242130
$AL + LD$	4.3502183
AD	4.2744313

Computation of AS —Triangle ADS —

$AD = 18811.8398$	4.2744313
$DS = 15775.6618$	4.1979876
$AD.DS$	8.4724189
$\sqrt{AD.DS}$	4.2362094
2	0.3010300
$\cos \frac{1}{2} ADS$	8.2288397
$AD + DS = 34587.5016$	4.5389192
	2.7660791
$\cos \theta (\theta = 89^\circ 01' 59''.79)$	8.2271599
$\sin \theta$	9.9999382
$AD + DS$	4.5389192
AS	4.5388574

Triangle ALS —

$AL = 13338.7791$	4.1251161
$LS = 23257.1176$	4.3665559
$AL.LS$	8.4916720
$\sqrt{AL.LS}$	4.2458360
2	0.3010300
$\cos \frac{1}{2} ALS$	9.5312643
$AL + LS = 36595.8967$	4.5634324
	4.0781303
$\cos \theta (\theta = 70^\circ 54' 22''.88)$	9.5146979
$\sin \theta$	9.9754250
$AL + LS$	4.5634324
AS	4.5388574
AS (by Primary Triang'n)	4.5272949
AS (provisional)	4.5388574
Ratio	1.9884375

Logarithms of corrected lengths of lines—

Line	Log.	Line	Log.
<i>CS</i>	4.1110206	<i>MD</i>	3.8943370
<i>CD</i>	3.8840425	<i>AL</i>	4.1135536
<i>LC</i>	4.0273426	<i>AM</i>	4.0392776
<i>LD</i>	3.9455507	<i>DS</i>	4.1864251
<i>LM</i>	3.8959982	<i>LS</i>	4.3549934

Triangle *AGM*—

Sph. angles ° ' "	Pl. angles "	Log.
<i>A</i> = 56 56 12.27	12.21	<i>AM</i>4.0392776
<i>G</i> = 78 31 19.50	19.43	sin <i>G</i>9.9912267
<i>M</i> = 44 32 28.42	28.36	4.0480509
<u>180 00 00.19</u>		sin <i>A</i>9.9232795
		sin <i>M</i>9.8459795
		<u><i>GM</i>.....3.9713304</u>
		<u><i>AG</i>.....3.8940304</u>

Triangle *ALS*—

<i>LS</i> = 23257.1176	
<i>AL</i> = 13338.7791	
<u><i>LS</i> + <i>AL</i> = 36595.8967.....</u>	<u>4.5634324</u>
<i>LS</i> - <i>AL</i> = 9918.3385.....	3.9964389
tan ½ (<i>A</i> + <i>S</i>) (= 19° 51' 59".895).....	<u>9.5579118</u>
° ' "	3.5543507
tan ½ (<i>A</i> - <i>S</i>) (= 5 35 35.426).....	<u>8.9909183</u>
<i>A</i> = 25 27 35.321	
<i>S</i> = 14 16 24.469	
<u><i>L</i> = 140 16 00.21</u>	
<u>180 00 00.000</u>	

COMPUTATION OF GEODETIC POSITIONS.

Formulae used in the computation—

$$x = \frac{s \sin \alpha_1}{N_1} \quad y = \frac{s \cos \alpha_1}{N_1}$$
$$\Delta \phi^1 = \frac{y}{\sin 1''} + \frac{y^3 \tan^2 \alpha_1}{3 \sin 1''} - \frac{x^2 \tan \phi^1}{2 \sin 1''}$$
$$\phi^1 = \phi_1 + \text{1st two terms.}$$
$$\Delta \phi = \Delta \phi^1 \frac{N_1}{R_m}$$
$$\Delta L = \frac{1}{\sin 1''} \quad x \sec \phi^1_2$$
$$\Delta \alpha = \Delta L \sin \phi_m \sec \frac{1}{2} \Delta \phi \quad \alpha_2 = \alpha_1 + 180^\circ + \Delta \alpha.$$

In these formulæ—

ϕ_1 = initial latitude

α_1 = initial azimuth

s = length of line

$\Delta\phi^1$ = diff. of latitude on a sphere of radius N_1

$\Delta\phi$ = diff. of latitude on the spheroid

ΔL = diff. of longitude

$\Delta\alpha$ = convergence of meridians

$\phi_2 = \phi_1 + \Delta\phi^1$

$\phi_m = \phi_1 + \frac{1}{2} \Delta\phi$

N_1 = length of normal for latitude ϕ_1

R_m = radius of curvature of meridian for latitude ϕ_m

α_2 = reverse azimuth

GEODETIC POSITIONS.

Line—Anson to Gull Lake.

		°	'	''
α Anson to Somerville	=	150	09	20.28
$\angle$ Somerville to Gull Lake	= -	68	38	57.17
α Anson to Gull Lake	=	81	30	23.11
		°	'	''
$\phi_1=44$	52	35.389		
$\Delta\phi =$	+37.336			
$\phi_2=44$	53	12.725		
$L_1=78$	52	05.935		
$-\Delta L =$	-5	53.096		
$L_2=78$	46	12.839		
s				Log.
N_1				3.8940304
s/N_1				6.8054315
$\sin \alpha_1$				3.0885989
$\cos \alpha_1$				9.9952105
x				9.1693763
y				3.0838094
$1/\sin 1''$				4.2579752
1st term	37''.3594			5.3144251
y^3				1.5724003
$\tan^2 \alpha_1$				12.7739256
$1/3 \sin 1''$				1.6516684
2nd term	0			4.83730
x^2				5.26289
$\tan \phi^1$	$\phi^1=44^\circ 53' 12''.748$			6.1676188
$1/2 \sin 1''$				9.9983272
-3rd term	-0''.1511			5.0133951
$\Delta\phi^1$	37''.2083			1.1793411
N_1				1.5706402
R_m				6.8054315
$\Delta\phi$	37''.3356			6.8039483
x				8.3760717
$1/\sin 1''$				1.5721234
$\sec \phi^{1_2}$	$\phi^{1_2}=44^\circ 53' 12''.597$			3.0838094
ΔL	353''.0965			5.3144251
$\sin \phi_m$	$\phi_m=44^\circ 52' 54''.057$			0.1496589
$\sec 1/2 \Delta\phi$				2.5478934
$\Delta\alpha$	249''.1608			9.8485863
				0
				2.3964797

Line—Anson to Miner's Bay.

α Anson to Somerville	= 150 09 20.28
$\angle$ Somerville to Miner's Bay	= - 11 42 44.90
α Anson to Miner's Bay	= 138 26 35.38

$\phi_1 = 44\ 52\ 35.389$	$\alpha_1 = 138\ 26\ 35.38$
$\Delta\phi = -4\ 25.491$	$180^\circ + \Delta\alpha = 180\ 03\ 52.98$
$\phi_2 = 44\ 48\ 09.898$	$\alpha_2 = 318\ 30\ 28.36$

$L_1 = 78\ 52\ 05.935$
$-\Delta L = -5\ 30.407$
$L_2 = 78\ 46\ 35.528$

		Log.
s		4.0392776
N_1		6.8054315
s/N_1		3.2338461
$\sin \alpha_1$		9.8217510
$\cos \alpha_1$		9.8740746 n
x		3.0555971
y		3.1079207 n
$1/\sin 1''$		5.3144251
1st term	-264''.4513	2.4223458 n
y^3		9.32376 n
$\tan^2 \alpha_1$		9.89535
$1/3 \sin 1''$		4.83730
2nd term	-0''.00011	4.05641 n
x^2		6.1111942
$\tan \phi^1$	$\phi^1 = 44^\circ\ 48'\ 10''.938$	9.9970141
$1/2 \sin 1''$		5.0133951
-3rd term	-0''.13231	1.1216034
$\Delta\phi^1$	-264''.58376	2.4225632 n
N_1		6.8054315
R_m		6.8039450
$\Delta\phi$	-265''.49092	9.2279947 n
x		3.0555971
$1/\sin 1''$		5.3144251
$\sec \phi^{1/2}$	$\phi^{1/2} = 44^\circ\ 48'\ 10''.805$	0.1490269
ΔL	330''.4069	2.5190491
$\sin \phi_m$	$\phi_m = 44^\circ\ 50'\ 22''.644$	9.8482660
$\sec 1/2 \Delta\phi$		0.0000001
$\Delta\alpha$	232''.9782	2.3673152

Line—Anson to Laxton.

		°	'	''	
α Anson to Somerville					=150 09 20.28
$\angle$ Somerville to Laxton					= 25 27 35.49
α Anson to Laxton					<u>=175 36 55.77</u>
		°	'	''	
$\phi_1=44$ 52 35.389					$\alpha_1=175$ 36 55.77
$\Delta\phi =$ -6 59.532					$180^\circ + \Delta\alpha =$ 180 00 31.82
$\phi_2=44$ 45 35.857					$\alpha_2=$ <u>355 37 27.59</u>
$L_1=78$ 52 05.935					
$-\Delta L =$ -45.147					
$L_2=78$ 51 20.788					
					Log.
s					4.1135536
N_1					<u>6.8054315</u>
s/N_1					3.3081221
$\sin \alpha_1$					8.8833743
$\cos \alpha_1$					<u>9.9987271n</u>
x					<u>4.1914964</u>
y					3.3068492n
$1/\sin 1''$					<u>5.3144251</u>
1st term	-418'' .0943				<u>2.6212743n</u>
y^3					<u>9.9205476n</u>
$\tan^2 \alpha_1$					7.7692944
$1/3 \sin 1''$					<u>4.83730</u>
2nd term					<u>6.52714n</u>
x^2					<u>8.3829928</u>
$\tan \phi^1$	$\phi^1=44^\circ 45' 37'' .295$				9.9963671
$\frac{1}{2} \sin 1''$					<u>5.0133951</u>
-3rd term	-0'' .00247				<u>3.3927550</u>
$\Delta\phi^1$	-418'' .0968				<u>2.6212769n</u>
N_1					<u>6.8054315</u>
R_m					<u>6.8039434</u>
					9.4267084n
$\Delta\phi$	-419'' .5319				<u>2.6227650n</u>
x					<u>4.1914964</u>
$1/\sin 1''$					5.3144251
$\sec \phi^{1_2}$	$\phi^{1_2}=44^\circ 45' 37'' .292$				<u>0.1487061</u>
ΔL	45'' .14686				1.6546276
$\sin \phi_m$	$\phi_m=44^\circ 49' 05'' .624$				9.8481028
$\sec \frac{1}{2} \Delta\phi$					<u>0.0000002</u>
$\Delta\alpha$	31'' .8222				<u>1.5027306</u>

Line—Laxton to Coboconk.

	°	'	''	
α Laxton to Anson	=	355	37	27.59
$\angle$ Anson to Coboconk	=	-202	02	04.92
α Laxton to Coboconk	=	153	35	22.67
	°	'	''	
$\phi_1=44$	45	35.857		
$\Delta\phi =$	-5	09.058		
$\phi_2=44$	40	26.799		
			$\alpha_1 =$	153 35 22.67
			$180 + \Delta\alpha =$	180 02 31.32
			$\alpha_2 =$	333 37 53.99
$L_1=78$	51	20.788		
$-\Delta L =$	-3	35.061		
$L_2=78$	47	45.727		
			Log.	
s				4.0273426
N_1				6.8054285
s/N_1				3.2219141
$\sin \alpha_1$				9.6481621
$\cos \alpha_1$				9.9521293 n
x				4.8700762
y				3.1740434 n
$1/\sin 1''$				5.3144251
1st term	-307''	.9417		2.4884685 n
y^3				9.5221302 n
$\tan^2\alpha_1$				9.3920656
$1/3 \sin 1''$				4.83730
2nd term	0''	.000056		5.75150 n
x^2				7.7401524
$\tan \phi^1$	$\phi^1=44^\circ$	40'	27''	.915
$1/2 \sin 1''$				5.0133951
-3rd term	-0''	.05605		2.7486117
$\Delta\phi^1$	-307''	.9979		2.4885477 n
N_1				6.8054285
R_m				6.8039356
				9.2939762 n
$\Delta\phi$	-309''	.0584		2.4900406 n
x				4.8700762
$1/\sin 1''$				5.3144251
$\sec \phi^1_2$	$\phi^1_2=44^\circ$	40'	27''	.859
ΔL	215''	.0613		2.3325623
$\sin \phi_m$	$\phi =44^\circ$	43'	01''	.328
$\sec 1/2 \Delta\phi$				0.0000001
$\Delta\alpha$	151''	.3185		2.1798919

Line—Coboconk to Somerville.

	° ' "	
α Coboconk to Laxton	=	333 37 53.99
∠ Laxton to Somerville	=	<u>-212 13 09.27</u>
α Coboconk to Somerville	=	<u>121 24 44.72</u>
° ' "		
ϕ ₁ = 44 40 26.799	α ₁ =	121 24 44.72
Δϕ = <u>-3 38.334</u>	180 + Δα =	<u>180 05 51.21</u>
ϕ ₂ = <u>44 36 48.465</u>	α ₂ =	<u>301 30 35.93</u>
L ₁ = 78 47 45.727		
-ΔL = <u>-8 19.802</u>		
L ₂ = <u>78 39 25.925</u>		
		Log.
s		4.1110206
N ₁		<u>6.8054263</u>
s/N ₁		3.3055943
sin α ₁		9.9311719
cos α ₁		<u>9.7170000n</u>
x		<u>3.2367662</u>
y		3.0225943n
1/sin 1''		<u>5.3144251</u>
1st term	-217''.2798	<u>2.3370194n</u>
y ³		9.0677829n
tan ² α ₁		0.4283438
1/3 sin 1''		<u>4.83730</u>
2nd term	-0''.00021	<u>4.33343n</u>
x ²		6.4735324
tan ϕ ¹	ϕ ¹ = 44° 36' 49''.519	9.9941445
½ sin 1''		<u>5.0133951</u>
-3rd term	-0''.30274	<u>1.4810720</u>
Δϕ ¹	-217''.5827	<u>2.3376244n</u>
N ₁		<u>6.8054263</u>
R _m		<u>6.8039299</u>
		9.1430507n
Δϕ	-218''.3337	<u>2.3391208n</u>
x		<u>3.2367662</u>
1/sin 1''		5.3144251
sec ϕ ¹ ₂	ϕ ¹ ₂ = 44° 36' 49''.216	<u>0.1476064</u>
ΔL	499''.8016	2.6987977
sin ϕ _m	ϕ _m = 44° 38' 37''.632	9.8467681
sec ½ Δϕ		<u>0.0000001</u>
Δα	351''.2092	2.5455659

Line—Miner's Bay to Dongola.

		'	''
α Miner's Bay to Anson	=	318	30 28.36
$\angle$ Anson to Dongola	=	-153	58 04.42
α Miner's Bay to Dongola	=	164	32 23.94
		°	' ''
$\phi_1=44$ 48 09.898	$\alpha_1=$	164	32 23.94
$\Delta\phi =$ -4 04.814	180 $+\Delta\alpha =$	180	01 06.89
$\phi_2=44$ 44 05.084	$\alpha_2=$	344	33 30.83
$L_1=78$ 46 35.528			
$-\Delta L =$ -1 34.984			
$L_2=78$ 45 00.544			
			Log.
s			3.8943370
N_1			6.8054296
s/N_1			3.0889074
$\sin \alpha_1$			9.4258045
$\cos \alpha_1$			9.9839945 n
x			4.5147119
y			3.0729019 n
$1/\sin 1''$			5.3144251
1st term	-243'' .96472		2.3873270 n
y^3			9.2187057 n
$\tan^2\alpha_1$			8.8826200
$1/3 \sin 1''$			4.83730
2nd term	-0.		6.93963 n
x^2			7.0294238
$\tan \phi^1$	$\phi^1=44^\circ$ 44' 05'' .933		9.9959823
$\frac{1}{2} \sin 1''$			5.0133951
-3rd term	0'' .01093		2.0388012
$\Delta\phi^1$	-243'' .97566		2.3873465 n
N_1			6.8054296
R_m			6.8039396
$\Delta\phi$	-244'' .81412		9.1927761 n
x			2.3888365 n
$1/\sin 1''$			4.5147119
$\sec \phi^{\frac{1}{2}}$	$\phi^{\frac{1}{2}}=44^\circ$ 44' 05'' .922		5.3144251
ΔL	94'' .98442		0.1485154
$\sin \phi_m$	$\phi_m=44^\circ$ 46' 07'' .491		1.9776524
$\sec \frac{1}{2} \Delta\phi$			9.8477251
$\Delta\alpha$	66'' .89252		0.0000001
			1.8253776

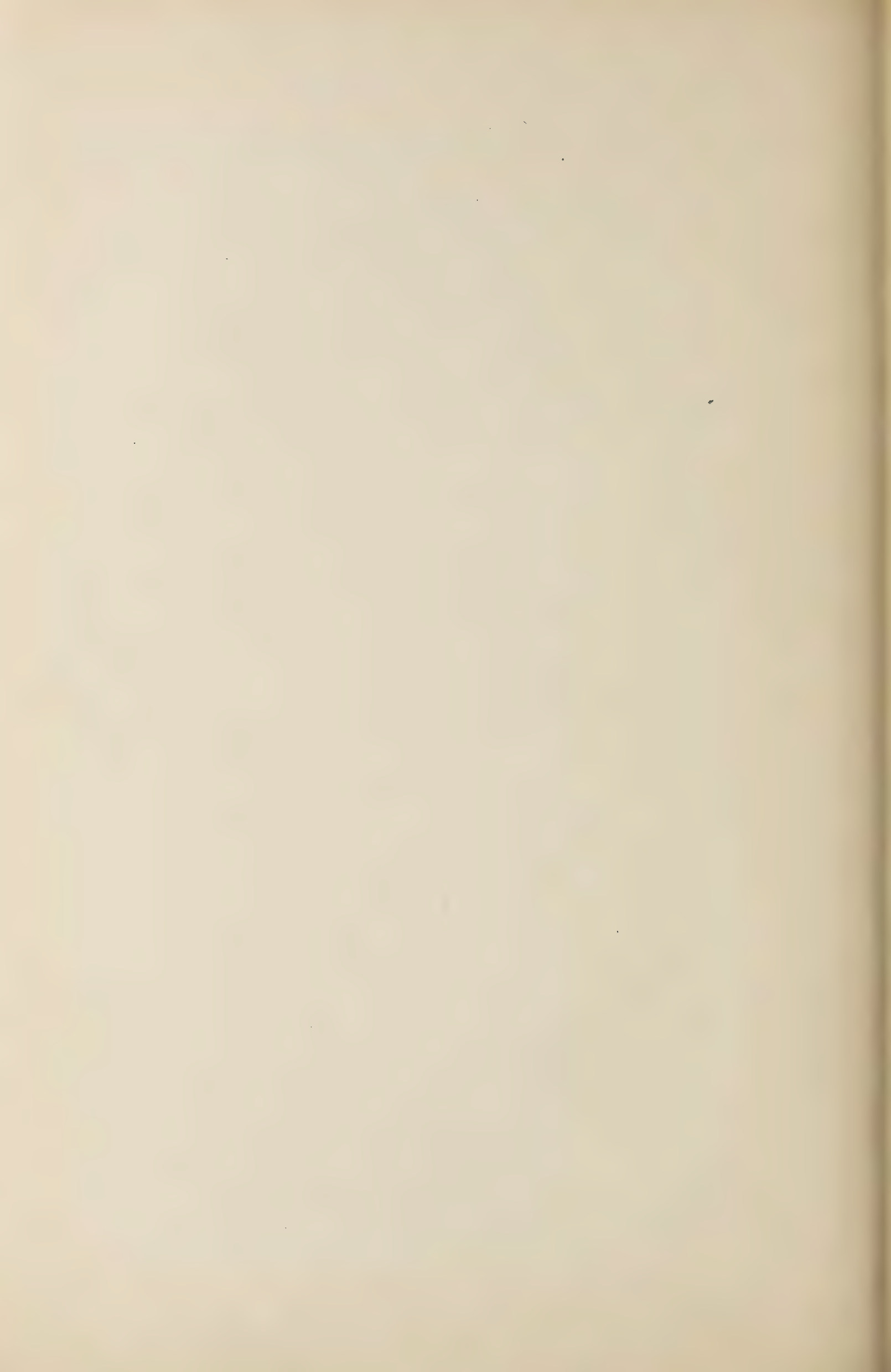
Line—Dongola to Somerville.

	°	'	''	
α Dongola to Miner's Bay	=	344	33	30.83
$\angle$ Miner's Bay to Somerville	=	-193	15	50.72
α Dongola to Somerville	=	151	17	40.11
	°	'	''	
$\phi_1 = 44$	44	05.084		
$\Delta\phi$	-7	16.619		
$\phi_2 = 44$	36	48.465		
	°	'	''	
$a_1 =$	151	17	40.11	
$180 + \Delta\alpha =$	180	03	55.26	
$\alpha_2 =$	331	21	35.37	
	°	'	''	
$L_1 = 78$	45	00.544		
$-\Delta L =$	-5	34.619		
$L_2 = 78$	39	25.925		
				Log.
s				4.1864251
N_1				6.8054279
s/N_1				3.3809972
$\sin \alpha_1$				9.6815199
$\cos \alpha_1$				9.9430490n
x				3.0625171
y				3.3240462n
$1/\sin 1''$				5.3144251
1st term	-434''	.9820		2.6384713n
y^3				9.9721386n
$\tan^2 \alpha_1$				9.4769418
$1/3 \sin 1''$				4.83730
2nd term	-0.	''00019		4.28638n
x^2				6.1250342
$\tan \phi^1$	$\phi^1 = 44^\circ$	36'	50''.102	9.9941469
$\frac{1}{2} \sin 1''$				5.0133951
-3rd term	-0''	.135699		1.1325762
$\Delta\phi^1$	-435''	.1179		2.6386066n
N_1				6.8054279
R_m				6.8039323
$\Delta\phi$	-436''	.61859		9.4440345n
x				2.6401022n
$1/\sin 1''$				3.0625171
$\sec \phi^1_2$	$\phi^1_2 = 44^\circ$	36'	49''.966	5.3144251
ΔL	334''	.61862		0.1476079
$\sin \phi_m$	$\phi_m = 44^\circ$	40'	26''.775	2.5245501
$\sec \frac{1}{2} \Delta\phi$				9.8470006
$\Delta\alpha$	235''	.26151		0.0000002
				2.3715509

The geographical positions of the triangulation stations may then be tabulated as follows:

Sta.	Lat.			Long.		
	°	'	"	°	'	"
Anson (G.S.C.)	44	52	35.389	78	52	05.935
Gull Lake	44	53	12.725	78	46	12.839
Miner's Bay	44	48	09.898	78	46	35.528
Laxton	44	45	35.857	78	51	20.788
Dongola	44	44	05.084	78	45	00.544
Coboconk	44	40	26.799	78	47	45.727
Somerville (G.S.C.)	44	36	48.465	78	39	25.926

The agreement between the values for the latitude and longitude of Somerville, given on pp. 22 and 24, serves as a final check on all the numerical work.



STEEL I-BEAMS HAUNCHED IN CONCRETE

By PETER GILLESPIE, *Professor of Civil Engineering*
and R. C. LESLIE, *Research Assistant*

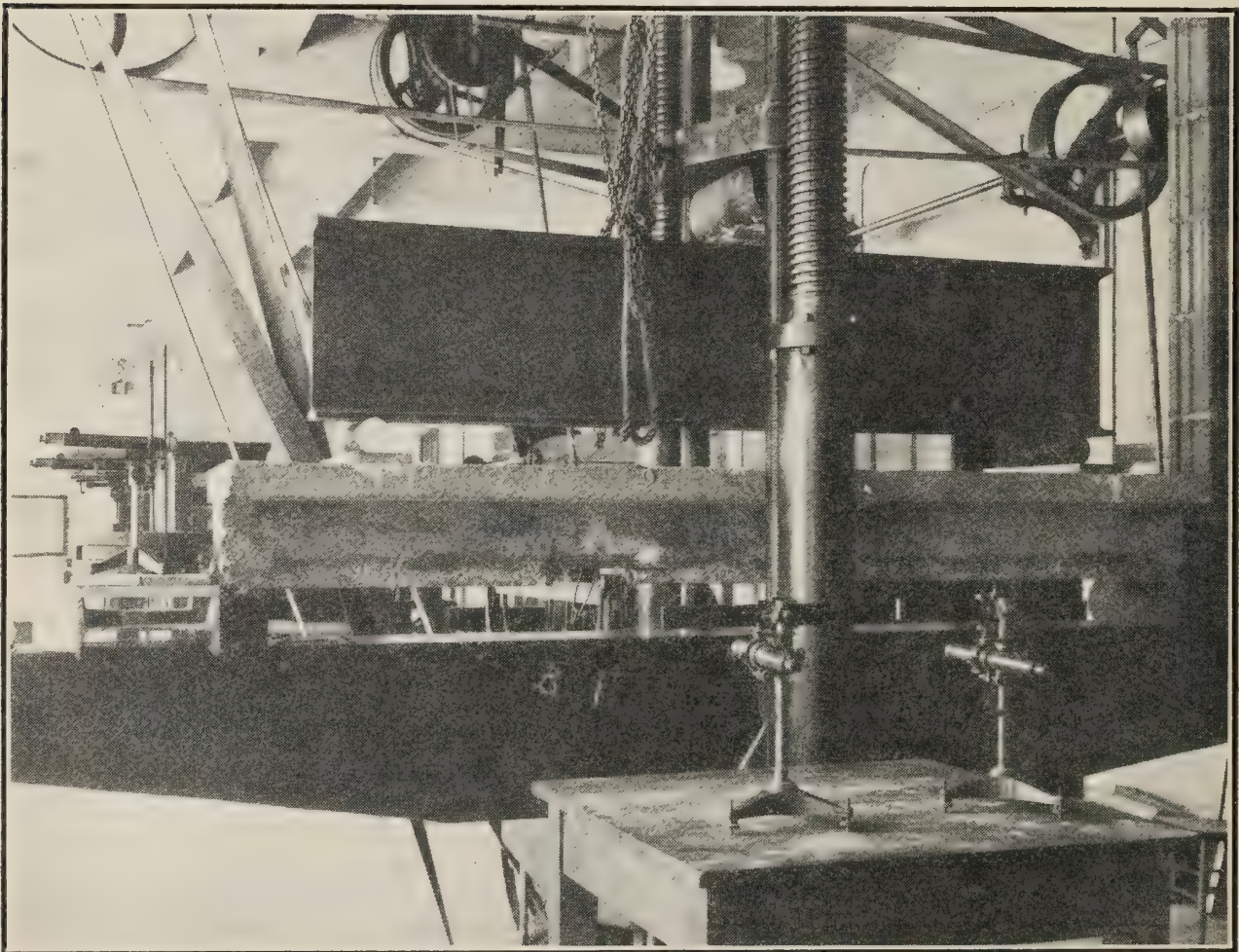
The steel frame building having reinforced concrete floors is a recognized competitor of the building constructed wholly of reinforced concrete. In its construction, the steel is erected in the ordinary way after which the concrete floor slabs and beam haunchings are poured into forms usually supported by the steel beams themselves. The practice of supporting the forms in this way eliminates much of the shoring otherwise necessary (facilitating thereby free movement for men and materials on the floor below) and is a saver of time and money generally. An additional economy would result were the two materials assumed to act together after the manner of reinforced concrete. This assumption, however, has not heretofore been generally made, the present practice being to consider the steel beam as supporting the entire dead and live load without assistance from the surrounding envelope of concrete. That is the attitude taken by the building departments of such American cities as Baltimore, Boston, Buffalo, Chicago, Minneapolis, Montreal, New York, Philadelphia and Toronto. In the United Kingdom also a similar custom prevails although a concession is sometimes made by allowing the haunched beam to be figured at 18,000 or 20,000 lb. per sq. in. where otherwise 16,000 lb. per sq. in. would be the maximum permissible stress.

A series of tests on haunched beams were conducted at the plant of the Dominion Bridge Company at Lachine in 1922-23.* Two floor panels, each 10 ft. by 16 ft., having a concrete slab four inches thick, reinforced with wire mesh, were constructed. These panels were supported by I-beams haunched in concrete fastened to girders by standard connections. Strain gauge readings were taken on the lower flanges of the encased I-beams before and after dead load was applied. Live load deformations were measured by Martens extensometers and deflections were observed. The two materials in these tests behaved very much as they do in ordinary reinforced concrete beams.

A test was also made in the Strength of Materials Laboratory, University of Toronto, during the winter of 1924. The beam tested in

* "Report on the Strength of I-Beams Haunched in Concrete", MacKay, Gillespie and Leluau, Engineering Journal, August, 1923.

this case was an 8-inch, 18.4-pound I. The width of haunching was 8" at the bottom of the stem and 9" at the bottom of the slab, which was 20" wide and $3\frac{1}{2}$ in. thick. The beam was 9' 4" long over all, with a $1\frac{3}{4}$ " bearing plate at each end. The span was 9' 2" and two loads were applied at points 13 inches from either support. Loads were of equal magnitude, thus giving a uniform bending moment and zero shear between their points of application. Extensometers of the Martens type were placed on the top of the slab and also on the lower flange of the I-beam, which was exposed for the purpose, at points approximately 1' 8" from one support. The maximum load applied was 135,000 lb., *i.e.*, 67,500 lb. at each concentration.



Haunched Beam under Test

For a load of 28,000 lb., computations based upon the observed strains between load points, gave a tension below the neutral axis of 16,770 lb. and a compression above it of 15,660 lb. of which 14,170 lb. existed in the concrete flange. The figured bending moment (14,000 $\times$ 13 in. lb.) did not agree exactly with either of the products, Tjd or Cjd where T and C denote aggregate tension and aggregate compression respectively. The area resisting horizontal shear was 9 $\times$ 14 sq. in., approximately, allowing for 1 in. extending past each end support.

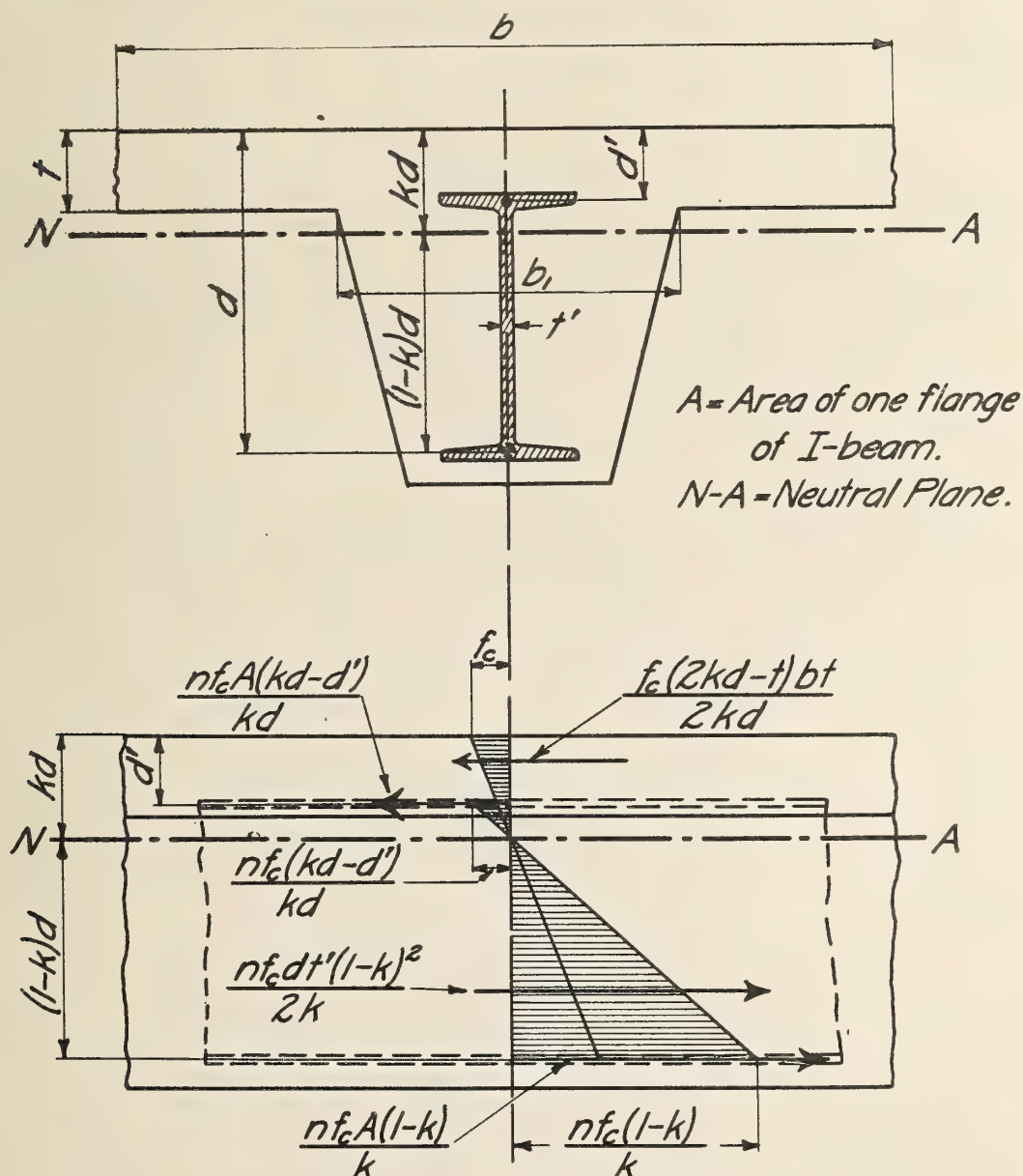
Hence, the horizontal shear,

$$f_v = \frac{14,170}{9 \times 14} = 112 \text{ lb. per sq. in.}$$

Assuming that as the load increases the shearing stress will increase in direct proportion, the value of f_v at maximum load, 135,000 lb., would be

$$\frac{135,000}{28,000} \text{ of } 112 = 542 \text{ lb. per sq. in.}$$

The intensity of the shearing stress for uniformly distributed loads increases uniformly from zero at the centre of the span to a maximum at the supports. In computing its value the critical area is considered as being horizontal and lying in the plane of the bottom of the slab. Failure, however, would probably take place along an area arching over the top of the I-beam flange. As this area is not much greater than



ANALYSIS OF BEAMS.

that assumed, the computation is approximately correct. The following formula, assuming the neutral axis to lie below the slab, gives the maximum shearing stress for such a case:

$$f_v = \frac{2bt \left\{ f_c + f_c \left(\frac{kd-t}{kd} \right) \right\}}{lb_1}$$

where,

f_v = horizontal shear at end of beam,

l = span,

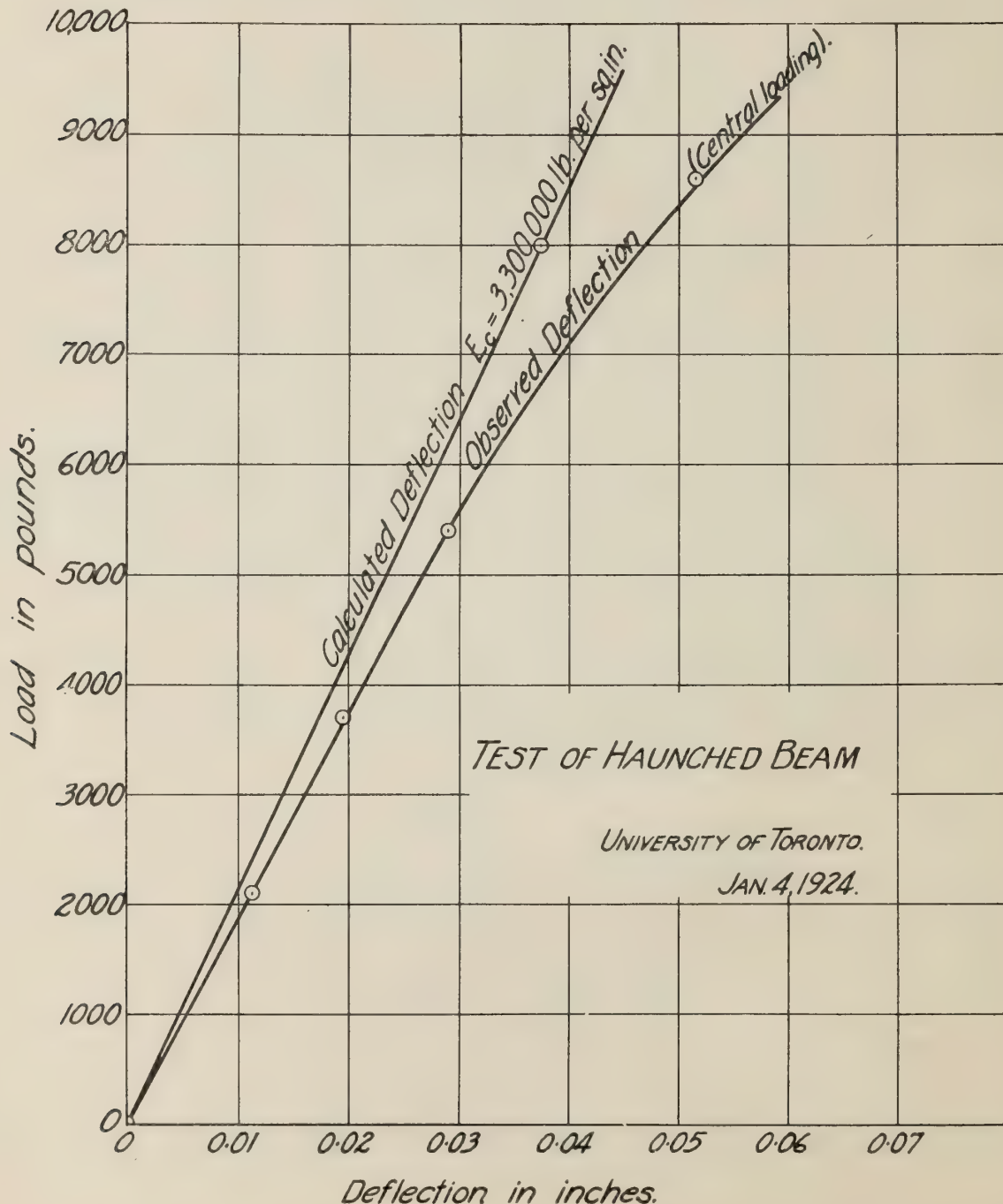
b_1 = width of haunch at bottom of floor slab,

t = thickness of slab,

b = width of slab acting as T-beam,

kd = depth to neutral axis, and

f_c = compressive stress in concrete at top of slab.



The test included measurements of the deflection of the beam under central load. The accompanying graphs and others studied show that the deflection of the composite beam follows the law for the homogeneous beam and that for a uniformly distributed load the following formula applies with approximate correctness:

$$\Delta = \frac{5}{384} \frac{Wl^3}{EI}$$

In the tables which follow there appears a coefficient of deflection, "c" for each beam. This is the deflection in inches for that particular haunched beam carrying a uniformly distributed load of one thousand pounds on a span of one foot. It is used in determining the deflection for any given span and load with the aid of the following formula:

$$\Delta = c \frac{Wl^3}{1000}$$

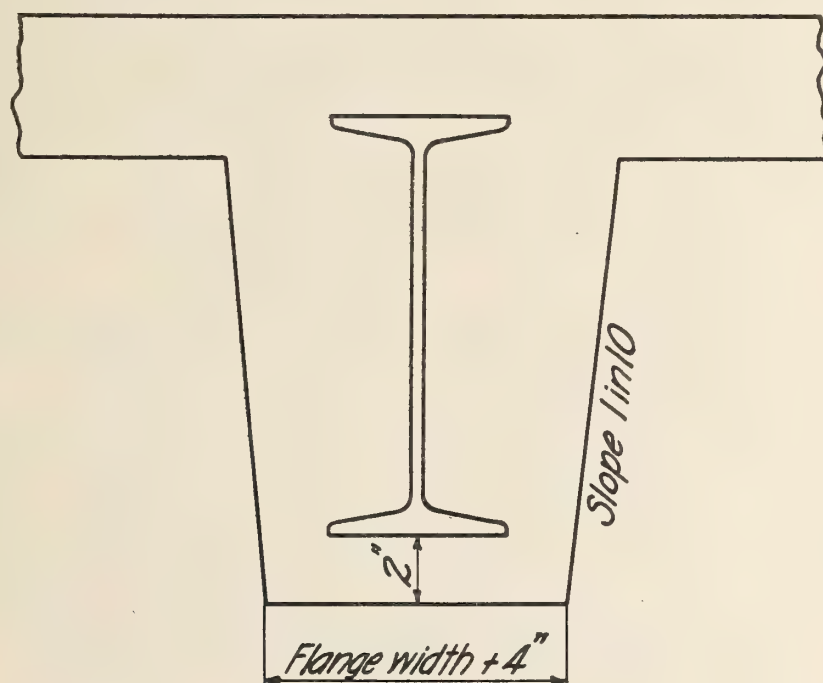
where, .

Δ = deflection in inches,

W = uniformly distributed load, and

l = span in feet.

When the forms for concrete are suspended from the steel beams, W in the above formula will denote the live load.



Plane of bottom of floor slab cuts web of I-beam at beginning of fillet.

**SUGGESTED STANDARD HAUNCHING
FOR STEEL I-BEAMS.**

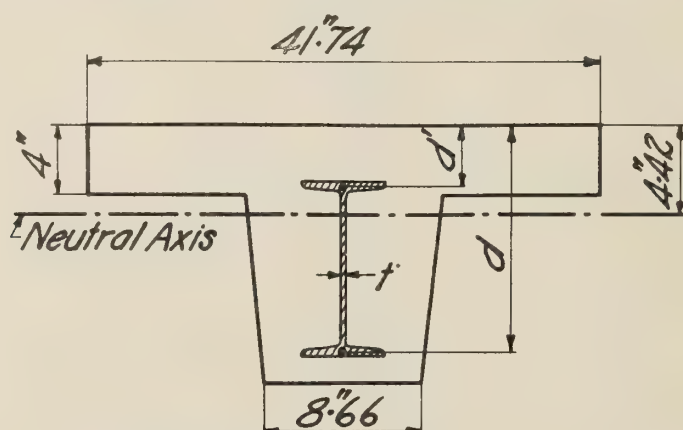
The tests cited appear to warrant the following assumptions:

- (1) The two materials act together as they do in reinforced concrete.
- (2) Live load deflections follow with necessary changes, approximately, the formulae for homogeneous beams.
- (3) A working stress of 240 lb. per sq. in. in horizontal shear is not excessive.

Messrs. Redpath, Brown and Co., Ltd., of Manchester, England, have published tables which give the strength of I-beams surrounded by concrete. In his book, *Steel Frame Structures*, Mr. W. C. Cocking, Consulting Engineer of London, England, gives tables for safe loads for British standard beams encased in concrete. So far as the writers have been able to learn there are no available data regarding the strength of haunched beams applicable to American shapes. In the belief that present practice penalizes unduly this type of construction, that tests have warranted the assumptions stated above and that results of computations for strength may be of value to the busy designer, the following tables have been prepared.

METHOD OF COMPUTATION*

In figuring the composite beam it was assumed that there is no tension in the concrete, that $n=15$, and that the effective width of the slab is equal to the average width of the haunching plus eight times the slab thickness. Working stresses in steel and concrete have been held to the conservative values of 16,000 and 650 lb. per sq. in. respectively.



10" I @ 25.4 lb. Standard Haunching.

$$I = 4384 \text{ in.}^4$$

$$S_c = 992 \text{ in.}^3$$

$$S_s = 35.5 \text{ in.}^3$$

$$M_s = 568,000 \text{ in.-lb.}$$

$$M_c = 644,500 \text{ in.-lb.}$$

*See "Report on the Strength of I-Beams Haunched in Concrete", Engineering Journal, August, 1923.

In the computations, the following symbols have been used:

- A_s = area of steel beam,
 I_s = moment of inertia of steel beam,
 I = moment of inertia, in terms of concrete units, of the composite beam about its neutral axis,
 A = area of one flange of steel beam,
 S_c = section modulus of the composite beam (concrete),
 S_s = section modulus of the composite beam (steel),
 S = section modulus of the naked steel beam,
 $R = \frac{S_s}{S}$,
 $n = \frac{E_s}{E_c}$,
 M_c = moment of resistance based on concrete,
 M_s = moment of resistance based on steel,
 d = depth to C.G. of bottom flange,
 d' = depth to C.G. of top flange,
 t' = thickness of web of steel beam,
 f_s = stress in steel, bottom flange,
 L = live load per beam,
 D = dead load per beam,
 W = tabulated load per beam.

When the normal forces acting on any section in the beam are equated the following, assuming the neutral axis to lie below the slab, is the result:

$$\frac{f_c(2kd-t)bt}{2kd} + \frac{f_c An(kd-d')}{kd} = \frac{nf_c dt'(1-k)^2}{2k} + \frac{f_c An(1-k)}{k}$$

whence

$$k^2 - \frac{2(bt+2An+ndt')k}{ndt'} + \frac{bt^2+2An(d+d')+nd^2t'}{nd^2t'} = 0.$$

In the following example, the data of a specific case are used in the solution of the above equation. The beam is a 10-in., 25.4-lb. I carrying a 4 in. concrete slab.

$$d' = 3.20''; d = 12.64''; t = 4; t' = .31''; A = 2.46 \text{ sq. in.}$$

$$b = 9.74'' + 32'' = 41.74''$$

$$\begin{array}{llll} bt = 41.74 \times 4 & = 167.0 & bt^2 = 167 \times 4 & = 668 \\ 2An = 30 \times 2.46 & = 73.8 & 2An(d+d') = 73.8 \times 15.84 & = 1168 \\ ndt' = 15 \times 12.64 \times .31 & = 58.7 & nd^2t' = 58.7 \times 12.64 & = 742 \\ & & 299.5 & 2578 \end{array}$$

$$k^2 - \frac{599.0}{58.7} k + \frac{2578}{742} = 0 \text{ or } k^2 - 10.20 k + 3.47 = 0$$

$$k = .35 \text{ and } kd = .35 \times 12.64'' = 4.42 \text{ in.}$$

MOMENT OF INERTIA ABOUT THE NEUTRAL AXIS

$$I = \frac{1}{12} bt^3 + bt \left(kd - \frac{t}{2} \right)^2 + n \left\{ I_s + A_s \left(\frac{d+d'}{2} - kd \right)^2 \right\}$$

$$\frac{1}{12} \times 41.74 \times 4^3 = 222$$

$$167 \times 2.42^2 = 977$$

$$15 \times 122.1 = 1832$$

$$15 \times 7.38 \times 3.5^2 = 1353$$

$$I = 4384 \text{ in.}^4$$

$$S_s = \frac{4384}{15 \times 8.22} = 35.5 \text{ in.}^3$$

$$S_c = \frac{4384}{4.42} = 992 \text{ in.}^3$$

$$M_s = 16,000 \times 35.5 = 568,000 \text{ in.-lb.}$$

$$M_c = 650 \times 992 = 644,500 \text{ in.-lb.}$$

The smaller of the two moments of resistance is the one governing the strength. For a bending moment of that magnitude,

$$f_c = \frac{M_s}{S_c} = \frac{568,000}{992} = 572 \text{ lb. per sq. in.}$$

$$\text{Stress at bottom of the slab is } \frac{.42}{4.42} \text{ of } 572 = 54 \text{ lb. per sq. in.}$$

If the span be 120 in. and $b_1 = 10.82$ in.,

$$f_v = \frac{2 \times 41.74 \times 4(572 + 54)}{120 \times 10.82} = 161 \text{ lb. per sq. in.}$$

It will be noticed that in some cases the capacity of a haunched beam as listed in the tables following is somewhat less than the capacity of the naked beam as given in the Carnegie Handbook. This is true of large sections with thin slabs only. For example, the 24-in., 100-lb. I figured for a 4" concrete slab has a capacity on a 10 ft. span of 210,000 lb. The capacity of the same beam naked is 211,500 lb. This is explained by the fact that when the concrete adjacent to the steel is working at its full capacity, 650 lb. per sq. in., the accompanying stress in the steel is 650×15 or 9,750 lb. per sq. in., which is 6,250 lb. per sq. in. below its permissible value. This beam with its haunching is, however, a class "B" beam, that is, it is over-reinforced. A computation shows that for the span assumed, a dead load of 50,000 lb. might have been applied to and supported by the naked beam, and that after the concrete had set an additional load of 210,000 lb. could still be supported without overstressing either material. The obvious thing to do in such cases is to suspend the forms from the steel shapes.

In some cases it will be observed that a certain beam, when haunched, is apparently stronger than the next heavier beam of the same or related class similarly haunched. In some instances the "supplementary" beams are in this category, these having profiles somewhat different from those of the American standard sections. In some instances the apparent increase is due to the fact that while both kd and I decrease when the lighter beam is introduced, the relative reduction in the former exceeds that in the latter. This results in an increase in the concrete section modulus and its derived moment of resistance. The case of 24-in. beams weighing 85 lb. and 79.9 lb. respectively with 4½-in. slab is one in point. Passing from the heavier to the lighter, it will be seen that the depths, as defined previously, are practically the same, but that kd decreases 3.7% and that I decreases 3.2%. In consequence of this, S_c is ½ of one per cent. greater for the lighter than for the heavier beam.

SAVING ACCOMPLISHED BY HAUNCHING

The saving in metal resulting from using haunched beams is illustrated in the following examples:

- (1) Floor panel 15' × 15'—beams framing into girders at third points.
Live load = 100 lb. per sq. ft.
Thickness of slab, 3 in.
Dead load tributary to beams = 60 lb. per sq. ft.
Dead load tributary to girder = concentrations + 175 lb. per lineal ft.

When figured as naked beams, the following sections are required:

Beams— 9-in., 21.8-lb.
Girder—12-in., 50.0-lb.

When figured as haunched beams with suspended forms, the following sections are adequate:

Beams— 8-in., 17.5-lb.
Girder—12-in., 40.8-lb.

Total saving in weight = 1.47 lb. per sq. ft. of panel.

- (2) Floor panel 18' × 18'—beams as in (1).

Live load = 100 lb. per sq. ft.

Thickness of slab, 3½ in.

Dead load tributary to beams = 70 lb. per sq. ft.

Dead load tributary to girder = concentrations + 300 lb. per lineal ft.

When figured as naked beams the following sections are required:

Beams— 10-in., 40-lb.
Girder—18-in., 60-lb.

When figured as haunched beams with suspended forms, the following sections are adequate:

Beams—10-in., 30-lb.

Girder—18-in., 48.2-lb.

Total saving in weight = 2.3 lb. per sq. ft. of panel.

In a building of 10 stories, $135' \times 135'$ designed according to (1) the steel saved would amount to

$$135 \times 135 \times 10 \times 1.47 = 268,000 \text{ lb.} = 134 \text{ tons.}$$

It will be apparent that the upper flange of the steel beam contributes but little to the flexural strength of the composite beam and that it might with profit be eliminated. The steel mills were consulted to ascertain whether the T-shape could be economically rolled as a substitute for the present I-shape, if the demand were sufficiently great to warrant its production. It seems to be the opinion of those consulted that for any tee over 6" in depth the cost of rolling would be greater than that of the I-beam of equivalent depth but that if a bulb were put on the end of the stem it might be economically produced. Such a structural shape, if used in the composite beam, would effect a saving in the weight of metal required.

USE OF DATA

The tables are headed: "Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also Horizontal Shear Produced by these Loads. Forms for Concrete Assumed Supported from below."

The information supplied at the head of each column is so arranged that those values which are entirely independent of stress are grouped together while those which depend upon working stresses, M_c and M_s , are also grouped. While the loads and shears tabulated are worked out on the basis of a working stress of 16,000 lb. per sq. in. for steel and 650 lb. per sq. in. for concrete, with $n=15$, the values of k , I , S_c and S_s are dependent on $n=15$ but are independent of stress and may be used with stresses other than those assumed to compute new values of M_c and M_s . The flexural capacity of a beam is arrived at from the following equations:

$$M_c = f_c S_c; \quad M_s = f_s S_s$$

The smaller moment of resistance governs.

When the forms for the concrete are suspended from the steel skeleton the loads given in the tables are modified somewhat, in accordance with the rules given hereafter. The value of the shear is also affected.

Beams when haunched are of two classes:

Class (A) Under-reinforced.

Class (B) Over-reinforced.

RULES AND EXAMPLES

Case I, Class "A" or class "B".

Forms shored from below.

$$L = W - D \dots \dots \text{Rule 1}$$

Illustration from the tables

Beam to span 20 ft. Dead load = 14,000 lb.; 4" concrete slab.

From table for 18" I-beams-4" slabs, we find that for a 20' span, and an 18-in., 54.7-lb. I, $W = 59,050$ lb.

$$L = W - D = 59,050 - 14,000 = 45,050 \text{ lb.}$$

$$\text{Horizontal shear} = 103.5 \text{ lb. per sq. in.}$$

Case II

Forms suspended from the steel beams.

Class "A"

$$L = W - RD \dots \dots \text{Rule 2}$$

Illustration

Span, 20 ft.; 4 in. slab; $D.L. = 10,000$ lb.

$$L = W - RD$$

Try 18-in., 48.2-lb. I-4" slab.

$$R = 1.30; W = 56,850$$

$$L = 56,850 - 1.30 \times 10,000 = 43,850 \text{ lb.}$$

Class "B" type 1

When the dead load fibre stress in the naked beams is equal to or less than $\frac{M_s - M_c}{M_s}$ of 16,000

$$L = W \dots \dots \text{Rule 3}$$

Illustration

$3\frac{1}{2}$ " slab, span 20 ft., $D.L. = 8,000$ lb.

Try 15-in., 70-lb. I- $3\frac{1}{2}$ " slab.

$$S = 87.9$$

$$\text{Dead load stress in steel} = \frac{8000 \times 20 \times 12}{8 \times 87.9} = 2730 \text{ lb. per sq. in.}$$

$$\text{But } \frac{M_s - M_c}{M_s} \text{ of } 16,000 = \frac{1,866,000 - 1,361,000}{1,866,000} \text{ of } 16,000 = 4330 \text{ lb. per sq. in.}$$

$$L = W = 45,370 \text{ lb.}$$

Class "B" type 2

When the dead load fibre stress in the naked beam is greater than $\frac{M_s - M_c}{M_s}$ of 16,000

$$L = \frac{8M_s}{l} - RD \dots \dots \text{Rule 4}$$

Illustration

$4\frac{1}{2}$ " slab; span 20 ft.; $D.L. = 16,000$ lb.

Try 18-in., 60-lb. I—a class "B" beam; $S = 93.1$; $R = 1.37$.

$$\text{Dead load stress in steel} = \frac{16,000 \times 20 \times 12}{8 \times 93.1} = 5,155 \text{ lb. per sq. in.}$$

$$\text{But } \frac{M_s - M_c}{M_s} \text{ of } 16,000 = \frac{2,039,000 - 1.986,000}{2,039,000} \text{ of } 16,000 \\ = 416 \text{ lb. per sq. in.}$$

$$\text{Then } L = \frac{8 \times 2,039,000}{240} - 1.37 \times 16,000 = 46,100 \text{ lb.}$$

Shear

When forms are shored from below, the shearing stress given in the tables is correct.

When the forms are supported from the steel beams the maximum shearing stress will be $\frac{\text{Safe live load}}{W}$ of the tabulated shearing stress.

Illustration

Use example given above of class "B" beam, type (2), where $L = 46,100$ lb. and $f_v = 122.0$ lb. per sq. in. $W = 66,200$ lb.

$$\text{Then max. shearing stress} = \frac{46,100}{66,200} \text{ of } 122.$$

The following table gives the weight per foot of concrete haunching plus included beam. The weights given are average values for each size of beam.

Beam	Weight per foot
27"	520 lb.
24"	425 lb.
21"	375 lb.
20"	340 lb.
18"	300 lb.
15"	235 lb.
12"	175 lb.
10"	140 lb.
9"	120 lb.
8"	105 lb.
7"	90 lb.
6"	75 lb.
5"	50 lb.
4"	50 lb.

Tables of loadings follow.

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

	27-INCH BEAM 5-INCH SLAB		27-INCH BEAM 4½-INCH SLAB		27-INCH BEAM 4-INCH SLAB		24-INCH BEAMS 5-INCH SLABS			
	27-in., 90-lb. I S = 219.1		27-in., 90-lb. I S = 219.1		27-in., 90-lb. I S = 219.1		24-in., 115-lb. I S = 245.0		24-in., 110-lb. I S = 239.1	
	k = .38 I = 80020 S_c = 6965 S_s = 284.3 R = 1.30 c = .000000141		k = .40 I = 75240 S_c = 6320 S_s = 281.0 R = 1.28 c = .000000149		k = .415 I = 70550 S_c = 5810 S_s = 274.5 R = 1.25 c = .000000159		k = .425 I = 72480 S_c = 6463 S_s = 318.1 R = 1.30 c = .000000155		k = .42 I = 70940 S_c = 6400 S_s = 308.7 R = 1.29 c = .000000159	
	M_c = 4,528,000 M_s = 4,547,000 "B"		M_c = 4,108,000 M_s = 4,490,000 "B"		M_c = 3,778,000 M_s = 4,390,000 "B"		M_c = 4,202,000 M_s = 5,090,000 "B"		M_c = 4,160,000 M_s = 4,935,000 "B"	
	Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.
10	301860	256	273860	221	251860	187	280100	275	277300	273
11	274000	232	248800	201	228800	170	254700	250	252000	248
12	251200	213	228000	184	209700	156	233300	229	231000	228
13	232000	196	210500	170	193500	144	215300	211	213100	210
14	215200	182	195500	158	179700	133	200000	196	198000	195
15	201000	170	182500	147	167700	124	186700	183	184700	182
16	188500	160	171000	138	157300	117	175000	171	173200	171
17	177400	150	161000	130	148000	110	164700	161	163000	161
18	167500	142	152000	123	139750	104	155600	152	154000	152
19	158700	134	144000	116	132500	98	147400	144	145900	144
20	150930	128	136930	110	125930	93	140050	137	138650	136
21	143500	121	130300	105	119800	89	133400	130	132000	130
22	137000	116	124400	100	114400	85	127300	125	126000	124
23	131100	111	119000	96	109400	81	121800	119	120500	119
24	125600	106	114100	92	104800	78	116700	114	115500	114
25	120600	102	109500	88	100700	74	112000	110	110850	109
26	116000	98	105300	85	96800	72	107700	105	106650	105
27	111700	94	101400	82	93200	69	103750	101	102700	101
28	107700	91	97700	79	89800	66	100000	98	99000	97
29	104000	88	94400	76	86800	64	96600	94	95600	94
30	100500	85	91200	73	83900	62	93400	91	92400	91
31	97300	82	88300	71	81200	60	90400	88	89450	88
32	94200	80	85500	69	78700	58	87500	85	86650	85
33	91500	77	83000	67	76300	56	84900	83	84000	82
34	88800	75	80500	65	74000	55	82400	80	81550	80
35	86200	73	78200	63	71900	53	80000	78	79200	78
36	83800	71	76000	61	70000	52	77800	76	77000	76
37	81600	69	74000	59	68000	50	75700	74	74950	73
38	79400	67	72000	58	66200	49	73700	72	73000	72
39	77400	65	70200	56	64600	48	71800	70	71100	70

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

24-INCH Beams 5-INCH Slabs										
24-in., 105.9-lb. I S = 234.3			24-in., 100-lb. I S = 197.6		24-in., 95-lb. I S = 191.8		24-in., 90-lb. I S = 185.8		24-in., 85-lb. I S = 180.0	
$k = .41$ $I = 69660$ $S_c = 6430$ $S_s = 297.7$ $R = 1.27$ $c = .000000161$			$k = .41$ $I = 63380$ $S_c = 5775$ $S_s = 267.6$ $R = 1.35$ $c = .000000178$		$k = .405$ $I = 61780$ $S_c = 5697$ $S_s = 258.4$ $R = 1.35$ $c = .000000182$		$k = .395$ $I = 60090$ $S_c = 5685$ $S_s = 247.1$ $R = 1.33$ $c = .000000187$		$k = .385$ $I = 58380$ $S_c = 5660$ $S_s = 236.3$ $R = 1.31$ $c = .000000193$	
$M_c = 4,180,000$ $M_s = 4,762,000$ "B"			$M_c = 3,752,000$ $M_s = 4,279,000$ "B"		$M_c = 3,700,000$ $M_s = 4,130,000$ "B"		$M_c = 3,695,000$ $M_s = 3,950,000$ "B"		$M_c = 3,680,000$ $M_s = 3,778,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
9	309500	302	278000	309	274000	312	273800	310	272500	308
10	278660	272	250140	278	246660	281	246300	279	245300	277
11	253200	247	227400	253	224100	255	224000	253	223000	251
12	232100	227	208400	232	205400	234	205200	232	204400	230
13	214200	209	192500	214	189600	216	189500	214	188600	213
14	199000	194	178700	199	176100	200	176000	199	175200	197
15	185700	181	166800	185	164400	187	164250	186	163500	184
16	174100	170	156300	174	154100	175	154000	174	153300	173
17	163800	160	147100	163	145000	165	144900	164	144250	162
18	154800	151	139000	154	137000	156	136900	155	136250	153
19	146600	143	131600	146	129800	147	129600	146	129100	145
20	139330	136	125070	139	123330	140	123150	139	122650	138
21	132650	129	119100	132	117400	133	117300	132	116800	131
22	126600	123	113700	126	112100	127	112000	126	111500	125
23	121200	118	108800	121	107200	122	107100	121	106600	120
24	116100	113	104300	116	102750	117	102650	116	102200	115
25	111400	109	100100	111	98700	112	98600	111	98200	110
26	107200	104	96200	107	94900	108	94800	107	94400	106
27	103200	100	92700	103	91400	104	91300	103	90900	102
28	99500	97	89300	99	88150	100	88050	99	87600	98
29	96100	94	86250	96	85100	96	85000	96	84600	95
30	92800	90	83400	92	82300	93	82200	93	81800	92
31	89850	87	80700	89	79600	90	79500	90	79150	89
32	87050	85	78200	87	77100	87	77000	87	76700	86
33	84400	82	75800	84	74800	85	74700	84	74350	83
34	81950	80	73600	81	72600	82	72500	82	72200	81
35	79600	77	71500	79	70550	80	70450	79	70500	79
36	77400	75	69500	77	68600	78	68500	77	68200	76
37	75300	73	67600	75	66700	75	66600	75	66300	74
38	73300	71	65800	73	65000	73	64900	73	64600	72

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

24-INCH BEAMS 5-INCH SLABS					24-INCH BEAMS 4½-INCH SLABS					
24-in., 79.9-lb. I S = 173.9			24-in., 74.2-lb. I S = 162.5		24-in., 115-lb. I S = 245.0		24-in., 110-lb. I S = 239.1		24-in., 105.9 lb.I S = 234.3	
$k = .38$ $I = 56620$ $S_c = 5567$ $S_s = 227.7$ $R = 1.31$ $c = .000000199$			$k = .365$ $I = 56360$ $S_c = 5620$ $S_s = 215.0$ $R = 1.32$ $c = .000000200$		$k = .435$ $I = 68130$ $S_c = 6048$ $S_s = 310.2$ $R = 1.27$ $c = .000000165$		$k = .43$ $I = 66670$ $S_c = 5987$ $S_s = 301.0$ $R = 1.26$ $c = .000000169$		$k = .425$ $I = 65580$ $S_c = 5960$ $S_s = 293.0$ $R = 1.25$ $c = .000000172$	
$M_c = 3,618,000$ $M_s = 3,635,000$ "B"			$M_c = 3,650,000$ $M_s = 3,435,000$ "A"		$M_c = 3,930,000$ $M_s = 4,960,000$ "B"		$M_c = 3,890,000$ $M_s = 4,811,000$ "B"		$M_c = 3,874,000$ $M_s = 4,690,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
9	268000	307	254300	263	291000	262	288100	261	287000	261
10	241200	276	229000	236	262000	236	259320	235	258260	235
11	219200	251	208100	215	238000	214	235800	213	234800	213
12	201000	230	190800	197	218200	196	216000	195	215100	195
13	185500	212	176100	182	201500	181	199400	180	198600	180
14	172200	197	163500	169	187100	168	185200	167	184400	167
15	160800	184	152700	157	174600	157	172900	156	172100	156
16	150800	172	143100	147	163700	147	162000	146	161400	146
17	141900	162	134700	139	154100	138	152500	138	151900	138
18	134000	153	127200	131	145500	131	144000	130	143500	130
19	127000	145	120500	124	137800	124	136500	123	135800	123
20	120600	138	114500	118	131000	118	129660	117	129130	117
21	114900	131	109000	112	124650	112	123400	111	122950	111
22	109600	125	104100	107	119100	107	117850	106	117400	106
23	104900	120	99600	102	113900	102	112700	102	112250	102
24	100500	115	95400	98	109150	98	108000	97	107600	97
25	96500	110	91600	94	104800	94	103700	94	103300	94
26	92800	106	88050	91	100700	90	99700	90	99350	90
27	89400	102	84800	87	97000	87	96000	87	95650	87
28	86200	98	81800	84	93500	84	92600	84	92250	84
29	83200	95	79000	81	90300	81	89400	81	89100	81
30	80450	92	76300	78	87300	78	86400	78	86100	78
31	77850	89	73900	76	84500	76	83600	75	83300	75
32	75400	86	71600	74	81850	73	81000	73	80700	73
33	73100	83	69400	71	79300	71	78550	71	78300	71
34	71000	81	67350	69	77000	69	76300	69	76000	69
35	68950	79	65400	67	74800	67	74100	67	73800	67
36	67050	76	63600	65	72800	65	72000	65	71700	65
37	65200	74	61900	64	70800	63	70100	63	69800	63
38	63500	72	60300	62	68900	62	68250	61	68000	61

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

24-INCH BEAMS 4½-INCH SLABS										
24-in., 100-lb. I S = 197.6			24-in., 95-lb. I S = 191.8		24-in., 90-lb. I S = 185.8		24-in., 85-lb. I S = 180.0		24-in., 79.9 lb. I S = 173.9	
$k = .425$ $I = 59320$ $S_c = 5320$ $S_s = 262.0$ $R = 1.33$ $c = .000000190$			$k = .42$ $I = 57800$ $S_c = 5245$ $S_s = 252.8$ $R = 1.32$ $c = .000000195$		$k = .41$ $I = 56250$ $S_c = 5223$ $S_s = 241.7$ $R = 1.30$ $c = .000000200$		$k = .41$ $I = 54830$ $S_c = 5090$ $S_s = 235.5$ $R = 1.31$ $c = .000000205$		$k = .395$ $I = 53050$ $S_c = 5120$ $S_s = 222.3$ $R = 1.28$ $c = .000000212$	
$M_c = 3,460,000$ $M_s = 4,195,000$ "B"			$M_c = 3,410,000$ $M_s = 4,042,000$ "B"		$M_c = 3,395,000$ $M_s = 3,865,000$ "B"		$M_c = 3,308,000$ $M_s = 3,768,000$ "B"		$M_c = 3,330,000$ $M_s = 3,558,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
9	256100	266	252700	268	251500	266	245000	266	246700	264
10	230660	240	227340	242	226300	240	220500	240	222000	238
11	209700	218	206800	220	205800	218	200500	218	201800	216
12	192100	200	189500	201	188700	200	183700	200	185000	198
13	177300	184	174900	186	174200	184	169600	184	170700	183
14	164700	171	162400	172	161700	171	157500	171	158500	170
15	153700	160	151600	161	151000	160	147000	160	148000	158
16	144100	150	142100	151	141500	150	137800	150	138700	148
17	135600	141	133800	142	133250	141	129700	141	130500	140
18	128100	133	126300	134	125800	133	122500	133	123300	132
19	121400	126	119700	127	119150	126	116100	126	116800	125
20	115330	120	113670	121	113150	120	110250	120	111000	119
21	109750	114	108300	115	107800	114	105000	114	105700	113
22	104800	109	103300	110	102900	109	100250	109	100800	108
23	100250	104	98800	105	98400	104	95850	104	96600	103
24	96100	100	94700	100	94300	100	91900	100	92500	99
25	92250	96	90900	96	90500	96	88200	96	88800	95
26	88700	92	87400	93	87050	92	84850	92	85400	91
27	85450	88	84200	89	83850	88	81700	88	82300	88
28	82400	85	81200	86	80850	85	78800	85	79300	85
29	79550	82	78400	83	78050	82	76050	82	76600	82
30	76900	80	75800	80	75450	80	73500	80	74050	79
31	74400	77	73300	78	73000	77	71150	77	71650	76
32	72100	75	71050	75	70750	75	68900	75	69400	74
33	69900	72	68900	73	68600	72	66800	72	67300	72
34	67800	70	66800	71	66600	70	64900	70	65300	70
35	65900	68	65000	69	64700	68	63000	68	63400	68
36	64100	66	63200	67	62900	66	61250	66	61700	66
37	62300	64	61500	65	61200	64	59600	64	60000	64

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

Span ft.	24-INCH BEAM 4½-INCH SLAB		24-INCH BEAMS 4-INCH SLABS							
	24-in., 74.2-lb. I S = 162.5		24-in., 115-lb. I S = 245.0		24-in., 110-lb. I S = 239.1		24-in., 105.9-lb. I S = 234.3		24-in., 100-lb. I S = 197.6	
	$k = .38$ $I = 52750$ $S_c = 5150$ $S_s = 210.2$ $R = 1.29$ $c = .000000213$		$k = .455$ $I = 64250$ $S_c = 5560$ $S_s = 309.6$ $R = 1.26$ $c = .000000175$		$k = .45$ $I = 62850$ $S_c = 5503$ $S_s = 299.8$ $R = 1.25$ $c = .000000179$		$k = .445$ $I = 61820$ $S_c = 5470$ $S_s = 292.2$ $R = 1.25$ $c = .000000182$		$k = .445$ $I = 55550$ $S_c = 4845$ $S_s = 259.0$ $R = 1.31$ $c = .000000203$	
	$M_c = 3,348,000$ $M_s = 3,364,000$ "B"		$M_c = 3,616,000$ $M_s = 4,952,000$ "B"		$M_c = 3,578,000$ $M_s = 4,795,000$ "B"		$M_c = 3,555,000$ $M_s = 4,675,000$ "B"		$M_c = 3,150,000$ $M_s = 4,140,000$ "B"	
	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
9	248000	242	268500	221	265000	220	263300	221	233200	228
10	223200	218	241060	199	238500	198	237000	199	210000	206
11	203000	198	219500	181	216800	180	215500	181	191000	187
12	186000	182	201000	166	198600	165	197500	165	175000	171
13	171700	167	185500	153	183500	152	182300	153	161500	158
14	159500	156	172500	142	170300	141	169400	142	150000	147
15	148900	145	160800	133	159000	132	158000	132	140000	137
16	139500	136	150750	124	149000	124	148200	124	131300	128
17	131400	128	141800	117	140300	116	134500	117	123600	121
18	124000	121	134000	110	132500	110	131700	110	116700	114
19	117500	114	126900	105	125500	104	129800	104	110500	108
20	111600	109	120530	99	119250	99	118500	99	105000	103
21	106300	104	114800	95	113550	94	112800	94	100000	98
22	101500	99	109600	90	108400	90	107700	90	95500	93
23	97100	95	104900	86	103700	86	103100	86	91300	89
24	93100	91	100450	83	99400	82	98750	82	87500	85
25	89300	87	96400	79	95400	79	94800	79	84000	82
26	85900	84	92750	76	91700	76	91100	76	80800	79
27	82700	80	88300	74	88300	73	87750	73	77800	76
28	79750	78	86100	71	85200	71	84600	71	75000	73
29	77000	75	83150	68	82250	68	81700	68	72450	71
30	74450	72	80400	66	79500	66	79000	66	70000	68
31	72050	70	77800	64	76950	64	76450	64	68800	66
32	69800	68	75350	62	74550	62	74050	62	65600	64
33	67650	66	73100	60	72250	60	71800	60	63600	62
34	65700	64	71000	58	70150	58	69700	58	61800	60
35	63800	62	69000	57	68100	56	67700	56	60000	58
36	62000	60	67100	55	66200	55	65800	55	58350	57
37	60400	59	65300	53	64450	53	64100	53	56750	55

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

24-INCH BEAMS 4-INCH SLABS										
24-in., 95-lb. I S = 191.8			24 in., 90-lb. I S = 185.8		24-in., 85-lb. I S = 180.0		24-in., 79.9 lb. I S = 173.9		24-in., 74.2-lb. I S = 162.5	
$k = .44$ $I = 54150$ $S_c = 4775$ $S_s = 250.1$ $R = 1.30$ $c = .000000208$			$k = .435$ $I = 52760$ $S_c = 4705$ $S_s = 241.2$ $R = 1.30$ $c = .000000213$		$k = .425$ $I = 51260$ $S_c = 4685$ $S_s = 230.8$ $R = 1.28$ $c = .000000219$		$k = .41$ $I = 49620$ $S_c = 4695$ $S_s = 217.5$ $R = 1.25$ $c = .000000227$		$k = .42$ $I = 49670$ $S_c = 4468$ $S_s = 215.6$ $R = 1.33$ $c = .000000226$	
$M_c = 3,103,750$ $M_s = 4,000,000$ "B"			$M_c = 3,055,000$ $M_s = 3,860,000$ "B"		$M_c = 3,045,000$ $M_s = 3,692,000$ "B"		$M_c = 3,052,000$ $M_s = 3,480,000$ "B"		$M_c = 2,904,000$ $M_s = 3,448,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
9	229800	227	226200	226	225600	226	226000	224	215000	209
10	206900	204	203660	204	203000	203	203400	201	193600	188
11	188000	185	185100	185	184500	184	185000	183	175800	171
12	172300	170	169700	170	169100	169	169500	167	161200	157
13	159100	157	156600	156	156100	156	156500	155	148800	145
14	147600	146	145500	145	144900	145	145300	143	138200	134
15	137800	136	135800	136	135300	135	135600	134	129000	125
16	129200	127	127300	127	126800	127	127200	125	120800	117
17	121600	120	119800	120	119400	119	119700	118	113800	110
18	114800	113	113100	113	112700	113	113000	111	107500	104
19	108800	107	107200	107	106700	107	107100	106	101800	99
20	103450	102	101830	102	101500	101	101700	100	96800	94
21	98400	97	97000	97	96650	96	96800	96	92100	89
22	94000	92	92600	92	92250	92	92400	91	87900	85
23	89900	88	88550	88	88300	88	88400	87	84100	82
24	86200	85	84800	85	84600	84	84750	84	80600	78
25	82700	81	81450	81	81200	81	81350	80	77400	75
26	79500	78	78300	78	78100	78	78200	77	74400	72
27	76600	75	75400	75	75200	75	75300	74	71650	69
28	73800	73	72700	72	72500	72	72600	72	69100	67
29	71300	70	70200	70	70000	70	70100	69	66700	65
30	68900	68	67900	68	67700	67	67800	67	64500	62
31	66700	65	65700	65	65500	65	65600	65	62500	60
32	64600	63	63650	63	63400	63	63600	63	60500	58
33	62700	62	61700	61	61500	61	61650	61	58700	57
34	60800	60	59900	60	59700	59	59800	59	56900	55
35	59100	58	58200	58	58000	58	58100	57	55300	53
36	57450	56	56600	56	56400	56	56400	56	53800	52
37	55900	55	55000	55	54800	55	55000	54	52300	51

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

21-INCH BEAM 5-INCH SLAB			21-INCH BEAM 4½-INCH SLAB		21-INCH BEAM 4-INCH SLAB		20-INCH BEAMS 5-INCH SLABS			
21-in., 60.4-lb. I S = 117.7			21-in., 60.4-lb. I S = 117.7		21-in., 60.4-lb. I S = 117.7		20-in., 100-lb. I S = 164.8		20-in., 95-lb. I S = 160.0	
k = .35 I = 38120 S _c = 4440 S _s = 159.0 R = 1.35 c = .000000295			k = .37 I = 35460 S _c = 3980 S _s = 155.6 R = 1.32 c = .000000317		k = .385 I = 33040 S _c = 3640 S _s = 151.8 R = 1.29 c = .000000340		k = .425 I = 44120 S _c = 4596 S _s = 226.3 R = 1.37 c = .000000255		k = .42 I = 43010 S _c = 4535 S _s = 218.8 R = 1.37 c = .000000262	
M _c = 2,885,000 M _s = 2,540,000 "A"			M _c = 2,586,000 M _s = 2,488,000 "A"		M _c = 2,365,000 M _s = 2,428,000 "B"		M _c = 2,986,000 M _s = 3,620,000 "B"		M _c = 2,947,000 M _s = 3,500,000 "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
8	211600	277	207300	267	197100	236	248800	350	245400	350
9	188100	246	184300	237	175300	210	221100	311	218200	311
10	169340	222	165860	213	157660	189	199060	280	196460	280
11	153800	201	150700	194	143400	171	181000	255	178500	254
12	141000	185	138150	178	131450	157	165800	234	163600	233
13	130150	170	127550	164	121350	145	153100	216	151000	215
14	120850	158	118450	152	112700	135	142100	200	140300	200
15	112800	148	110500	142	105150	126	132600	187	130800	186
16	105750	138	103600	133	98600	118	124400	175	122800	174
17	99500	130	97500	125	92800	111	117100	165	115500	164
18	94000	123	92100	118	87600	105	110500	156	109100	155
19	89050	116	87250	112	83000	99	104750	147	103300	147
20	84670	111	82930	106	78830	94	99530	140	98230	140
21	80600	105	78950	101	75100	90	94800	133	93500	133
22	76900	100	75350	97	71700	86	90500	127	89250	127
23	73600	96	72100	92	68600	82	86500	122	85400	121
24	70500	92	69100	89	65700	78	82900	117	81800	116
25	67700	88	66300	85	63100	75	79600	112	78600	112
26	65100	85	63750	82	60650	72	76550	108	75500	107
27	62700	82	61400	79	58400	70	73700	104	72700	103
28	60500	79	59200	76	56300	67	71100	100	70100	100
29	58400	76	57200	73	54350	65	88650	96	67700	96
30	56400	74	55300	71	52500	63	66300	93	65500	93
31	54600	71	53500	68	50850	61	64200	90	63350	90
32	52900	69	51800	66	49250	59	62200	87	61400	87
33	51300	67	50250	64	47750	57	60300	85	59500	84
34	49800	65	48800	62	46300	56	58500	82	57800	82
35	48400	63	47300	61	45000	54	56900	80	56100	80
36	47200	62	46100	59	43700	52	55300	78	54600	77

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

Span ft.	20-INCH BEAMS 5-INCH SLABS									
	20-in., 90-lb. I S = 155.0		20-in., 85-lb. I S = 150.2		20-in., 81.4-lb. I S = 146.6		20-in., 75-lb. I S = 126.3		20-in., 70-lb. I S = 121.4	
	$k = .41$		$k = .40$		$k = .395$		$k = .39$		$k = .375$	
	$I = 41790$		$I = 40630$		$I = 39750$		$I = 36790$		$I = 35450$	
	$S_c = 4510$		$S_c = 4495$		$S_c = 4457$		$S_c = 4115$		$S_c = 4123$	
	$S_s = 208.7$		$S_s = 199.7$		$S_s = 193.7$		$S_s = 175.3$		$S_s = 165.0$	
	$R = 1.35$		$R = 1.33$		$R = 1.32$		$R = 1.39$		$R = 1.36$	
	$c = .00000027$		$c = .000000277$		$c = .000000283$		$c = .000000306$		$c = .000000317$	
	$M_c = 2,931,000$		$M_c = 2,921,000$		$M_c = 2,896,000$		$M_c = 2,675,000$		$M_c = 2,680,000$	
	$M_s = 3,336,000$ "B"		$M_s = 3,193,000$ "B"		$M_s = 3,096,700$ "B"		$M_s = 2,803,000$ "B"		$M_s = 2,639,000$ "A"	
	Load	Shear	Load	Shear	Load	Shear	Load	Shear	Load	Shear
	lb.	lb./sq. in.	lb.	lb./sq. in.	lb.	lb./sq. in.	lb.	lb./sq. in.	lb.	lb./sq. in.
8	244100	348	243300	347	241300	345	222800	354	217600	350
9	217000	310	216200	308	214400	306	198000	315	193500	311
10	195400	279	194700	277	193060	276	178300	283	174100	280
11	177600	253	177000	252	175500	250	162000	257	158300	254
12	162800	232	162200	231	160800	230	148500	236	145100	233
13	150200	214	149700	213	148500	212	137100	218	133900	215
14	139600	199	139100	198	137800	197	127300	202	124400	200
15	130250	186	129800	185	128700	184	118800	189	116100	186
16	122100	174	121600	173	120600	172	111400	177	108800	175
17	114900	164	114500	163	113500	162	104800	166	102400	164
18	108500	155	108100	154	107200	153	99000	157	96750	155
19	102800	146	102500	146	101600	145	93800	149	91650	147
20	97700	139	97350	138	96530	138	89150	141	87050	140
21	93050	132	92700	132	91950	131	84900	135	82900	133
22	88800	126	88500	126	87750	125	81000	128	78150	127
23	84900	121	84650	120	83950	120	77500	123	75700	121
24	81400	116	81100	115	80450	115	74250	118	72550	116
25	78150	111	77900	111	77200	110	71300	113	69650	112
26	75150	107	74900	106	74250	106	68500	109	67000	107
27	72350	103	72100	102	71500	102	66000	105	64500	103
28	69800	99	69500	99	69000	98	63700	101	62200	100
29	67400	96	67100	95	66600	95	61450	97	60000	96
30	65100	93	64900	92	64350	92	59400	94	58000	93
31	63000	90	62800	89	62300	89	57500	91	56200	90
32	61100	87	60850	86	60350	86	55700	88	54400	87
33	59200	84	59000	84	58500	83	54000	85	52800	84
34	57500	82	57300	81	56800	81	52450	83	51200	82
35	55800	79	55600	79	55200	78	50900	81	49750	80
36	54300	77	54100	77	53600	76	49500	78	48400	77

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

Span ft.	20-INCH BEAM 5-INCH SLAB		20-INCH BEAMS 4½-INCH SLABS							
	20-in., 65.4-lb. I S = 116.9		20-in., 100-lb. I S = 164.8		20-in., 95-lb. I S = 160.0		20-in., 90-lb. I S = 155.0		20-in., 85-lb. I S = 150.2	
	$k = .37$ $I = 34280$ $S_c = 4040$ $S_s = 158.2$ $R = 1.35$ $c = .000000328$		$k = .44$ $I = 41120$ $S_c = 4232$ $S_s = 221.7$ $R = 1.35$ $c = .000000273$		$k = .435$ $I = 40120$ $S_c = 4180$ $S_s = 214.2$ $R = 1.34$ $c = .00000028$		$k = .425$ $I = 38980$ $S_c = 4150$ $S_s = 204.7$ $R = 1.32$ $c = .000000289$		$k = .415$ $I = 37910$ $S_c = 4135$ $S_s = 195.5$ $R = 1.30$ $c = .000000297$	
	$M_c = 2,625,000$ $M_s = 2,530,000$ "A"		$M_c = 2,751,000$ $M_s = 3,545,000$ "B"		$M_c = 2,717,000$ $M_s = 3,425,000$ "B"		$M_c = 2,697,000$ $M_s = 3,272,000$ "B"		$M_c = 2,690,000$ $M_s = 3,128,000$ "B"	
	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
8	210900	335	229100	303	226500	303	224800	302	224100	301
9	187500	297	203700	269	201150	269	199800	268	199250	267
10	168660	268	183400	242	181100	242	179800	241	179340	241
11	153400	243	166600	220	164700	220	163500	219	163000	219
12	140500	223	152600	202	151000	202	149800	201	149400	200
13	129700	206	141000	186	139400	186	138300	185	137900	185
14	120500	191	130900	173	129400	173	128400	172	128100	172
15	112500	178	122200	161	120800	161	119800	161	119500	160
16	105500	167	114500	151	113200	151	112400	151	112000	150
17	99200	157	107800	142	106600	142	105750	142	105500	141
18	93700	148	101800	134	100600	134	99900	134	99600	133
19	88800	141	96500	127	95350	127	94600	127	94400	126
20	84330	134	91700	121	90550	121	89900	120	89670	120
21	80300	127	87250	115	86250	115	85600	115	85400	114
22	76650	121	83300	110	82300	110	81700	109	81500	109
23	73300	116	79700	105	78750	105	78200	105	78000	104
24	70300	111	76350	101	75500	101	74900	100	74700	100
25	67450	107	73300	97	72500	97	71900	96	71700	96
26	64850	103	70500	93	69650	93	69150	92	69000	92
27	62500	99	67900	89	67100	89	66600	89	66400	89
28	60250	95	65450	86	64700	86	64200	86	64000	86
29	58150	92	63200	83	62450	83	62000	83	61800	83
30	56200	89	61100	80	60400	80	59900	80	59800	80
31	54400	86	59100	78	58400	78	58000	77	57800	77
32	52700	83	57300	75	56600	75	56200	75	56000	75
33	51100	81	55600	73	54900	73	54500	73	54300	73
34	49600	78	53900	71	53300	71	52900	71	52700	70
35	48200	76	52400	69	51750	69	51400	69	51200	68
36	46850	74	50950	67	50300	67	49950	67	49800	66

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

20-INCH BEAMS 4½-INCH SLABS							20-INCH BEAM 4-INCH SLAB			
20-in., 81.4-lb. I S = 146.6			20-in., 75-lb. I S = 126.3		20-in., 70-lb. I S = 121.4		20-in., 65.4-lb. I S = 116.9		20-in., 100-lb. I S = 164.8	
$k = .41$ $I = 37110$ $S_c = 4098$ $S_s = 189.7$ $R = 1.29$ $c = .000000303$			$k = .405$ $I = 34190$ $S_c = 3763$ $S_s = 170.8$ $R = 1.35$ $c = .000000329$		$k = .395$ $I = 32990$ $S_c = 3723$ $S_s = 162.2$ $R = 1.34$ $c = .000000341$		$k = .385$ $I = 32630$ $S_c = 3775$ $S_s = 157.6$ $R = 1.35$ $c = .000000345$		$k = .455$ $I = 38320$ $S_c = 3902$ $S_s = 217.0$ $R = 1.32$ $c = .000000294$	
$M_c = 2,662,000$ $M_s = 3,035,000$ "B"			$M_c = 2,446,000$ $M_s = 2,732,000$ "B"		$M_c = 2,420,000$ $M_s = 2,594,000$ "B"		$M_c = 2,455,000$ $M_s = 2,520,000$ "B"		$M_c = 2,538,000$ $M_s = 3,472,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
8	221900	299	203800	307	201700	306	204700	316	211500	256
9	197100	266	181200	273	179200	272	181900	281	188000	228
10	177460	239	163060	246	161300	245	163660	253	169200	205
11	161250	217	148200	223	146600	222	148800	230	153750	186
12	147800	199	135800	205	134400	204	136400	210	141000	171
13	136400	184	125400	189	124000	188	125900	194	130100	158
14	126700	170	116500	175	115200	175	116900	180	120800	146
15	118200	159	108650	164	107500	163	109100	168	112800	137
16	110800	149	101900	153	100800	153	102300	158	105700	128
17	104300	140	95900	144	94850	144	96300	148	99500	120
18	98500	132	90600	136	89600	136	90950	140	94000	114
19	93300	126	85800	129	84900	128	86200	133	89000	108
20	88730	119	81530	123	80650	122	81830	126	84600	102
21	84400	114	77650	117	76800	116	78000	120	80500	97
22	80600	108	74100	111	73300	111	74400	115	76900	93
23	77100	104	70900	107	70100	106	71200	110	73500	89
24	73900	99	67950	102	67200	102	68200	105	70500	85
25	70900	95	65250	98	64500	98	65500	101	67650	82
26	68200	92	62700	94	62000	94	63000	97	65050	79
27	65700	88	60400	91	59700	90	60650	93	62650	76
28	63300	85	58250	87	57600	87	58500	90	60400	73
29	61100	82	56200	84	55600	84	56450	87	58300	70
30	59100	79	54350	82	53750	81	54600	84	56400	68
31	57200	77	52600	79	52000	79	52800	81	54550	66
32	55450	74	50900	76	50400	76	51100	79	52850	64
33	53800	72	49400	74	48900	74	49600	76	51250	62
34	52200	70	47950	72	47400	72	48100	74	49800	60
35	50700	68	46600	70	46100	70	46750	72	48350	58
36	49300	66	45300	68	44700	68	45500	70	47000	57

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

20-INCH BEAMS 4-INCH SLABS										
20-in., 95-lb. I S = 160.0			20-in., 90-lb. I S = 155.0		20-in., 85-lb. I S = 150.2		20-in., 81.4-lb. I S = 146.6		20-in., 75-lb. I S = 126.3	
$k = .45$ $I = 37390$ $S_c = 3850$ $S_s = 209.7$ $R = 1.31$ $c = .000000301$			$k = .44$ $I = 36340$ $S_c = 3822$ $S_s = 200.5$ $R = 1.29$ $c = .00000031$		$k = .435$ $I = 35400$ $S_c = 3765$ $S_s = 193.4$ $R = 1.29$ $c = .000000318$		$k = .425$ $I = 34590$ $S_c = 3767$ $S_s = 185.6$ $R = 1.27$ $c = .000000326$		$k = .42$ $I = 31700$ $S_c = 3442$ $S_s = 166.2$ $R = 1.32$ $c = .000000355$	
$M_c = 2,502,500$ $M_s = 3,353,000$ "B"			$M_c = 2,484,000$ $M_s = 3,205,000$ "B"		$M_c = 2,448,000$ $M_s = 3,091,000$ "B"		$M_c = 2,448,000$ $M_s = 2,968,000$ "B"		$M_c = 2,238,000$ $M_s = 2,657,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
8	208500	256	207000	255	204000	256	204000	254	186500	261
9	185300	228	184000	227	181250	227	181250	226	165750	232
10	166800	205	165600	204	163200	205	163200	203	149200	209
11	151600	186	150500	185	148400	186	148400	185	135600	190
12	139000	171	138000	170	136000	170	136000	169	124400	174
13	128250	158	127400	157	125500	157	125500	156	114800	160
14	119100	146	118250	146	116550	146	116550	145	106600	149
15	111200	136	110400	136	108800	136	108800	135	99500	139
16	104200	128	103500	127	102000	128	102000	127	93250	130
17	98100	120	97400	120	96100	120	96100	119	87800	123
18	92600	114	92000	113	90700	113	90700	113	82900	116
19	87800	108	87200	107	85900	107	85900	107	78550	110
20	83400	102	82800	102	81600	102	81600	101	74600	104
21	79350	97	78850	97	77750	97	77750	97	71050	99
22	75750	93	75250	93	74200	93	74200	92	67850	95
23	72450	89	72000	88	71000	89	71000	88	64900	90
24	69450	85	69000	85	68000	85	68000	84	62200	87
25	66700	82	66200	81	65300	82	65300	81	59700	83
26	64100	79	63700	78	62800	78	62800	78	57400	80
27	61700	76	61350	75	60450	75	60450	75	55250	77
28	59500	73	59150	73	58300	73	58300	72	53300	74
29	57500	70	57100	70	56300	70	56300	70	51450	72
30	55550	68	55200	68	54400	68	54400	67	49750	69
31	53750	66	53400	66	52650	66	52650	65	48150	67
32	52100	64	51700	63	51000	64	51000	63	46600	65
33	50500	62	50150	62	49500	62	49500	61	45200	63
34	49100	60	48700	60	48000	60	48000	59	43900	61
35	47700	58	47300	58	46600	58	46600	58	42600	59
36	46350	57	46000	56	45350	56	45350	56	41500	58

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

Span ft.	20-INCH BEAMS 4-INCH SLABS				18-INCH BEAMS 4½-INCH SLABS					
	20-in., 70-lb. I S = 121.4		20 in., 65.4-lb. I S = 116.9		18-in., 90-lb. I S = 139.6		18-in., 85-lb. I S = 135.2		18-in., 80-lb. I S = 130.8	
	k = .41 I = 30610 S _c = 3401 S _s = 156.6 R = 1.29 c = .000000368		k = .405 I = 29660 S _c = 3340 S _s = 151.5 R = 1.30 c = .000000379		k = .435 I = 31870 S _c = 3642 S _s = 186.9 R = 1.34 c = .000000353		k = .425 I = 30930 S _c = 3615 S _s = 178.4 R = 1.32 c = .000000364		k = .415 I = 30010 S _c = 3594 S _s = 169.8 R = 1.30 c = .000000375	
	M _c = 2,211,000 M _s = 2,505,000 "B"		M _c = 2,172,000 M _s = 2,425,000 "B"		M _c = 2,368,000 M _s = 2,989,000 "B"		M _c = 2,350,000 M _s = 2,852,000 "B"		M _c = 2,337,000 M _s = 2,712,000 "B"	
	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
7	210500	297	206800	297	225200	343	223800	341	222600	340
8	184200	260	181000	259	197200	300	195800	298	194800	297
9	163700	231	160900	231	175300	267	174000	265	173200	264
10	147400	208	144800	208	157860	240	156660	238	155800	238
11	133900	189	131600	189	143500	218	142500	217	141600	216
12	122750	173	120600	173	131500	200	130500	199	129800	198
13	113300	160	111400	160	121300	184	120500	183	119800	183
14	105200	148	103400	148	112700	171	111900	170	111300	170
15	98200	139	96500	138	105200	160	104400	159	103900	158
16	92100	130	90500	130	98600	150	97900	149	97400	148
17	86700	122	85200	122	92800	141	92100	140	91650	140
18	81900	115	80400	115	87600	133	87000	132	86550	132
19	77500	109	76200	109	83000	126	82450	125	82000	125
20	73700	104	72400	104	78930	120	78330	119	77900	119
21	70200	99	68900	99	75100	114	74600	113	74200	113
22	67000	94	65800	94	71700	109	71200	108	70800	108
23	64100	90	62950	90	68600	104	69100	103	67700	103
24	61400	86	60300	86	65700	100	65300	99	64900	99
25	58950	83	57900	83	63100	96	62700	95	62300	95
26	56700	80	55700	80	60700	92	60300	91	59900	91
27	54600	77	53600	77	58450	89	58000	88	57700	88
28	52600	74	51700	74	56300	85	55950	85	55650	85
29	50800	71	49900	71	54400	82	54000	82	53700	82
30	49100	69	48250	69	52600	80	52200	79	51900	79
31	47500	67	46700	67	50900	77	50500	77	50250	76
32	46100	65	45300	65	49300	75	48950	74	48700	74
33	44650	63	43900	63	47850	72	47500	72	47200	72
34	43350	61	42600	61	46450	70	46100	70	45850	70
35	42100	59	41400	59	45100	68	44750	68	44500	68

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

18-INCH BEAMS 4½-INCH SLABS											
18-in., 75.6-lb. I S=126.9			18-in., 70-lb. I S=101.9		18-in., 65-lb. I S=97.5		18-in., 60-lb. I S=93.1		18-in., 54.7-lb. I S=88.4		
$k = .405$ $I = 29180$ $S_c = 3585$ $S_s = 162.8$ $R = 1.28$ $c = .000000386$			$k = .405$ $I = 26310$ $S_c = 3152$ $S_s = 143.0$ $R = 1.40$ $c = .000000428$		$k = .395$ $I = 25300$ $S_c = 3108$ $S_s = 135.2$ $R = 1.39$ $c = .000000445$		$k = .385$ $I = 24280$ $S_c = 3057$ $S_s = 127.6$ $R = 1.37$ $c = .000000463$		$k = .375$ $I = 23150$ $S_c = 2990$ $S_s = 119.8$ $R = 1.36$ $c = .000000486$		
$M_c = 2,330,000$ $M_s = 2,600,000$ "B"			$M_c = 2,050,000$ $M_s = 2,287,000$ "B"		$M_c = 2,020,000$ $M_s = 2,160,000$ "B"		$M_c = 1,986,000$ $M_s = 2,039,000$ "B"		$M_c = 1,944,000$ $M_s = 1,915,000$ "A"		
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	
7	221800	336	195200	351	192400	349	189100	348	182300	339	
8	194000	294	170800	307	168300	305	165500	305	159500	297	
9	172500	261	151800	273	149600	271	147100	271	141800	264	
10	155300	235	136660	246	134660	244	132400	244	127660	238	
11	141100	214	124100	223	122400	222	120300	221	116000	216	
12	129300	196	113900	205	112200	203	110300	203	106400	198	
13	119400	181	105100	189	103600	188	101800	187	98200	183	
14	110800	168	97600	175	96200	174	94600	174	91200	170	
15	103500	157	91100	164	89800	163	88200	162	85100	158	
16	97000	147	85400	153	84200	152	82700	152	79800	148	
17	91300	138	80400	144	79200	143	77900	143	75100	140	
18	86200	130	75900	136	74800	135	73500	135	70900	132	
19	81700	124	71900	129	70850	128	69700	128	67200	125	
20	77650	117	68330	123	67330	122	66200	122	63830	119	
21	73900	112	65000	117	64100	116	63000	116	60800	113	
22	70500	107	62100	111	61200	111	60200	110	58000	108	
23	67500	102	59400	106	58500	106	57500	106	55500	103	
24	64700	98	56900	102	56100	101	55200	101	53200	99	
25	62100	94	54650	98	53850	97	52950	97	51050	95	
26	59700	90	52500	94	51800	94	50900	93	49100	91	
27	57500	87	50600	91	49900	90	49000	90	47300	88	
28	55400	84	48800	87	48100	87	47250	87	45600	85	
29	53500	81	47100	84	46400	84	45650	84	44000	82	
30	51700	78	45550	82	44900	81	44150	81	42550	79	
31	50050	76	44100	79	43400	79	42700	78	41200	76	
32	48500	73	42700	76	42100	76	41400	76	39900	74	
33	47100	71	41400	74	40800	74	40100	74	38700	71	
34	45700	69	40200	72	39600	72	38900	72	37500	70	
35	44400	67	39000	70	38400	70	37800	70	36500	68	

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

18-INCH BEAM 4½-INCH SLAB			18-INCH BEAMS 4-INCH SLABS							
18-in., 48.2-lb. I S = 81.9			18-in., 90-lb. I S = 139.6		18-in., 85-lb. I S = 135.2		18-in., 80-lb. I S = 130.8		18-in., 75.6-lb. I S = 126.9	
$k = .35$ $I = 22820$ $S_c = 3070$ $S_s = 110.3$ $R = 1.35$ $c = .000000493$			$k = .45$ $I = 29620$ $S_c = 3355$ $S_s = 183.0$ $R = 1.31$ $c = .000000038$		$k = .44$ $I = 28740$ $S_c = 3331$ $S_s = 174.3$ $R = 1.29$ $c = .0000000392$		$k = .43$ $I = 27900$ $S_c = 3302$ $S_s = 166.0$ $R = 1.27$ $c = .0000000403$		$k = .425$ $I = 27180$ $S_c = 3255$ $S_s = 160.6$ $R = 1.27$ $c = .0000000414$	
$M_c = 1,966,000$ $M_s = 1,764,000$ "A"			$M_c = 2,181,000$ $M_s = 2,928,000$ "B"		$M_c = 2,165,000$ $M_s = 2,789,000$ "B"		$M_c = 2,147,000$ $M_s = 2,655,000$ "B"		$M_c = 2,115,000$ $M_s = 2,570,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
7	168000	277	207800	291	206100	290	204400	289	201400	289
8	147000	242	181700	254	180300	254	178800	253	176250	253
9	130700	215	161500	226	160300	226	159000	225	156700	225
10	117600	194	145400	204	144300	203	143100	202	141000	202
11	106900	176	132100	185	131100	185	130100	184	128200	184
12	98000	161	121100	170	120200	169	119200	168	117500	168
13	90400	149	111800	157	111000	156	110100	155	108500	155
14	84000	138	103800	145	103000	145	102200	144	100700	144
15	78400	129	96900	136	96200	135	95400	135	94000	134
16	73500	121	90800	127	90200	127	89450	126	88100	126
17	69200	114	85500	120	84900	119	84200	119	82900	119
18	65300	107	80750	113	80200	113	79500	112	78300	112
19	61900	102	76500	107	75900	107	75300	106	74200	106
20	58800	97	72700	102	72150	101	71550	101	70500	101
21	56000	92	68200	97	68700	96	68200	96	67100	96
22	53450	88	66100	92	65600	92	65050	92	64100	92
23	51100	84	63200	88	62700	88	62200	88	61300	88
24	49000	80	60600	85	60100	84	59650	84	58700	84
25	47050	77	58100	81	57700	81	57250	81	56400	81
26	45200	74	55900	78	55500	78	55000	78	54200	77
27	43550	71	53800	75	53450	75	53000	75	52200	75
28	42000	69	51900	72	51500	72	51100	72	50300	72
29	40550	67	50100	70	49750	70	49350	69	48600	69
30	39200	64	48450	68	48100	67	47700	67	47000	67
31	37950	62	46900	65	46550	65	46150	65	45500	65
32	36750	60	45400	63	45100	63	44700	63	44050	63
33	35650	58	44050	61	43750	61	43350	61	42700	61
34	34600	57	42750	60	42450	59	42100	59	41500	59

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

18-INCH BEAMS 4-INCH SLABS										
18-in., 70-lb. I S = 101.9			18-in., 65-lb. I S = 97.5		18-in., 60-lb. I S = 93.1		18-in., 54.7-lb. I S = 88.4		18-in., 48.2-lb. I S = 81.9	
$k = .42$ $I = 24220$ $S_c = 2865$ $S_s = 138.2$ $R = 1.36$ $c = .000000464$			$k = .415$ $I = 23350$ $S_c = 2796$ $S_s = 132.2$ $R = 1.36$ $c = .000000482$		$k = .40$ $I = 22390$ $S_c = 2780$ $S_s = 123.6$ $R = 1.33$ $c = .000000503$		$k = .39$ $I = 21380$ $S_c = 2725$ $S_s = 116.2$ $R = 1.32$ $c = .000000526$		$k = .365$ $I = 21050$ $S_c = 2780$ $S_s = 106.6$ $R = 1.30$ $c = .000000535$	
$M_c = 1,862,000$ $M_s = 2,210,000$ "B"			$M_c = 1,817,000$ $M_s = 2,114,000$ "B"		$M_c = 1,807,000$ $M_s = 1,975,000$ "B"		$M_c = 1,772,000$ $M_s = 1,856,000$ "B"		$M_c = 1,807,000$ $M_s = 1,706,000$ "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
7	177300	299	173000	300	172000	291	168700	295	162400	255
8	155200	262	151400	262	150500	260	147700	258	142100	223
9	137900	232	134600	233	133800	231	131300	230	126300	198
10	124100	209	121100	210	120460	208	118100	207	113700	179
11	112800	190	110100	190	109400	189	107300	188	103400	162
12	103400	174	100900	175	100300	173	98400	172	94800	149
13	95500	161	93200	161	92600	160	90850	159	87500	137
14	88650	149	86500	150	86000	148	84400	147	81250	127
15	82750	139	80700	140	80250	138	78700	138	75800	119
16	77600	130	75700	131	75200	130	73800	129	71100	111
17	73000	123	71200	123	70800	122	69450	121	66900	105
18	68950	116	67300	116	66900	115	65600	115	63200	99
19	65300	110	63700	110	63300	109	62200	109	59900	94
20	62050	104	60550	105	60230	104	59050	103	56850	89
21	59100	99	57700	100	57300	99	56200	98	54150	85
22	56400	95	55050	95	54700	94	53700	94	51700	81
23	54000	91	52650	91	52300	90	51350	90	49450	77
24	51700	87	50450	87	50150	86	49200	86	47400	74
25	49650	83	48450	84	48150	83	47250	82	45500	71
26	47700	80	46600	80	46300	80	45400	79	43750	68
27	45950	77	44850	77	44600	77	43750	76	42150	66
28	44350	74	43250	75	43000	74	42200	73	40600	63
29	42800	72	41750	72	41550	71	40700	71	39200	61
30	41350	69	40400	70	40150	69	39350	69	37900	59
31	40000	67	39100	67	38850	67	38100	66	36700	57
32	38700	65	37800	65	37600	65	36800	65	35500	56
33	37600	63	36600	64	36400	63	35700	63	34400	54
34	36400	61	35600	62	35300	61	34600	61	33400	52

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

18-INCH BEAMS 3½-INCH SLABS											
18-in., 90-lb. I S = 139.6			18-in., 85-lb. I S = 135.2		18-in., 80-lb. I S = 130.8		18-in., 75.6-lb. I S = 126.9		18-in., 70-lb. I S = 101.9		
$k = .465$ $I = 27540$ $S_c = 3095$ $S_s = 179.5$ $R = 1.29$ $c = .000000409$			$k = .455$ $I = 26720$ $S_c = 3072$ $S_s = 171.0$ $R = 1.26$ $c = .000000421$		$k = .45$ $I = 25970$ $S_c = 3015$ $S_s = 164.6$ $R = 1.26$ $c = .000000434$		$k = .445$ $I = 25330$ $S_c = 2975$ $S_s = 159.2$ $R = 1.26$ $c = .000000444$		$k = .445$ $I = 22340$ $S_c = 2560$ $S_s = 136.8$ $R = 1.34$ $c = .000000503$		
$M_c = 2,011,700$ $M_s = 2,872,000$ "B"			$M_c = 1,997,000$ $M_s = 2,734,000$ "B"		$M_c = 1,960,000$ $M_s = 2,632,000$ "B"		$M_c = 1,934,000$ $M_s = 2,547,000$ "B"		$M_c = 1,664,000$ $M_s = 2,189,000$ "B"		
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	
7	191500	241	190100	240	186600	240	184100	240	158500	249	
8	167600	211	166300	210	163300	210	161100	210	138600	218	
9	149000	187	147800	187	145200	187	143200	186	123300	194	
10	134100	169	133100	168	130660	168	128900	168	110980	174	
11	121800	153	121000	153	118800	153	117100	152	100800	158	
12	111700	140	110900	140	108900	140	107400	140	92500	145	
13	103100	129	102350	129	100500	129	99100	129	85400	134	
14	95800	120	95050	120	93400	120	92000	120	79300	124	
15	89400	112	88700	112	87150	112	85850	112	74000	116	
16	83800	105	83200	105	81700	105	80500	105	69350	109	
17	78900	99	78300	99	76900	99	75750	98	65250	102	
18	74500	93	73950	93	72600	93	71650	93	61650	97	
19	70600	88	70100	88	68800	88	67800	88	58400	91	
20	67050	84	66550	84	65330	84	64450	84	55490	87	
21	63900	80	63400	80	62250	80	61350	80	52800	83	
22	60950	76	60500	76	59400	76	58600	76	50450	79	
23	58300	73	57900	73	56850	73	56000	73	48250	75	
24	55900	70	55450	70	54500	70	53700	70	46250	72	
25	53650	67	53250	67	52300	67	51500	67	44350	69	
26	51600	65	51200	64	50300	64	49550	64	42650	67	
27	49700	62	49300	62	48400	62	47700	62	41100	64	
28	47900	60	47550	60	46700	60	46000	60	39600	62	
29	46250	58	45900	58	45100	58	44400	57	38250	60	
30	44700	56	44350	56	43600	56	42950	56	37000	58	
31	43250	54	42900	54	42150	54	41600	54	35700	56	
32	41900	52	41600	52	40850	52	40300	52	34600	54	
33	40650	51	40300	51	39600	51	39100	50	33700	53	

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

18-INCH BEAMS 3½-INCH SLABS									15-INCH BEAM 4½-INCH SLAB	
18-in., 65-lb. I S=97.5			18-in., 60-lb. I S=93.1		18-in., 54.7-lb. I S=88.4		18-in., 48.2-lb. I S=81.9		15-in., 75-lb. I S=91.6	
$k = .43$ $I = 21460$ $S_c = 2546$ $S_s = 127.8$ $R = 1.31$ $c = .000000524$			$k = .42$ $I = 20610$ $S_c = 2504$ $S_s = 120.8$ $R = 1.30$ $c = .000000546$		$k = .405$ $I = 19680$ $S_c = 2475$ $S_s = 112.3$ $R = 1.27$ $c = .000000572$		$k = .385$ $I = 19390$ $S_c = 2488$ $S_s = 103.9$ $R = 1.27$ $c = .00000058$		$k = .43$ $I = 19070$ $S_c = 2568$ $S_s = 129.0$ $R = 1.41$ $c = .000000590$	
$M_c = 1,654,900$ $M_s = 2,043,500$ "B"			$M_c = 1,627,000$ $M_s = 1,932,000$ "B"		$M_c = 1,608,000$ $M_s = 1,797,000$ "B"		$M_c = 1,615,000$ $M_s = 1,664,000$ "B"		$M_c = 1,668,000$ $M_s = 2,065,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
7	157500	248	154800	247	153100	246	153700	227	158900	349
8	137800	217	135500	216	133900	215	134500	198	139000	306
9	122500	192	120500	192	119000	191	119600	176	123500	272
10	110300	173	108460	173	107200	172	107660	159	111200	244
11	100250	157	98600	157	97400	156	97800	144	101100	222
12	91900	144	90400	144	89300	143	89700	132	92650	203
13	84800	133	83400	133	82400	132	82800	122	85500	188
14	78750	123	77500	123	76500	123	76900	113	79400	174
15	73500	115	72300	115	71400	114	71750	106	74100	163
16	68900	108	67800	108	67000	107	67250	99	69500	152
17	64850	102	63800	101	63000	101	63300	93	65400	143
18	61250	96	60300	96	59500	95	59800	88	61800	135
19	58000	91	57100	91	56400	90	56650	83	58500	128
20	55150	86	54230	86	53600	86	53830	79	55600	122
21	52500	82	51600	82	51000	82	51250	75	52900	116
22	50100	78	49250	78	48700	78	48900	72	50500	111
23	47950	75	47100	75	46600	74	46800	69	48350	106
24	45950	72	45150	72	44650	71	44850	66	46300	102
25	44100	69	43350	69	42850	68	43050	63	44500	97
26	42400	66	41650	66	41200	66	41400	61	42750	94
27	40850	64	40150	64	39700	63	39850	58	41200	90
28	39350	62	38700	61	38300	61	38450	56	39700	87
29	38000	59	37400	59	36950	59	37100	54	38200	84
30	36800	57	36150	57	35700	57	35900	53	36900	81
31	35500	56	34900	56	34600	55	34700	51	35800	79
32	34400	54	33800	54	33500	54	33600	50	34600	76

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

15-INCH BEAMS 4½-INCH SLABS										
15-in., 70-lb. I S = 87.9			15-in., 65-lb. I S = 84.3		15-in., 60.8-lb. I S = 81.2		15-in., 55-lb. I S = 67.8		15-in., 50-lb. I S = 64.2	
$k = .42$ $I = 18400$ $S_c = 2535$ $S_s = 122.3$ $R = 1.39$ $c = .000000612$			$k = .41$ $I = 17720$ $S_c = 2500$ $S_s = 115.7$ $R = 1.37$ $c = .000000635$		$k = .40$ $I = 17130$ $S_c = 2472$ $S_s = 109.8$ $R = 1.35$ $c = .000000657$		$k = .39$ $I = 15990$ $S_c = 2305$ $S_s = 98.3$ $R = 1.45$ $c = .000000704$		$k = .38$ $I = 15220$ $S_c = 2252$ $S_s = 91.8$ $R = 1.43$ $c = .000000739$	
$M_c = 1,648,000$ $M_s = 1,956,000$ "B"			$M_c = 1,625,000$ $M_s = 1,850,000$ "B"		$M_c = 1,606,800$ $M_s = 1,758,000$ "B"		$M_c = 1,498,200$ $M_s = 1,570,000$ "B"		$M_c = 1,464,000$ $M_s = 1,468,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
7	157000	348	154800	345	153000	343	142700	346	139400	344
8	137300	304	135400	302	133900	300	124800	303	122000	301
9	122000	270	120300	268	119000	267	111000	269	108400	267
10	109860	243	108340	241	107120	240	99880	242	97600	241
11	99900	221	98500	219	97400	218	90800	220	88700	219
12	91600	203	90300	201	89200	200	83200	202	81300	200
13	84500	187	83300	185	82400	185	76800	186	75100	185
14	78450	174	77400	172	76500	172	71300	173	69700	172
15	73200	162	72200	161	71400	160	66550	161	65050	160
16	68650	152	67650	151	66900	150	62400	151	61000	150
17	64600	143	63700	142	63000	141	58750	142	57400	141
18	61000	135	60150	134	59500	133	55450	134	54200	133
19	57800	128	57000	127	56350	126	52550	127	51350	126
20	54930	121	54170	120	53560	120	49940	121	48800	120
21	52300	116	51550	115	51000	114	47550	115	46450	114
22	49900	110	49200	109	48700	109	45400	110	44350	109
23	47750	105	47050	105	46550	104	43400	105	42450	104
24	45750	101	45100	100	44600	100	41600	101	40650	100
25	43900	97	43300	96	42850	96	39950	97	39050	96
26	42250	93	41650	92	41200	92	38400	93	37500	92
27	40700	90	40100	89	39650	89	37000	89	36150	89
28	39300	87	38700	86	38200	86	35700	86	34800	86
29	37900	84	37300	83	36800	83	34400	83	33600	83
30	36600	81	36100	80	35600	80	33300	81	32500	80
31	35400	78	34900	78	34400	77	32300	78	31400	78

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

15-INCH BEAMS 4½-INCH SLABS						15-INCH BEAMS 4-INCH SLABS					
15-in., 45-lb. I S = 60.5			15-in., 42.9-lb. I S = 58.9			15-in., 37.3-lb. I S = 54.1			15-in., 75-lb. I S = 91.6		
$k = .365$ $I = 14380$ $S_c = 2212$ $S_s = 84.9$ $R = 1.40$ $c = .000000783$			$k = .36$ $I = 14020$ $S_c = 2187$ $S_s = 82.2$ $R = 1.39$ $c = .000000803$			$k = .335$ $I = 13760$ $S_c = 2238$ $S_s = 75.2$ $R = 1.39$ $c = .000000818$			$k = .445$ $I = 17490$ $S_c = 2341$ $S_s = 125.0$ $R = 1.36$ $c = .000000643$		
$M_c = 1,438,000$ $M_s = 1,356,000$ "A"			$M_c = 1,421,500$ $M_s = 1,312,000$ "A"			$M_c = 1,454,700$ $M_s = 1,203,000$ "A"			$M_c = 1,521,600$ $M_s = 2,000,000$ "B"		
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.
		in.									in.
7	129100	319	124900	311	114500	252	144900	299	143200	298	
8	113000	279	109400	272	100250	221	126750	261	125300	260	
9	100400	248	97200	242	89100	196	112700	232	111400	231	
10	90400	223	87460	218	80200	177	101440	209	100260	208	
11	82150	203	79500	198	72900	160	92200	190	91100	189	
12	75300	186	72900	181	66800	147	84500	174	83500	173	
13	69500	172	67300	167	61700	136	78000	161	77100	160	
14	64550	159	62500	155	57300	126	72450	149	71600	149	
15	60250	149	58300	145	53500	118	67600	139	66800	139	
16	56500	139	54700	136	50100	110	63400	131	62650	130	
17	53150	131	51500	128	47200	104	59650	123	58950	122	
18	50200	124	48600	121	44550	98	56350	116	55700	115	
19	47550	117	46050	114	42200	93	53400	110	52700	109	
20	45200	111	43730	109	40100	88	50720	104	50130	104	
21	43000	106	41650	103	38200	84	48300	99	47700	99	
22	41050	101	39750	99	36450	80	46100	95	45550	94	
23	39300	97	38050	94	34850	77	44100	91	43550	90	
24	37650	93	36450	90	33400	73	42250	87	41750	86	
25	36150	89	35000	87	32100	70	40550	83	40100	83	
26	34750	86	33600	83	30850	68	39000	80	38550	80	
27	33500	82	32400	80	29700	65	37550	77	37100	77	
28	32300	79	31200	78	28700	63	36000	75	35700	74	
29	31100	77	30200	75	27600	61	34900	72	34400	72	
30	30100	74	29200	73	26800	59	33700	70	33300	69	

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

15-INCH BEAMS 4-INCH SLABS										
15-in., 65-lb. I S = 84.3			15-in., 60.8-lb. I S = 81.2		15-in., 55-lb. I S = 67.8		15-in., 50-lb. I S = 64.2		15-in., 45-lb. I S = 60.5	
$k = .425$			$k = .415$		$k = .41$		$k = .395$		$k = .38$	
$I = 16280$			$I = 15750$		$I = 14620$		$I = 13900$		$I = 13170$	
$S_c = 2280$			$S_c = 2255$		$S_c = 2061$		$S_c = 2035$		$S_c = 2002$	
$S_s = 112.2$			$S_s = 106.9$		$S_s = 95.5$		$S_s = 88.4$		$S_s = 81.8$	
$R = 1.33$			$R = 1.32$		$R = 1.41$		$R = 1.38$		$R = 1.35$	
$c = .000000691$			$c = .000000714$		$c = .000000769$		$c = .000000810$		$c = .000000855$	
$M_c = 1,482,000$			$M_c = 1,466,000$		$M_c = 1,339,600$		$M_c = 1,322,000$		$M_c = 1,301,000$	
$M_s = 1,796,000$			$M_s = 1,708,000$		$M_s = 1,527,000$		$M_s = 1,415,000$		$M_s = 1,308,000$	
"B"			"B"		"B"		"B"		"B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	164600	346	162800	344	148800	349	146800	346	144500	342
7	141100	296	139500	295	127500	299	125800	297	123800	293
8	123500	259	122100	258	111600	262	110100	260	108400	256
9	109800	230	108500	229	99200	232	97900	231	96300	228
10	98800	207	97700	206	89300	209	88100	208	86700	205
11	89800	188	88800	187	81150	190	80100	189	78800	186
12	82300	173	81400	172	74400	174	73400	173	72200	171
13	76000	159	75100	159	68650	161	67750	160	66650	158
14	70550	148	69750	147	63800	149	62900	148	61900	146
15	65850	138	65100	137	59500	139	58700	138	57750	137
16	61700	129	61050	129	55800	131	55050	130	54150	128
17	58100	122	57450	121	52500	123	51800	122	51000	120
18	54900	115	54250	114	49600	116	48900	115	48150	114
19	52000	109	51400	108	47000	110	46350	109	45600	108
20	49400	103	48850	103	44650	104	44050	104	43350	102
21	47000	98	46500	98	42500	99	41950	99	41250	97
22	44900	94	44400	94	40600	95	40050	94	39400	93
23	42950	90	42450	89	38850	91	38300	90	37700	89
24	41150	86	40700	86	37200	87	36700	86	36100	85
25	39500	83	39050	82	35700	83	35250	83	34650	82
26	38000	79	37550	79	34300	80	33800	80	33300	79
27	36600	76	36150	76	33100	77	32600	77	32100	76
28	35300	74	34800	74	31900	75	31400	74	30900	73
29	34100	71	33600	71	30700	72	30300	72	29900	71

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

Span ft.	15-INCH BEAMS 4-INCH SLABS				15-INCH BEAMS 3½-INCH SLABS					
	15-in., 42.9-lb. I S=58.9		15-in., 37.3-lb. I S=54.1		15-in., 75-lb. I S=91.6		15-in., 70-lb. I S=87.9		15-in., 65-lb. I S=84.3	
	$k = .375$		$k = .35$		$k = .46$		$k = .455$		$k = .445$	
	$I = 12840$		$I = 12610$		$I = 16070$		$I = 15550$		$I = 14960$	
	$S_c = 1978$		$S_c = 2020$		$S_c = 2145$		$S_c = 2093$		$S_c = 2064$	
	$S_s = 79.1$		$S_s = 72.5$		$S_s = 121.8$		$S_s = 116.8$		$S_s = 110.4$	
	$R = 1.34$		$R = 1.34$		$R = 1.33$		$R = 1.33$		$R = 1.31$	
	$c = .000000877$		$c = .000000892$		$c = .000000700$		$c = .000000723$		$c = .000000752$	
	$M_c = 1,285,000$		$M_c = 1,313,000$		$M_c = 1,394,000$		$M_c = 1,361,000$		$M_c = 1,342,000$	
	$M_s = 1,265,000$		$M_s = 1,158,000$		$M_s = 1,950,000$		$M_s = 1,866,000$		$M_s = 1,765,000$	
	"A"		"A"		"B"		"B"		"B"	
	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	140500	335	128600	275	154800	290	151200	291	149100	289
7	120400	287	110300	236	132700	248	129600	249	127800	248
8	105400	252	96500	206	116100	217	113400	218	111800	217
9	93600	223	85800	183	103200	193	100800	194	99400	193
10	84300	201	77200	165	92900	174	90740	174	89460	173
11	76600	183	70200	150	84400	158	82450	158	81300	158
12	70200	167	64350	137	77400	145	75600	145	74500	144
13	64800	154	59400	127	71400	134	69800	134	68800	133
14	60200	143	55150	118	66300	124	64800	124	63900	124
15	56200	134	51450	110	61900	116	60500	116	59600	115
16	52650	125	48250	103	58000	108	56700	109	55900	108
17	49550	118	45400	97	54600	102	53350	102	52600	102
18	46800	111	42900	91	51600	96	50400	97	49700	96
19	44350	106	40650	87	48850	91	47750	92	47050	91
20	42150	100	38600	82	46450	87	45370	87	44730	86
21	40100	96	36750	78	44200	82	43200	83	42600	82
22	38300	91	35100	75	42200	79	41200	79	40650	79
23	36650	87	33550	71	40400	75	39450	76	38900	75
24	35100	83	32100	69	38700	72	37800	72	37250	72
25	33700	80	30800	66	37150	69	36300	69	35750	69
26	32400	77	29700	63	35720	67	34900	67	34400	66
27	31200	74	28500	61	34400	64	33500	64	33100	64
28	30100	72	27500	59	33300	62	32400	62	32000	62

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

15-INCH BEAMS 3½-INCH SLABS										
Span ft.	15-in., 60.8-lb. I S=81.2		15-in., 55-lb. I S=67.8		15-in., 50-lb. I S=64.2		15-in., 45-lb. I S=60.5		15-in., 42.9-lb. I S=58.9	
	$k = .435$		$k = .43$		$k = .415$		$k = .40$		$k = .395$	
	$I = 14480$		$I = 13310$		$I = 12680$		$I = 12010$		$I = 11740$	
	$S_c = 2040$		$S_c = 1843$		$S_c = 1817$		$S_c = 1787$		$S_c = 1768$	
	$S_s = 105.0$		$S_s = 92.8$		$S_s = 86.2$		$S_s = 79.3$		$S_s = 76.8$	
	$R = 1.29$		$R = 1.37$		$R = 1.34$		$R = 1.31$		$R = 1.30$	
	$c = .000000777$		$c = .000000845$		$c = .000000888$		$c = .000000936$		$c = .000000958$	
	$M_c = 1,326,000$		$M_c = 1,197,500$		$M_c = 1,181,000$		$M_c = 1,162,000$		$M_c = 1,149,000$	
	$M_s = 1,676,000$		$M_s = 1,484,000$		$M_s = 1,377,000$		$M_s = 1,268,000$		$M_s = 1,228,000$	
	"B"		"B"		"B"		"B"		"B"	
	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	147300	289	133000	292	131100	291	129100	289	127700	288
7	126250	247	114000	251	112400	249	110700	247	109400	247
8	110500	216	99800	219	98400	218	96800	216	95700	216
9	98200	192	88700	195	87400	194	86000	192	85100	192
10	88400	173	79800	175	78700	174	77460	173	76600	173
11	80350	157	72500	159	71500	158	70400	157	69600	157
12	73650	144	66500	146	65500	145	64550	144	63800	144
13	68000	133	61350	135	60500	134	59600	133	58900	133
14	63100	123	57000	125	56200	124	55300	123	54700	123
15	58900	115	53200	117	52400	116	51600	115	51050	115
16	55200	108	49850	109	49150	109	48400	108	47850	108
17	52000	102	46900	103	46250	102	45550	102	45050	101
18	49100	96	44300	97	43700	97	43000	96	42550	96
19	46500	91	42000	92	41400	92	40750	91	40300	91
20	44200	86	39900	87	39350	87	38730	86	38300	86
21	42100	82	38000	83	37450	83	36900	82	36450	82
22	40150	78	36250	79	35750	79	35200	78	34800	78
23	38400	75	34680	76	34200	76	33650	75	33300	75
24	36850	72	33240	73	32750	72	32250	72	31900	72
25	35350	69	31900	70	31400	70	30900	69	30600	69
26	34000	67	30700	67	30300	67	29700	67	29400	67
27	32700	64	29500	65	29100	65	28700	64	28400	64

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

15-INCH BEAM 3½-INCH SLAB			12-INCH BEAMS 4½-INCH SLABS							
15-in., 37.3-lb. I S=54.1			12-in., 55-lb. I S=53.2		12-in., 50-lb. I S=50.3		12-in., 45-lb. I S=47.3		12-in., 40.8-lb. I S=44.8	
$k = .37$ $I = 11520$ $S_c = 1795$ $S_s = 70.3$ $R = 1.30$ $c = .000000976$			$k = .42$ $I = 10420$ $S_c = 1692$ $S_s = 81.5$ $R = 1.53$ $c = .00000108$		$k = .405$ $I = 9890$ $S_c = 1663$ $S_s = 75.5$ $R = 1.50$ $c = .00000114$		$k = .39$ $I = 9360$ $S_c = 1632$ $S_s = 69.5$ $R = 1.47$ $c = .00000120$		$k = .38$ $I = 8880$ $S_c = 1588$ $S_s = 64.8$ $R = 1.44$ $c = .00000127$	
$M_c = 1,167,000$ $M_s = 1,124,000$ "A"			$M_c = 1,100,000$ $M_s = 1,303,000$ "B"		$M_c = 1,081,000$ $M_s = 1,207,000$ "B"		$M_c = 1,062,000$ $M_s = 1,112,000$ "B"		$M_c = 1,032,000$ $M_s = 1,037,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	124800	255	122300	400	120100	395	118000	388	114600	383
7	107000	218	104800	343	103000	338	101100	333	98250	329
8	93600	191	91700	300	90100	296	88500	291	86000	287
9	83200	170	81500	266	80100	263	78650	259	76450	255
10	74940	153	73300	240	72100	237	70800	233	68800	230
11	68100	139	66650	218	65500	215	64350	211	62500	209
12	62400	127	61100	200	60100	197	59000	194	57300	191
13	57600	117	56400	184	55450	182	54450	179	52900	177
14	53500	109	52350	171	51450	169	50550	166	49130	164
15	49950	102	48900	160	48050	158	47200	155	45850	153
16	46800	95	45800	150	45050	148	44250	145	43000	143
17	44050	90	43100	141	42400	139	41650	137	40450	135
18	41600	85	40700	133	40050	131	39300	129	38200	128
19	39400	80	38580	126	37950	124	37250	122	36200	121
20	37470	76	36650	120	36050	118	35400	116	34400	115
21	35650	72	34900	114	34300	112	33700	111	32750	109
22	34050	69	33300	109	32750	107	32170	105	31250	104
23	32550	66	31870	104	31350	103	30750	101	29900	100
24	31200	63	30500	100	30100	99	29500	97	28700	96
25	29900	61	29300	96	28900	95	28400	93	27600	92
26	28700	59	28200	92	27800	91	27300	90	26500	88

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

	12-INCH BEAMS 4½-INCH SLABS						12-INCH BEAMS 4-INCH SLABS								
	12-in., 35-lb. I S = 37.8			12-in., 31.8-lb. I S = 36.0			12-in., 27.9-lb. I S = 33.2			12-in., 55-lb. I S = 53.2			12-in., 50-lb. I S = 50.3		
	$k = .355$			$k = .345$			$k = .33$			$k = .435$			$k = .42$		
	$I = 8160$			$I = 7740$			$I = 7660$			$I = 9390$			$I = 8930$		
	$S_c = 1530$			$S_c = 1493$			$S_c = 1498$			$S_c = 1523$			$S_c = 1498$		
	$S_s = 56.2$			$S_s = 52.5$			$S_s = 49.2$			$S_s = 78.1$			$S_s = 72.4$		
$R = 1.49$			$R = 1.46$			$R = 1.48$			$R = 1.47$			$R = 1.44$			
$c = .00000138$			$c = .00000145$			$c = .00000147$			$c = .00000120$			$c = .00000126$			
$M_c = 995,000$			$M_c = 970,500$			$M_c = 974,000$			$M_c = 990,000$			$M_c = 974,000$			
$M_s = 899,500$			$M_s = 840,000$			$M_s = 786,500$			$M_s = 1,248,000$			$M_s = 1,158,500$			
“ A ”			“ A ”			“ A ”			“ B ”			“ B ”			
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.					
6	99900	338	93200	319	87400	278	110000	346	108300	343					
7	85600	290	79900	273	74900	238	94200	296	92800	294					
8	74950	254	69900	239	65550	209	82500	259	81200	257					
9	66600	225	62100	212	58250	185	73300	230	72200	228					
10	59950	203	55950	191	52450	167	66000	207	65000	205					
11	54500	184	50850	174	47700	152	60000	188	59050	187					
12	49950	169	46600	159	43730	139	55000	172	54100	171					
13	46100	156	43000	147	40360	128	50750	159	49950	158					
14	42800	145	39950	136	37500	119	47150	148	46400	147					
15	39950	135	37300	127	34950	111	44000	138	43300	137					
16	37480	127	34950	119	32800	104	41250	129	40600	128					
17	35250	119	32900	112	30860	98	38800	122	38200	121					
18	33300	112	31080	106	29150	92	36650	115	36100	114					
19	31550	106	29420	100	27600	87	34720	109	34200	108					
20	29970	101	27950	95	26200	83	33000	103	32470	102					
21	28550	96	26650	91	24950	79	31400	98	30930	98					
22	27250	92	25800	87	23850	76	30000	94	29500	93					
23	26050	88	24300	83	22800	72	28700	90	28250	89					
24	25000	84	23300	80	21800	70	27500	86	27100	85					

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

12-INCH BEAMS 4-INCH SLABS										
12-in., 45-lb. I S=47.3			12-in., 40.8-lb. I S=44.8		12-in., 35-lb. I S=37.8		12-in., 31.8-lb. I S=36.0		12-in., 27.9-lb. I S=33.2	
$k = .40$ $I = 8440$ $S_c = 1487$ $S_s = 66.1$ $R = 1.40$ $c = .00000133$			$k = .39$ $I = 8030$ $S_c = 1448$ $S_s = 61.7$ $R = 1.38$ $c = .00000140$		$k = .37$ $I = 7360$ $S_c = 1372$ $S_s = 53.7$ $R = 1.42$ $c = .00000153$		$k = .36$ $I = 7000$ $S_c = 1341$ $S_s = 50.2$ $R = 1.39$ $c = .00000161$		$k = .34$ $I = 6940$ $S_c = 1360$ $S_s = 46.9$ $R = 1.41$ $c = .00000162$	
$M_c = 967,000$ $M_s = 1,057,000$ "B"			$M_c = 942,000$ $M_s = 987,500$ "B"		$M_c = 892,000$ $M_s = 859,000$ "A"		$M_c = 872,000$ $M_s = 803,700$ "A"		$M_c = 884,000$ $M_s = 750,000$ "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	107400	336	104600	334	95450	318	89250	300	83250	255
7	92000	288	89650	286	81800	273	76500	257	71350	219
8	80500	252	78500	250	71600	238	66950	225	62450	191
9	71600	224	69750	222	63650	212	59500	200	55500	170
10	64450	202	62800	200	57300	191	53550	180	49950	153
11	58600	183	57080	182	52080	173	48670	164	45400	139
12	53700	168	52300	167	47700	159	44600	150	41600	127
13	49550	155	48300	154	44050	146	41180	138	38400	117
14	46000	144	44830	143	40900	136	38250	128	35680	109
15	42950	134	41850	133	38180	127	35700	120	33300	102
16	40500	126	39220	125	35800	119	33460	112	31200	95
17	37900	118	36920	118	33680	112	31500	106	29380	90
18	35800	112	34880	111	31800	106	29730	100	27730	85
19	33900	106	33030	105	30150	100	28180	94	26300	80
20	32200	101	31370	100	28640	95	26770	90	24970	76
21	30680	96	29900	95	27270	91	25500	85	23800	73
22	29280	91	28520	91	26000	86	24330	82	22700	69
23	28000	87	27300	87	24900	83	23280	78	21700	66
24	26850	84	26150	83	23850	79	22300	75	20800	63

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

12-INCH BEAMS 3½-INCH SLABS										
12-in., 55-lb. I S=53.2			12-in., 50-lb. I S=50.3		12-in., 45-lb. I S=47.3		12-in., 40.8-lb. I S=44.8		12-in., 35-lb. I S=37.8	
k =.45 I =8450 S _c =1372 S _s =74.9 R =1.41 c =.00000133			k =.435 I =8040 S _c =1352 S _s =69.4 R =1.38 c =.00000140		k =.42 I =7630 S _c =1325 S _s =63.9 R =1.35 c =.00000147		k =.41 I =7260 S _c =1292 S _s =59.9 R =1.34 c =.00000155		k =.39 I =6630 S _c =1214 S _s =51.7 R =1.37 c =.00000170	
M _c = 892,000 M _s =1,197,500 "B"			M _c = 879,000 M _s =1,110,000 "B"		M _c = 862,000 M _s =1,023,000 "B"		M _c =840,000 M _s =957,270 "B"		M _c =789,000 M _s =827,000 "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	99200	291	97600	289	95800	286	93250	285	87700	283
7	85000	249	83700	247	82050	245	79900	244	75200	243
8	74400	218	73200	216	71800	215	69950	213	65750	212
9	66100	194	65100	192	63800	191	62200	190	58450	189
10	59500	174	58600	173	57450	172	55950	171	52600	170
11	54100	158	53300	157	52200	156	50850	155	47800	154
12	49600	145	48850	144	47850	143	46600	142	43800	141
13	45750	134	45100	133	44200	132	43000	131	40420	130
14	42500	124	41850	123	41000	122	39950	122	37550	121
15	39650	116	39060	115	38300	114	37300	114	35060	113
16	37200	109	36650	108	35900	107	34950	106	32850	106
17	35000	102	34500	102	33800	101	32900	100	30920	100
18	33050	97	32550	96	31900	95	31050	95	29200	94
19	31300	91	30850	91	30400	90	29400	90	27670	89
20	29750	87	29300	86	28700	86	27950	85	26300	85
21	28330	83	27900	82	27350	81	26600	81	25020	81
22	27050	79	26630	78	26100	78	25400	77	23900	77
23	25850	76	25450	75	24950	74	24300	74	22850	73
24	24800	72	24400	72	23950	71	23300	71	21900	70

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

12-INCH BEAMS 3½-INCH SLABS			12-INCH BEAMS 3-INCH SLABS			12-INCH BEAMS 3-INCH SLABS			12-INCH BEAMS 3-INCH SLABS		
12-in., 31.8-lb. I S = 36.0			12-in., 27.9-lb. I S = 32.2			12-in., 55-lb. I S = 53.2			12-in., 50-lb. I S = 50.3		
$k = .37$ $I = 6310$ $S_c = 1217$ $S_s = 47.7$ $R = 1.33$ $c = .00000178$			$k = .36$ $I = 6270$ $S_c = 1201$ $S_s = 45.1$ $R = 1.36$ $c = .00000180$			$k = .465$ $I = 7600$ $S_c = 1241$ $S_s = 71.9$ $R = 1.35$ $c = .00000148$			$k = .45$ $I = 7240$ $S_c = 1221$ $S_s = 66.6$ $R = 1.32$ $c = .00000155$		
$M_c = 791,000$ $M_s = 762,500$ "A"			$M_c = 780,000$ $M_s = 722,000$ "A"			$M_c = 808,000$ $M_s = 1,149,000$ "B"			$M_c = 793,600$ $M_s = 1,065,000$ "B"		
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	
6	84700	267	80200	242	89800	236	88100	235	86400	233	
7	72600	229	68800	207	77000	202	75500	201	74050	200	
8	63550	200	60200	186	67350	177	66100	176	64800	175	
9	56500	178	53500	161	59850	157	58750	156	57600	156	
10	50800	160	48150	145	53900	141	52850	141	51800	140	
11	46200	145	43750	132	49000	128	48050	128	47100	127	
12	42350	133	40100	121	44900	118	44050	117	43150	116	
13	39100	123	37000	111	41450	109	40650	108	39850	107	
14	36300	114	34400	104	38500	101	37750	100	37000	100	
15	33900	107	32100	97	35900	94	35250	94	34580	93	
16	31770	100	30100	90	33700	88	33050	88	32380	87	
17	29900	94	28300	85	31700	83	31100	82	30480	82	
18	28250	89	26740	80	29920	78	29370	78	28800	77	
19	26760	84	25320	76	28360	74	27820	74	27280	73	
20	25420	80	24080	72	26930	70	26420	70	25900	70	
21	24200	76	22900	69	25650	67	25180	67	24680	66	
22	23100	73	21900	66	24500	64	24020	64	23550	63	
23	22100	69	20900	63	23400	61	22950	61	22500	61	
24	21150	66	20050	60	22400	59	22000	59	21600	58	

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

12-INCH BEAMS 3-INCH SLABS							10-INCH BEAM 4½-INCH SLAB			
12-in., 40.8-lb. I S = 44.8			12-in., 35-lb. I S = 37.8		12-in., 31.8-lb. I S = 36.0		12-in., 27.9-lb. I S = 33.2		10-in., 40-lb. I S = 31.6	
$k = .425$ $I = 6550$ $S_c = 1165$ $S_s = 57.4$ $R = 1.28$ $c = .00000172$			$k = .41$ $I = 5950$ $S_c = 1074$ $S_s = 49.9$ $R = 1.32$ $c = .00000189$		$k = .395$ $I = 5480$ $S_c = 1028$ $S_s = 44.7$ $R = 1.24$ $c = .00000205$		$k = .375$ $I = 5620$ $S_c = 1070$ $S_s = 42.9$ $R = 1.29$ $c = .00000200$		$k = .40$ $I = 6350$ $S_c = 1211$ $S_s = 53.8$ $R = 1.70$ $c = .00000177$	
$M_c = 757,000$ $M_s = 918,000$ "B"			$M_c = 698,000$ $M_s = 797,000$ "B"		$M_c = 668,000$ $M_s = 715,000$ "B"		$M_c = 695,200$ $M_s = 686,000$ "A"		$M_c = 788,000$ $M_s = 860,000$ "B"	
Span	Load	Shear	Load	Shear	Load	Shear	Load	Shear	Load	Shear
ft.	lb.	lb./sq. Shear in.	lb.	lb./sq. Shear in.	lb.	lb./sq. Shear in.	lb.	lb./sq. Shear in.	lb.	lb./sq. Shear in.
6	84100	233	77500	233	74250	231	76200	213	87500	381
7	72100	200	66450	200	63650	198	65300	183	75000	326
8	63100	175	58150	175	55700	173	57150	160	65650	285
9	56100	155	51700	155	49500	154	50800	142	58300	253
10	50500	140	46500	140	44550	138	45700	128	52540	228
11	45880	127	42300	127	40500	126	41550	116	47750	207
12	42050	116	38750	116	37100	115	38100	106	43800	190
13	38800	107	35780	107	34250	106	35150	98	40400	175
14	36050	100	33200	100	31800	99	32650	91	37500	163
15	33650	93	31000	93	29700	92	30500	85	35000	152
16	31550	87	29080	87	27850	86	28580	80	32800	142
17	29680	82	27350	82	26200	81	26900	75	30900	134
18	28020	77	25850	77	24740	77	25400	71	29200	126
19	26570	73	24480	73	23440	72	24050	67	27650	120
20	25220	70	23250	70	22280	69	22850	64	26270	114
21	24200	66	22150	66	21200	66	21750	61	25000	108
22	22930	63	21150	63	20250	63	20750	58	23880	103
23	21950	60	20200	60	19350	60	19850	55	22850	99
24	21000	58	19300	58	18500	57	19000	53	21900	95

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

10-INCH BEAMS 4½-INCH SLABS									10-INCH BEAM 4-INCH SLAB	
10-in., 35-lb. I S = 29.2			10-in., 30-lb. I S = 26.7		10-in., 25.4-lb. I S = 24.4		10-in., 22.4-lb. I S = 22.7		10-in., 40-lb. I S = 31.6	
$k = .38$ $I = 5890$ $S_c = 1183$ $S_s = 48.3$ $R = 1.65$ $c = .00000191$			$k = .36$ $I = 5410$ $S_c = 1146$ $S_s = 42.8$ $R = 1.60$ $c = .00000208$		$k = .34$ $I = 4910$ $S_c = 1100$ $S_s = 37.8$ $R = 1.55$ $c = .00000229$		$k = .321$ $I = 4910$ $S_c = 1125$ $S_s = 35.5$ $R = 1.56$ $c = .00000229$		$k = .415$ $I = 5620$ $S_c = 1073$ $S_s = 50.7$ $R = 1.60$ $c = .00000200$	
$M_c = 769,000$ $M_s = 772,000$ "B"			$M_c = 745,000$ $M_s = 685,400$ "A"		$M_c = 715,000$ $M_s = 604,000$ "A"		$M_c = 731,000$ $M_s = 568,000$ "A"		$M_c = 698,000$ $M_s = 811,000$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	85400	368	76100	327	67100	290	63100	244	77600	332
7	73200	315	65200	280	57500	248	54100	209	66500	285
8	64100	276	57100	245	50350	217	47340	183	58200	249
9	56950	245	50750	218	44750	193	42060	163	51700	221
10	51260	221	45680	196	40260	174	37860	146	46500	199
11	46600	200	41500	178	36600	158	34400	133	42250	181
12	42700	184	38050	163	33530	145	31540	122	38740	166
13	39400	169	35100	151	30970	133	29100	113	35760	153
14	36600	157	32600	140	28740	124	27020	104	33200	142
15	34150	147	30450	130	26850	116	25220	98	31000	133
16	32020	138	28520	122	25150	108	23650	91	29050	124
17	30130	129	26850	115	23670	102	22270	86	27350	117
18	28470	122	25360	109	22360	96	21000	81	25820	110
19	26970	116	24020	103	21200	91	19900	77	24480	105
20	25630	110	22840	98	20130	87	18930	73	23250	99
21	24400	105	21750	93	19150	82	18000	70	22130	95
22	23300	100	20750	89	18300	79	17200	66	21120	90
23	22300	96	19850	85	17500	75	16450	64	20200	86
24	21350	92	19000	81	16770	72	15770	61	19360	83

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

10-INCH BEAMS 4-INCH SLABS										10-INCH BEAM 3½-INCH SLAB
10-in., 35-lb. I S = 29.2			10-in., 30-lb. I S = 26.7		10-in., 25.4-lb. I S = 24.4		10-in., 22.4-lb. I S = 22.7		10-in., 40-lb. I S = 31.6	
k = .395 I = 5230 S _c = 1048 S _s = 45.6 R = 1.56 c = .00000215			k = .375 I = 4810 S _c = 1015 S _s = 40.6 R = 1.52 c = .00000234		k = .35 I = 4384 S _c = 992 S _s = 35.5 R = 1.45 c = .00000257		k = .33 I = 4390 S _c = 1015 S _s = 33.3 R = 1.47 c = .00000256		k = .43 I = 4960 S _c = 951.5 S _s = 48.0 R = 1.52 c = .00000227	
M _c = 682,000 M _s = 729,000 "B"			M _c = 659,500 M _s = 649,000 "A"		M _c = 644,500 M _s = 568,000 "A"		M _c = 660,000 M _s = 533,000 "A"		M _c = 618,000 M _s = 766,000 "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	75750	327	72100	313	63100	268	59200	226	68600	284
7	64950	280	61800	269	54100	230	50700	194	58850	244
8	56800	245	54050	235	47300	201	44400	170	51450	213
9	50500	218	48100	209	42050	179	39460	151	45750	189
10	45460	196	43260	188	37860	161	35520	136	41200	170
11	41340	178	39300	171	34400	146	32300	123	37400	155
12	37900	163	36050	156	31550	134	29600	113	34300	142
13	34950	150	33280	144	29100	123	27320	104	31650	131
14	32460	140	30900	134	27040	115	25380	97	29400	122
15	30300	130	28840	125	25230	107	23680	90	27440	113
16	28400	122	27040	117	23650	100	22200	85	25730	106
17	26750	115	25450	110	22270	94	20900	80	24200	100
18	25250	109	24020	104	21020	89	19740	75	22870	94
19	23920	103	22770	99	19920	84	18700	71	21670	89
20	22730	98	21630	94	18930	80	17760	68	20600	85
21	21650	93	20600	89	18030	76	16910	64	19600	81
22	20680	89	19650	85	17200	73	16150	61	18700	77
23	19760	85	18800	81	16450	70	15450	59	17900	74
24	18940	81	18030	78	15770	67	14800	56	17150	71

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

Span ft.	10-INCH BEAMS 3½-INCH SLABS								10-INCH BEAM 3-INCH SLAB	
	10-in., 35-lb. I S=29.2		10-in., 30-lb. I S=26.7		10-in., 25.4-lb. I S=24.4		10-in., 22.4-lb. I S=22.7		10-in., 40-lb. I S=31.6	
	$k = .415$		$k = .39$		$k = .365$		$k = .345$		$k = .445$	
	$I = 4630$		$I = 4260$		$I = 3900$		$I = 3910$		$I = 4361$	
	$S_c = 920$		$S_c = 901$		$S_c = 881$		$S_c = 900$		$S_c = 843.5$	
	$S_s = 43.5$		$S_s = 38.4$		$S_s = 33.7$		$S_s = 31.6$		$S_s = 45.1$	
	$R = 1.49$		$R = 1.44$		$R = 1.38$		$R = 1.39$		$R = 1.43$	
	$c = .00000243$		$c = .00000264$		$c = .00000288$		$c = .00000288$		$c = .00000258$	
	$M_c = 598,000$		$M_c = 586,000$		$M_c = 572,000$		$M_c = 585,000$		$M_c = 548,000$	
	$M_s = 695,500$ "B"		$M_s = 614,000$ "B"		$M_s = 539,000$ "A"		$M_s = 506,000$ "A"		$M_s = 722,000$ "B"	
	Load	Shear	Load	Shear	Load	Shear	Load	Shear	Load	Shear
	lb.	lb./sq. in.	lb.	lb./sq. in.	lb.	lb./sq. in.	lb.	lb./sq. in.	lb.	lb./sq. in.
6	66400	282	65100	275	59900	250	56200	214	60800	233
7	56900	241	55800	236	51300	214	48200	183	52150	200
8	49800	211	48820	206	44900	188	42200	160	45600	175
9	44280	188	43400	183	39920	167	37500	142	40550	155
10	39860	169	39060	165	35940	150	33740	128	36500	140
11	36200	153	35500	150	32660	136	30670	116	33170	127
12	33200	141	32550	137	29930	125	28100	107	30400	116
13	30620	130	30050	127	27630	115	25950	98	28070	107
14	28450	120	27900	118	25670	107	24100	91	26070	100
15	26550	112	26050	110	23950	100	22490	85	24320	93
16	24900	105	24400	103	22450	93	21090	80	22800	87
17	23420	99	23000	97	21120	88	19850	75	21460	82
18	22120	93	21700	91	19950	83	18740	71	20270	77
19	20960	89	20550	86	18900	79	17750	67	19200	73
20	19930	84	19530	82	17970	75	16870	64	18250	70
21	18960	80	18600	78	17100	71	16060	61	17370	66
22	18100	76	17750	75	16340	68	15340	58	16590	63
23	17320	73	16980	71	15620	65	14670	55	15860	60
24	16600	70	16280	68	14960	62	14050	53	15200	58

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

10-INCH BEAMS 3-INCH SLABS										9-INCH BEAM 4½-INCH SLAB
10-in., 35-lb. I S = 29.2			10-in., 30-lb. I S = 26.7		10-in., 25.4-lb. I S = 24.4		10-in., 22.4-lb. I S = 22.7		9-in., 35-lb. I S = 24.7	
k = .43 I = 4079 S _c = 816 S _s = 41.0 R = 1.40 c = .00000276			k = .41 I = 3772 S _c = 791 S _s = 36.7 R = 1.37 c = .00000298		k = .38 I = 3460 S _c = 783 S _s = 32.0 R = 1.31 c = .00000325		k = .365 I = 3479 S _c = 788 S _s = 30.2 R = 1.33 c = .00000324		k = .40 I = 4910 S _c = 1007 S _s = 44.8 R = 1.81 c = .00000229	
M _c = 530,500 M _s = 656,500 "B"			M _c = 514,000 M _s = 586,000 "B"		M _c = 509,000 M _s = 511,000 "B"		M _c = 512,000 M _s = 482,500 "A"		M _c = 654,000 M _s = 716,000 "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	58900	232	57100	229	56500	222	53600	199	72700	372
7	50500	198	48930	196	48400	190	45900	170	62300	319
8	44200	174	42800	172	42350	167	40200	149	54550	279
9	39300	154	38080	152	37650	148	35720	132	48500	248
10	35360	139	34260	137	33900	133	32160	119	43650	223
11	32150	126	31150	125	30840	121	29220	108	39700	203
12	29470	116	28550	114	28250	111	26800	99	36380	186
13	27200	107	26350	105	26100	102	24720	91	33600	171
14	25260	99	24480	98	24220	95	22950	85	31150	159
15	23580	92	22850	91	22600	89	21420	79	29100	149
16	22100	87	21400	86	21200	83	20100	74	27300	139
17	20800	81	20150	80	19950	78	18900	70	25680	131
18	19650	77	19040	76	18840	74	17860	66	24250	124
19	18600	73	18040	72	17850	70	16920	62	22970	117
20	17680	69	17130	68	16950	66	16080	59	21820	111
21	16850	66	16320	65	16150	63	15300	56	20800	106
22	16070	63	15580	62	15400	60	14600	54	19850	101
23	15380	60	14900	59	14750	58	13970	51	18970	97
24	14740	58	14280	57	14130	55	13400	49	18200	93

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

9-INCH BEAMS 4½-INCH SLABS							9-INCH BEAMS 4-INCH SLABS				
9-in., 30-lb. I S = 22.5			9-in., 25-lb. I S = 20.3		9-in., 21.8-lb. I S = 18.9		9-in., 35-lb. I S = 24.7		9-in., 30-lb. I S = 22.5		
$k = .38$ $I = 4500$ $S_c = 970$ $S_s = 39.7$ $R = 1.76$ $c = .0000025$			$k = .355$ $I = 4050$ $S_c = 936$ $S_s = 34.3$ $R = 1.69$ $c = .00000278$		$k = .34$ $I = 3740$ $S_c = 902$ $S_s = 30.9$ $R = 1.64$ $c = .00000301$		$k = .41$ $I = 4300$ $S_c = 897$ $S_s = 41.5$ $R = 1.68$ $c = .00000262$		$k = .39$ $I = 3950$ $S_c = 867$ $S_s = 36.9$ $R = 1.64$ $c = .00000285$		
$M_c = 630,000$ $M_s = 635,000$ "B"			$M_c = 608,000$ $M_s = 548,000$ "A"		$M_c = 586,000$ $M_s = 494,500$ "A"		$M_c = 583,000$ $M_s = 664,000$ "B"		$M_c = 564,000$ $M_s = 591,000$ "B"		
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	
6	70000	360	60900	309	54900	276	64750	326	62650	318	
7	60000	309	52150	265	47050	237	55500	280	53700	273	
8	52500	270	45650	232	41200	207	48550	245	46950	239	
9	46700	240	40550	206	36600	184	43150	217	41750	212	
10	42000	216	36500	185	32950	166	38850	196	37600	191	
11	38150	196	33200	168	29950	151	35300	178	34150	173	
12	35000	180	30450	154	27450	138	32350	163	31300	159	
13	32300	166	28100	143	25350	128	29900	150	28900	147	
14	30000	154	26100	132	23530	119	27750	140	26850	136	
15	28000	144	24350	123	21960	111	25900	130	25050	127	
16	26250	135	22820	116	20600	104	24300	122	23500	119	
17	24700	127	21480	109	19370	98	22850	115	22100	112	
18	23330	120	20280	103	18300	92	21600	108	20880	106	
19	22100	113	19220	97	17350	87	20450	103	19780	100	
20	21000	108	18250	92	16475	83	19420	98	18780	95	
21	20000	103	17380	88	15700	79	18500	93	17900	91	
22	19100	98	16600	84	14980	75	17650	89	17100	86	
23	18250	94	15870	80	14330	72	16900	85	16350	83	

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

9-INCH BEAMS 4-INCH SLABS					9-INCH BEAMS 3½-INCH SLABS							
9-in., 25-lb. I S = 20.3					9-in., 21.8-lb. I S = 18.9		9-in., 35-lb. I S = 24.7		9-in., 30-lb. I S = 22.5		9-in., 25-lb. I S = 20.3	
$k = .365$ $I = 3570$ $S_c = 835$ $S_s = 32.0$ $R = 1.58$ $c = .00000315$					$k = .35$ $I = 3310$ $S_c = 806$ $S_s = 28.9$ $R = 1.53$ $c = .0000034$		$k = .425$ $I = 3760$ $S_c = 789$ $S_s = 38.9$ $R = 1.57$ $c = .00000299$		$k = .405$ $I = 3460$ $S_c = 763$ $S_s = 34.7$ $R = 1.54$ $c = .00000325$		$k = .38$ $I = 3140$ $S_c = 736$ $S_s = 30.1$ $R = 1.48$ $c = .00000358$	
$M_c = 542,500$ $M_s = 511,000$ "A"					$M_c = 524,000$ $M_s = 462,000$ "A"		$M_c = 513,000$ $M_s = 623,000$ "B"		$M_c = 496,000$ $M_s = 554,000$ "B"		$M_c = 478,000$ $M_s = 480,500$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
6	56750	289	51300	261	57000	281	55100	276	53050	269		
7	48650	248	44000	224	48850	240	47220	236	45450	231		
8	42550	217	38500	196	42720	210	41330	207	39780	202		
9	37850	193	34200	174	38000	187	36750	184	35350	179		
10	34050	173	30780	156	34200	168	33080	165	31800	161		
11	30950	158	27980	142	31100	153	30050	150	28920	147		
12	28400	144	25650	130	28500	140	27550	138	26520	134		
13	26200	133	23680	120	26300	129	25450	127	24480	124		
14	24330	124	21980	112	24420	120	23600	118	22720	115		
15	22700	115	20520	104	22800	112	22050	110	21220	107		
16	21300	108	19240	98	21370	105	20670	103	19900	101		
17	20050	102	18100	92	20100	99	19450	97	18730	95		
18	18930	96	17100	87	19000	93	18370	92	17680	89		
19	17930	91	16200	82	18000	88	17400	87	16750	85		
20	17030	86	15390	78	17100	84	16540	82	15920	80		
21	16200	82	14650	74	16280	80	15750	78	15140	77		
22	15470	79	13980	71	15550	76	15030	75	14450	73		
23	14800	75	13380	68	14800	73	14350	72	13800	70		

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

9-INCH BEAM 3½-INCH SLAB			9-INCH BEAMS 3-INCH SLABS							
9-in., 21.8-lb. I S=18.9			9-in., 35-lb. I S=24.7		9-in., 30-lb. I S=22.5		9-in., 25-lb. I S=20.3		9-in., 21.8-lb. I S=18.9	
$k = .355$ $I = 2910$ $S_c = 731$ $S_s = 26.8$ $R = 1.42$ $c = .00000387$			$k = .44$ $I = 3270$ $S_c = 695$ $S_s = 36.3$ $R = 1.47$ $c = .00000344$		$k = .425$ $I = 3020$ $S_c = 664$ $S_s = 32.8$ $R = 1.46$ $c = .00000373$		$k = .395$ $I = 2740$ $S_c = 649$ $S_s = 28.2$ $R = 1.39$ $c = .00000411$		$k = .38$ $I = 2560$ $S_c = 628$ $S_s = 25.6$ $R = 1.35$ $c = .00000440$	
$M_c = 475,000$ $M_s = 428,500$ "A"			$M_c = 451,500$ $M_s = 581,000$ "B"		$M_c = 431,500$ $M_s = 524,000$ "B"		$M_c = 422,000$ $M_s = 451,000$ "B"		$M_c = 408,000$ $M_s = 409,500$ "B"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
5	57150	277	60180	277	57500	277	56250	269	54390	265
6	47600	230	50150	231	47900	230	46850	224	45300	221
7	40800	198	43000	198	41100	197	40200	192	38850	189
8	35720	173	37600	173	35950	173	35150	168	34000	165
9	31750	153	33450	154	31950	153	31250	149	30200	147
10	28580	138	30100	138	28750	138	28130	134	27200	132
11	25950	126	27350	126	26120	125	25570	122	24720	120
12	23800	115	25070	115	23950	115	23420	112	22650	110
13	21970	106	23150	106	22100	106	21630	103	20900	102
14	20400	99	21500	99	20530	98	20100	96	19420	94
15	19050	92	20060	92	19170	92	18750	89	18130	88
16	17850	86	18800	86	17960	86	17580	84	17000	82
17	16800	81	17700	81	16900	81	16550	79	16000	78
18	15860	77	16720	77	15970	76	15630	74	15100	73
19	15030	72	15830	73	15140	72	14800	70	14320	69
20	14280	69	15050	69	14370	69	14070	67	13600	66
21	13600	66	14300	66	13700	66	13400	64	13000	63
22	12980	63	13700	63	13100	63	12800	61	12400	60

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

8-INCH BEAMS 4½-INCH SLABS										
8-in., 25.5-lb. I S = 17.0			8-in., 23-lb. I S = 16.0		8-in., 20.5-lb. I S = 15.1		8-in., 18.4-lb. I S = 14.2		8-in., 17.5-lb. I S = 14.6	
k = .375 I = 3390 S _c = 799 S _s = 32.0 R = 1.88 c = .00000332			k = .362 I = 3190 S _c = 781 S _s = 29.5 R = 1.84 c = .00000353		k = .348 I = 2990 S _c = 760 S _s = 27.0 R = 1.79 c = .00000376		k = .334 I = 2800 S _c = 741 S _s = 24.8 R = 1.75 c = .00000402		k = .323 I = 2980 S _c = 790 S _s = 25.2 R = 1.73 c = .00000378	
M _c = 519,500 M _s = 512,000 "A"			M _c = 507,000 M _s = 472,000 "A"		M _c = 494,000 M _s = 431,500 "A"		M _c = 481,500 M _s = 396,500 "A"		M _c = 513,000 M _s = 402,500 "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
5	68260	406	62920	371	57520	340	52860	310	53700	270
6	56900	339	52450	309	47900	284	44050	258	44700	225
7	48750	290	44950	265	41100	243	37750	221	38350	193
8	42650	254	39350	232	35930	213	33050	193	33550	169
9	37900	226	34970	206	31940	189	29350	172	29800	150
10	34130	203	31460	186	28760	170	26430	155	26830	135
11	31050	185	28600	169	26100	155	24020	140	24400	123
12	28450	169	26200	155	23950	142	22020	129	22350	113
13	26250	156	24200	143	22100	131	20320	119	20620	104
14	24400	145	22480	133	20520	122	18880	110	19160	96
15	22750	135	20980	124	19150	113	17620	103	17880	90
16	21340	127	19660	116	17950	106	16520	96	16770	85
17	20100	119	18500	109	16900	100	15550	91	15780	80
18	18970	113	17470	103	15960	95	14680	86	14900	75
19	17960	107	16550	98	15130	89	13900	81	14120	71
20	17070	102	15730	93	14360	85	13220	77	13420	68
21	16250	97	14980	88	13680	81	12580	74	12770	65
22	15520	92	14300	84	13060	77	12020	70	12200	62

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

8-INCH BEAMS 4-INCH SLABS										
8-in., 25.5-lb. I S=17.0			8-in., 23-lb. I S=16.0		8-in., 20.5-lb. I S=15.1		8-in., 18.4-lb. I S=14.2		8-in., 17.5-lb. I S=14.6	
$k = .385$ $I = 2940$ $S_c = 707.5$ $S_s = 29.5$ $R = 1.73$ $c = .00000383$			$k = .375$ $I = 2780$ $S = 686$ $S_s = 27.4$ $R = 1.71$ $c = .00000405$		$k = .359$ $I = 2600$ $S_c = 669$ $S_s = 25.0$ $R = 1.66$ $c = .00000433$		$k = .345$ $I = 2450$ $S_c = 655$ $S_s = 23.0$ $R = 1.62$ $c = .00000459$		$k = .333$ $I = 2620$ $S_c = 703$ $S_s = 23.4$ $R = 1.60$ $c = .0000043$	
$M_c = 459,500$ $M_s = 471,000$ "B"			$M_c = 446,000$ $M_s = 438,000$ "A"		$M_c = 435,000$ $M_s = 400,000$ "A"		$M_c = 426,000$ $M_s = 367,500$ "A"		$M_c = 457,000$ $M_s = 374,500$ "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
5	61260	368	58400	350	53320	322	49000	291	49900	255
6	51100	306	48650	291	44400	268	40850	243	41600	213
7	43800	262	41720	250	38100	230	35000	208	35650	182
8	38300	230	36500	218	33300	201	30620	182	31200	160
9	34050	204	32450	194	29600	179	27220	162	27700	142
10	30630	184	29200	175	26660	161	24500	146	24950	128
11	27850	167	26520	159	24220	146	22270	132	22680	116
12	25520	153	24300	145	22200	134	20400	121	20800	106
13	23560	141	22450	134	20500	124	18850	112	19180	98
14	21880	131	20850	125	19050	115	17500	104	17820	91
15	20420	122	19450	116	17770	107	16330	97	16630	85
16	19150	115	18250	109	16660	101	15320	91	15600	80
17	18000	108	17170	102	15680	95	14400	85	14670	75
18	17000	102	16200	97	14800	89	13600	81	13850	71
19	16130	96	15360	92	14030	85	12880	77	13130	67
20	15300	91	14600	87	13330	80	12250	73	12470	64
21	14580	87	13900	83	12700	77	11700	70	11870	60
22	13830	83	13260	79	12100	73	11100	66	11300	58

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

8-INCH BEAMS 3½-INCH SLABS										
8-in., 25.5-lb. I S = 17.0			8-in., 23-lb. I S = 16.0		8-in., 20.5-lb. I S = 15.1		8-in., 18.4-lb. I S = 14.2		8-in., 17.5-lb. I S = 14.6	
$k = .40$			$k = .385$		$k = .37$		$k = .35$		$k = .34$	
$I = 2540$			$I = 2400$		$I = 2260$		$I = 2130$		$I = 2280$	
$S_c = 617$			$S_c = 605$		$S_c = 592$		$S_c = 589$		$S_c = 629$	
$S_s = 27.5$			$S_s = 25.3$		$S_s = 23.2$		$S_s = 21.1$		$S_s = 21.6$	
$R = 1.62$			$R = 1.58$		$R = 1.54$		$R = 1.49$		$R = 1.48$	
$c = .00000443$			$c = .00000469$		$c = .00000498$		$c = .00000528$		$c = .00000494$	
$M_c = 401,000$			$M_c = 393,000$		$M_c = 385,000$		$M_c = 383,000$		$M_c = 409,000$	
$M_s = 439,000$			$M_s = 404,500$		$M_s = 370,500$		$M_s = 338,000$		$M_s = 345,500$	
"B"			"B"		"A"		"A"		"A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
5	53460	322	52400	316	49400	296	45060	260	46040	231
6	44550	268	43650	263	41150	246	37550	216	38400	193
7	38200	230	37400	225	35280	211	32200	185	32900	165
8	33400	201	32750	197	30870	185	28150	162	28780	145
9	29700	179	29100	175	27430	164	25050	144	25600	128
10	26730	161	26200	158	24700	148	22530	130	23020	116
11	24300	146	23800	143	22430	134	20500	118	20930	105
12	22270	134	21820	131	20570	123	18770	108	19200	96
13	20550	124	20150	121	19000	113	17330	100	17720	89
14	19100	115	18700	112	17640	105	16100	92	16450	82
15	17820	107	17460	105	16450	98	15020	86	15350	77
16	16700	100	16370	98	15430	92	14080	81	14400	72
17	15730	94	15400	92	14520	87	13250	76	13550	68
18	14850	89	14550	87	13720	82	12520	72	12800	64
19	14070	84	13780	83	13000	77	11850	68	12120	61
20	13370	80	13100	79	12350	74	11270	65	11520	57
21	12730	76	12470	75	11750	70	10730	61	10970	55
22	12150	73	11900	71	11200	67	10200	59	10470	52

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

8-INCH BEAMS 3-INCH SLABS										
8-in., 25.5-lb. I S = 17.0			8-in., 23-lb. I S = 16.0		8-in., 20.5-lb. I S = 15.1		8-in., 18.4-lb. I S = 14.2		8-in., 17.5-lb. I S = 14.6	
$k = .415$ $I = 2190$ $S_c = 539$ $S_s = 25.5$ $R = 1.50$ $c = .00000514$			$k = .40$ $I = 2070$ $S_c = 529$ $S_s = 23.5$ $R = 1.47$ $c = .00000544$		$k = .39$ $I = 1950$ $S_c = 511$ $S_s = 21.8$ $R = 1.44$ $c = .00000577$		$k = .365$ $I = 1840$ $S_c = 515$ $S_s = 19.7$ $R = 1.39$ $c = .00000611$		$k = .36$ $I = 1990$ $S_c = 543$ $S_s = 20.3$ $R = 1.39$ $c = .00000566$	
$M_c = 350,200$ $M_s = 407,200$ "B"			$M_c = 344,000$ $M_s = 375,700$ "B"		$M_c = 332,000$ $M_s = 348,500$ "B"		$M_c = 334,500$ $M_s = 315,500$ "A"		$M_c = 353,000$ $M_s = 325,000$ "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
5	46650	270	45860	266	44260	263	42050	238	43300	214
6	38900	225	38220	221	36900	219	35050	199	36080	178
7	33350	193	32750	190	31600	188	30050	170	30920	152
8	29150	169	28670	166	27670	164	26300	149	27080	133
9	25920	150	25500	147	24600	146	23350	132	24050	118
10	23350	135	22930	133	22130	131	21030	119	21650	107
11	21200	123	20850	120	20100	119	19120	108	19670	97
12	19450	112	19100	111	18440	109	17530	99	18050	89
13	17950	104	17640	102	17020	101	16180	91	16650	82
14	16660	96	16380	95	15800	94	15030	85	15470	76
15	15550	90	15300	88	14750	87	14020	79	14430	71
16	14580	84	14340	83	13830	82	13150	74	13530	66
17	13730	79	13500	78	13000	77	12370	70	12740	62
18	12960	75	12740	73	12290	73	11680	66	12030	59
19	12280	71	12070	70	11640	69	11070	62	11400	56
20	11670	67	11470	66	11060	65	10520	59	10830	53
21	11100	64	10920	63	10500	63	10000	57	10300	51

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

7-INCH BEAMS 4½-INCH SLABS							7-INCH BEAMS 4-INCH SLABS			
7-in., 20-lb. I S = 12.0			7-in., 17.5-lb. I S = 11.1		7-in., 15.3-lb. I S = 10.4		7-in., 20-lb. I S = 12.0		7-in., 17.5-lb. I S = 11.1	
k = .364 I = 2410 S _c = 637 S _s = 24.3 R = 2.02 c = .00000467			k = .347 I = 2220 S _c = 617.5 S _s = 21.8 R = 1.96 c = .00000507		k = .33 I = 2040 S _c = 596 S _s = 19.6 R = 1.88 c = .00000552		k = .374 I = 2060 S _c = 560 S _s = 22.1 R = 1.84 c = .00000547		k = .358 I = 1910 S _c = 538.5 S _s = 20.0 R = 1.80 c = .00000589	
M _c = 414,000 M _s = 389,200 "A"			M _c = 401,000 M _s = 349,000 "A"		M _c = 387,500 M _s = 313,000 "A"		M _c = 364,000 M _s = 354,000 "A"		M _c = 350,000 M _s = 320,200 "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
5	51880	363	46520	323	41740	287	47200	333	42680	304
6	43250	303	38750	269	34750	240	39300	277	35600	253
7	37050	260	33200	231	29800	205	33700	237	30500	217
8	32400	227	29100	202	26050	180	29500	208	26700	190
9	28800	202	25850	180	23150	160	26200	185	23720	169
10	25940	181	23260	162	20870	144	23600	166	21340	152
11	23600	165	21120	147	18950	130	21450	151	19400	138
12	21600	151	19370	135	17380	120	19670	138	17780	127
13	19950	140	17880	124	16050	110	18150	128	16400	117
14	18530	130	16600	115	14900	103	16850	119	15250	109
15	17300	121	15500	108	13900	96	15750	111	14230	101
16	16200	113	14530	101	13040	90	14750	104	13340	95
17	15250	107	13680	95	12270	85	13880	98	12550	90
18	14400	101	12920	90	11500	80	13120	92	11900	85
19	13650	95	12200	85	11000	75	12400	87	11200	80
20	13000	91	11600	81	10400	72	11800	83	10700	76

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

	7-INCH BEAM 4-INCH SLAB		7-INCH BEAMS 3½-INCH SLABS						7-INCH BEAM 3-INCH SLAB	
	7-in., 15.3-lb. I S = 10.4		7-in., 20-lb. I S = 12.0		7-in., 17.5-lb. I S = 11.1		7-in., 15.3-lb. I S = 10.4		7-in., 20-lb. I S = 12.0	
	$k = .341$ $I = 1760$ $S_c = 521$ $S_s = 18.0$ $R = 1.73$ $c = .00000639$		$k = .385$ $I = 1750$ $S_c = 485.5$ $S_s = 20.2$ $R = 1.68$ $c = .00000643$		$k = .369$ $I = 1630$ $S_c = 470$ $S_s = 18.3$ $R = 1.65$ $c = .00000690$		$k = .352$ $I = 1510$ $S_c = 455.5$ $S_s = 16.5$ $R = 1.59$ $c = .00000746$		$k = .40$ $I = 1490$ $S_c = 418.5$ $S_s = 18.6$ $R = 1.55$ $c = .00000755$	
	$M_c = 338,500$ $M_s = 287,800$ "A"		$M_c = 315,500$ $M_s = 323,500$ "B"		$M_c = 305,600$ $M_s = 292,500$ "A"		$M_c = 296,000$ $M_s = 263,800$ "A"		$M_c = 272,000$ $M_s = 297,500$ "B"	
	Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.
5	38380	272	42040	302	39000	280	35160	251	36260	258
6	32000	227	35050	251	32500	234	29300	209	30200	215
7	27400	194	30050	215	27850	200	25100	179	25900	184
8	24000	170	26300	188	24380	175	22000	157	22650	161
9	21320	151	23380	167	21680	156	19540	139	20150	143
10	19180	136	21020	151	19500	140	17580	125	18130	129
11	17440	123	19100	137	17720	127	15980	114	16480	117
12	15990	113	17500	125	16250	117	14650	105	15120	107
13	14750	105	16170	116	15000	108	13530	97	13950	99
14	13700	97	15000	107	13930	100	12550	90	12950	92
15	12780	91	14000	100	13000	93	11730	84	12080	86
16	11980	85	13140	94	12180	87	11000	78	11330	80
17	11300	80	12370	88	11500	82	10400	74	10600	76
18	10700	75	11700	84	10800	78	9800	69	10100	72

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

7-INCH BEAMS 3-INCH SLABS			6-INCH BEAMS 4½-INCH SLABS							
7-in., 17.5-lb. I S = 11.1			7-in., 15.3-lb. I S = 10.4		6-in., 17.25-lb. I S = 8.7		6-in., 14.75-lb. I S = 7.9		6-in., 12.5-lb. I S = 7.3	
$k = .385$ $I = 1390$ $S_c = 405.5$ $S_s = 16.9$ $R = 1.52$ $c = .00000809$			$k = .365$ $I = 1290$ $S_c = 396$ $S_s = 15.2$ $R = 1.46$ $c = .00000873$		$k = .366$ $I = 1790$ $S_c = 515$ $S_s = 19.8$ $R = 2.28$ $c = .00000628$		$k = .347$ $I = 1620$ $S_c = 492.5$ $S_s = 17.5$ $R = 2.22$ $c = .00000695$		$k = .326$ $I = 1460$ $S_c = 472$ $S_s = 15.2$ $R = 2.08$ $c = .00000770$	
$M_c = 263,600$ $M_s = 270,200$ "B"			$M_c = 257,200$ $M_s = 243,000$ "A"		$M_c = 335,000$ $M_s = 316,800$ "A"		$M_c = 320,000$ $M_s = 279,800$ "A"		$M_c = 306,900$ $M_s = 243,000$ "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
5	35140	253	32400	228	42240	348	37300	310	32400	267
6	29250	211	27000	190	35200	290	31100	258	27000	223
7	25100	181	23140	163	30200	249	26640	222	23130	191
8	21950	158	20250	142	26400	218	23300	194	20250	167
9	19520	140	18000	126	23500	193	20700	172	18000	148
10	17570	126	16200	114	21120	174	18650	155	16200	133
11	15970	115	14730	103	19200	158	16950	141	14730	121
12	14640	105	13500	95	17600	145	15550	129	13500	111
13	13500	97	12470	87	16250	134	14350	119	12460	103
14	12550	90	11570	81	15100	124	13330	111	11570	95
15	11720	84	10800	76	14100	116	12430	103	10800	89
16	10980	79	10130	71	13200	109	11650	97	10130	83
17	10300	75	9500	67	12430	102	10970	91	9500	78
18	9700	71	9000	63	11740	97	10370	86	9000	74

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

6-INCH BEAMS 4-INCH SLABS							6-INCH BEAMS 3½-INCH SLABS			
6-in., 17.25-lb. I S=8.7			6-in., 14.75-lb. I S=7.9		6-in., 12.5-lb. I S=7.3		6-in., 17.25-lb. I S=8.7		6-in., 14.75-lb. I S=7.9	
$k = .378$ $I = 1500$ $S_c = 442.5$ $S_s = 18.0$ $R = 2.07$ $c = .00000750$			$k = .358$ $I = 1370$ $S_c = 425.5$ $S_s = 15.8$ $R = 2.00$ $c = .00000822$		$k = .337$ $I = 1240$ $S_c = 407.5$ $S_s = 13.8$ $R = 1.89$ $c = .00000908$		$k = .39$ $I = 1250$ $S_c = 378.5$ $S_s = 16.1$ $R = 1.85$ $c = .00000901$		$k = .371$ $I = 1150$ $S_c = 364.5$ $S_s = 14.3$ $R = 1.81$ $c = .00000978$	
$M_c = 287,500$ $M_s = 287,000$ "A"			$M_c = 276,500$ $M_s = 252,500$ "A"		$M_c = 265,000$ $M_s = 221,200$ "A"		$M_c = 246,000$ $M_s = 258,000$ "B"		$M_c = 237,000$ $M_s = 229,200$ "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
4	47850	408	42100	359	36850	317	41000	360	38200	336
5	38280	328	33660	287	29500	254	32800	288	30560	268
6	31900	273	28050	239	24580	212	27300	240	25450	224
7	27340	234	24050	205	21080	181	23400	206	21830	192
8	23900	204	21050	179	18450	159	20500	180	19100	168
9	21250	182	18700	159	16400	141	18200	160	16970	149
10	19140	163	16830	144	14750	127	16400	143	15280	134
11	17400	149	15300	130	13400	115	14900	131	13880	122
12	15940	136	14020	120	12300	106	13670	120	12730	112
13	14700	126	12940	110	11350	98	12600	111	11750	103
14	13660	117	12000	102	10540	91	11700	103	10900	96
15	12750	109	11200	96	9830	85	10940	96	10180	90
16	11950	102	10500	90	9220	80	10250	90	9550	84
17	11250	96	9900	84	8680	75	9650	85	8980	79

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

	6-INCH BEAM 3½-INCH SLAB		6-INCH BEAMS 3-INCH SLABS				5-INCH BEAM 4-INCH SLAB			
	6-in., 12.5-lb. I S=7.3		6in., 17.25-lb. I S=8.7		6-in., 14.75-lb. I S=7.9		6-in., 12.5-lb. I S=7.3		5-in., 14.75-lb. I S=6.0	
	k =.35 I =1040 S _c =350 S _s =12.6 R =1.73 c =.0000108		k =.40 I =1040 S _c =326 S _s =14.5 R =1.67 c =.0000108		k =.385 I =953 S _c =310.0 S _s =13.0 R =1.65 c =.0000118		k =.362 I =869 S _c =300.5 S _s =11.4 R =1.56 c =.0000130		k =.385 I =1070 S _c =346 S _s =14.5 R =2.42 c =.0000105	
	M _c =227,500 M _s =201,000 “ A ”		M _c =211,800 M _s =231,000 “ B ”		M _c =201,500 M _s =207,200 “ B ”		M _c =195,300 M _s =181,700 “ A ”		M _c =225,000 M _s =231,300 “ B ”	
	Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.
4	33500	293	35250	307	33550	301	30300	267	37500	393
5	26800	234	28200	246	26860	241	24220	214	30000	314
6	22320	195	23500	205	22380	200	20180	178	25000	262
7	19150	167	20140	175	19170	172	17300	153	21420	225
8	16750	146	17620	153	16780	150	15140	134	18750	197
9	14900	130	15660	136	14900	133	13450	119	16670	175
10	13400	117	14100	123	13430	120	12110	107	15000	157
11	12180	106	12800	111	12200	109	11000	97	13640	143
12	11170	98	11750	102	11190	100	10100	89	12500	131
13	10300	90	10850	94	10330	92	9300	82	11540	121
14	9570	84	10080	87	9590	86	8650	76	10720	112
15	8930	78	9400	82	8950	80	8070	71	10000	105
16	8380	73	8800	76	8390	75	7560	67	9370	98

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

	5-INCH BEAMS 1-INCH SLABS				5-INCH BEAMS 3½-INCH SLABS					
	5-in., 12.25-lb. I S=5.4		5-in., 10-lb. I S=4.8		5-in., 14.75-lb. I S=6.0		5-in., 12.25-lb. I S=5.4		5-in., 10-lb. I S=4.8	
	$k = .36$		$k = .333$		$k = .397$		$k = .374$		$k = .345$	
	$I = 958$		$I = 840$		$I = 873$		$I = 784$		$I = 692$	
	$S_c = 330$		$S_c = 313.2$		$S_c = 291.8$		$S_c = 278$		$S_c = 265$	
	$S_s = 12.4$		$S_s = 10.4$		$S_s = 12.8$		$S_s = 11.0$		$S_s = 9.3$	
	$R = 2.30$		$R = 2.17$		$R = 2.13$		$R = 2.04$		$R = 1.94$	
	$c = .0000118$		$c = .0000134$		$c = .0000129$		$c = .0000144$		$c = .0000163$	
	$M_c = 214,500$		$M_c = 203,600$		$M_c = 189,500$		$M_c = 180,700$		$M_c = 172,300$	
	$M_s = 198,400$		$M_s = 166,400$		$M_s = 204,500$		$M_s = 176,600$		$M_s = 149,000$	
	“A”		“A”		“B”		“A”		“A”	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
4	33050	344	27730	284	31600	340	29450	319	24800	264
5	26440	275	22180	228	25280	273	23560	255	19860	211
6	22030	230	18490	190	21050	227	19640	212	16550	176
7	18880	197	15850	163	18050	195	16830	182	14180	151
8	16530	172	13870	142	15800	170	14730	159	12400	132
9	14680	153	12330	126	14050	152	13100	142	11030	117
10	13220	138	11090	114	12640	136	11780	127	9930	105
11	12000	125	10080	103	11480	124	10700	116	9020	96
12	11000	115	9240	95	10530	114	9820	106	8270	88
13	10170	106	8530	87	9720	105	9060	98	7640	81
14	9450	98	7920	81	9020	97	8420	91	7090	75
15	8820	92	7390	76	8420	91	7850	85	6620	70
16	8260	86	6930	71	7900	85	7360	80	6200	66

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

Span ft.	5-INCH BEAMS 3-INCH SLABS						4-INCH BEAMS 4-INCH SLABS			
	5-in., 14.75-lb. I $S=6.0$		5-in., 12.25-lb. I $S=5.4$		5-in., 10-lb. I $S=4.8$		4-in., 10.5-lb. I $S=3.5$		4-in., 9.5-lb. I $S=3.3$	
	$k = .411$		$k = .387$		$k = .361$		$k = .372$		$k = .359$	
	$I = 704$		$I = 635$		$I = 564$		$I = 677$		$I = 635$	
	$S_c = 243.5$		$S_c = 232.5$		$S_c = 221$		$S_c = 254.7$		$S_c = 248$	
	$S_s = 11.3$		$S_s = 9.8$		$S_s = 8.3$		$S_s = 10.1$		$S_s = 9.2$	
	$R = 1.88$		$R = 1.81$		$R = 1.73$		$R = 2.89$		$R = 2.79$	
	$c = .0000160$		$c = .0000177$		$c = .0000200$		$c = .0000166$		$c = .0000177$	
	$M_c = 158,200$		$M_c = 151,200$		$M_c = 143,700$		$M_c = 165,500$		$M_c = 161,000$	
	$M_s = 181,000$		$M_s = 156,700$		$M_s = 133,300$		$M_s = 161,000$		$M_s = 147,800$	
	"B"		"B"		"A"		"A"		"A"	
Load	Shear		Load	Shear	Load	Shear	Load	Shear	Load	Shear
lb.	lb./sq. in.		lb.	lb./sq. in.	lb.	lb./sq. in.	lb.	lb./sq. in.	lb.	lb./sq. in.
4	26350	293	25200	281	22200	245	26800	347	24600	316
5	21100	234	20160	225	17760	196	21460	278	19700	253
6	17580	195	16800	188	14800	163	17860	231	16420	210
7	15070	167	14400	160	12680	140	15320	199	14080	180
8	13180	146	12600	140	11100	123	13400	174	12320	157
9	11720	130	11200	125	9860	109	11920	154	10950	140
10	10550	117	10080	112	8880	98	10730	139	9850	127
11	9580	107	9170	102	8070	89	9750	126	8950	115
12	8790	98	8400	94	7400	82	8940	116	8200	105
13	8110	90	7750	86	6830	75	8250	107	7580	97
14	7530	84	7200	80	6340	70	7660	99	7040	90
15	7030	78	6720	75	5920	65	7150	93	6570	84
16	6590	73	6300	70	5550	61	6700	87	6150	79

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

4-INCH BEAMS 4-INCH SLABS					4-INCH BEAMS 3½-INCH SLABS					
4-in., 8.5-lb. I S=3.2			4-in., 7.7-lb. I S=3.0		4-in., 10.5-lb. I S=3.5		4-in., 9.5-lb. I S=3.3		4-in., 8.5-lb. I S=3.2	
$k = .344$ $I = 592$ $S_c = 240.5$ $S_s = 8.4$ $R = 2.62$ $c = .0000190$			$k = .329$ $I = 551$ $S_c = 234.5$ $S_s = 7.7$ $R = 2.57$ $c = .0000204$		$k = .386$ $I = 538$ $S_c = 210.2$ $S_s = 8.8$ $R = 2.51$ $c = .0000209$		$k = .374$ $I = 506$ $S_c = 204$ $S_s = 8.1$ $R = 2.45$ $c = .0000222$		$k = .356$ $I = 471$ $S_c = 198.7$ $S_s = 7.3$ $R = 2.28$ $c = .0000239$	
$M_c = 156,400$ $M_s = 134,500$ "A"			$M_c = 152,400$ $M_s = 122,300$ "A"		$M_c = 136,600$ $M_s = 140,500$ "B"		$M_c = 132,600$ $M_s = 129,600$ "A"		$M_c = 129,000$ $M_s = 117,300$ "A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
4	22400	289	20400	260	22780	309	21600	294	19550	264
5	17920	231	16320	208	18220	247	17280	235	15640	211
6	14840	192	13600	173	15180	206	14400	196	13030	176
7	12800	165	11650	148	13000	177	12350	168	11170	151
8	11200	144	10200	130	11380	155	10800	147	9770	132
9	9950	128	9060	115	10120	137	9600	131	8690	117
10	8960	116	8160	104	9110	124	8640	118	7820	106
11	8150	105	7420	94	8280	112	7860	107	7100	96
12	7470	96	6800	86	7590	103	7200	98	6520	89
13	6900	89	6280	80	7000	95	6650	90	6020	82
14	6400	82	5830	74	6500	88	6170	84	5580	77

Uniformly Distributed Safe Gross Loads for Steel Beams with Standard Haunching; also horizontal end Shear produced by these Loads. Forms for Concrete assumed Supported from below.

4-INCH BEAM 3½-INCH SLAB			4-INCH BEAMS 3-INCH SLABS							
4-in., 7.7-lb I S=3.0			4-in., 10.5-lb. I S=3.5		4-in., 9.5-lb. I S=3.3		4-in., 8.5-lb. I S=3.2		4-in., 7.7-lb. I S=3.0	
$k = .344$			$k = .401$		$k = .389$		$k = .372$		$k = .359$	
$I = 443$			$I = 421$		$I = 397$		$I = 372$		$I = 350$	
$S_c = 193.4$			$S_c = 171.1$		$S_c = 166.2$		$S_c = 162.5$		$S_c = 158.3$	
$S_s = 6.8$			$S_s = 7.6$		$S_s = 7.0$		$S_s = 6.4$		$S_s = 5.9$	
$R = 2.27$			$R = 2.17$		$R = 2.12$		$R = 2.00$		$R = 1.97$	
$c = .0000254$			$c = .0000267$		$c = .0000283$		$c = .0000302$		$c = .0000322$	
$M_c = 125,600$			$M_c = 111,300$		$M_c = 108,000$		$M_c = 105,600$		$M_c = 102,800$	
$M_s = 108,300$			$M_s = 121,800$		$M_s = 112,800$		$M_s = 102,800$		$M_s = 94,700$	
"A"			"B"		"B"		"A"		"A"	
Span ft.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.	Load lb.	Shear lb./sq. in.
4	18050	243	18550	264	18000	255	17130	242	15780	223
5	14440	195	14840	211	14400	204	13700	194	12620	178
6	12030	162	12370	176	12000	170	11400	162	10520	148
7	10320	139	10600	151	10280	146	9780	139	9020	128
8	9030	122	9270	132	9000	128	8560	121	7890	111
9	8030	108	8240	117	8000	113	7610	108	7010	99
10	7220	97	7420	106	7200	102	6850	97	6310	89
11	6560	88	6740	96	6540	93	6230	88	5740	81
12	6020	81	6180	88	6000	85	5710	81	5260	74
13	5550	75	5700	81	5530	78	5270	75	4850	68
14	5150	70	5300	75	5040	73	4890	69	4500	64

REINFORCED CONCRETE RESERVOIR WALLS

By PETER GILLESPIE, *Professor of Civil Engineering,*
and W. B. DUNBAR, *Research Assistant*

Concrete reservoir walls include gravity walls, plain reinforced walls and reinforced walls with counterforts or buttresses. The type of wall is determined to some extent by the type of roof which in part it supports. Of roofs, if the dome be excluded, there are three kinds—groined arch, flat slab and slab-and-beam.

The gravity wall is a massive structure depending largely on the weight of concrete and the width of its base for its stability and contains little or no reinforcing. It is often employed with the groined arch roof, the thrust of which it is normally well adapted to withstand. It is used for reservoirs of various depths, but mainly for the deeper structures.

When designed as a slab spanning from floor to roof, the plain reinforced wall is usually made thicker at the bottom than at the top and is reinforced vertically. In consequence of this, some type of roof, such as flat slab or slab-and-beam, must be used with it, which is capable of resisting the reaction at the top of the wall due to water pressure. The requirements of design, however, call for thicknesses that render this type too costly where great height is concerned.

The reinforced wall with counterforts, with which the second part of this study deals particularly, consists of a series of counterforts, spanning from roof to floor, together with a slab spanning from counterfort to counterfort. With this type also must be used a roof which is capable of resisting the reaction at the top of the counterfort, due to water pressure.

There is a diversity of opinion among engineers and contractors as to the relative cost of the various types of roof. The following are a few opinions gleaned from various articles written on the subject:

Mr. Allen Hazen, Consulting Engineer, New York, stated that in one case contractors were given a chance to substitute and after study concluded that no reinforced concrete design is cheaper than the groined arch. At Newton, Mass., bids for both were received and for the groined arch the tender was much less. He believed that centres for the groined arch are as cheaply made and as easily moved as those for the flat slab and that the groined arch has greater strength with less concrete. Moreover, the cost of steel in the latter is eliminated.

Mr. L. C. Wason, Aberthaw Construction Co., Boston, stated that some years ago, after a careful study, he came to the conclusion that the groined arch is cheaper. Five or ten years later a similar study was made, and the conclusion was reached that the groined arch is so much cheaper that there is no justification for using anything else.

Mr. J. R. McClintock, Fuller and McClintock, New York, believed that the groined arch is the cheapest form of reservoir cover.

Mr. John M. Rice, of Morris Knowles, Inc., Pittsburgh, stated that the groined arch is generally the cheaper. Developments in the flat slab, he believed, may modify this to some extent, especially where the fill is small or where it is omitted.

Mr. T. C. Atwood, District Plant Engineer, Emergency Fleet Corporation, New York, believed that the decision depends on the loads to be carried. If two to four feet of earth is to be supported, the groined arch is preferred. If less than two feet, the flat slab may be favoured. Where drainage of the roof enters the problem, the groined arch in his opinion presents the greater difficulty.

Mr. Nelson J. Bell, Dayton, Ohio, stated that although his experience was limited to the Dayton reservoir, his design (gravity wall and flat slab roof) was five or six times as strong as a groined arch and cost \$10,000 less than the lowest bid received for the latter. He claimed that the saving on forms and on pouring compensates for the cost of steel, and believed that had the roof been designed as a factory floor, the saving would have been doubled.

Mr. Chas. B. Burdick, Burdick and Alvord, Chicago, had employed the flat slab for reservoir roofs for fifteen years and had not been able to figure any advantage in the groined arch. He recognized the influence of roof on walls and had attempted in his experience to get away from plain concrete. He ignored the supporting power of back-filled earth and designed his walls as though they were unsupported thereby.

Mr. C. A. P. Turner, Minneapolis, used the flat slab roof in all cases. This, he believed, is not subject to the uncertainty of the groined arch where unbalanced loads occur.

It would seem as though the cost of the groined arch in general will not be much different from that of the flat slab. For given load and span, the volume of concrete is practically the same in both; the cost of material in the forms and the cost of placing forms is substantially the same in both. The extra cost of making forms for the groined arch will be about offset by the cost of steel and the placing of steel in the flat slab.

This study, which is primarily concerned with reservoir walls and only incidentally with roofs, was carried out in two parts. The first consisted in determining from plans of reservoir walls in actual existence,

the quantities of materials required per average foot of length. To these quantities, prices, considered fair at the time the computations were made, were applied, and thus a basis of comparison as between the different structures was established. Height of wall, where the roof was of the groined arch type, was arbitrarily considered to be equal to the distance from the floor level inside the wall to a point half way between water level and the level of the crown of the arch. This rule may be open to criticism, but on the whole, it is considered a fair one.

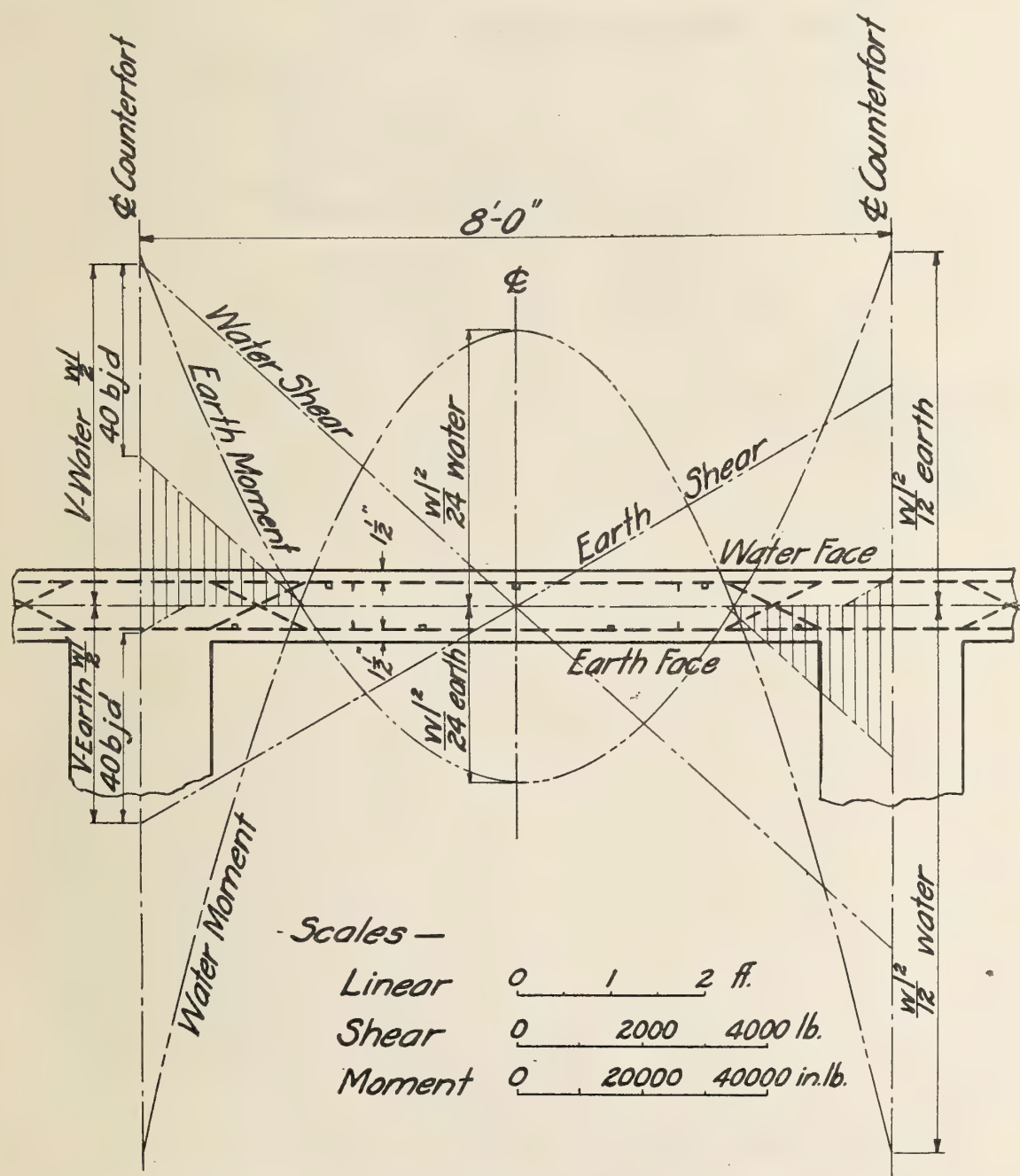


FIG. 1—Moments and Shears in Slab between Counterforts

The second part consisted in working out in considerable detail a series of designs in reinforced concrete for both plain and counterfort walls, determining the quantities of materials necessary therefore per lineal foot and applying to them also, the basic prices referred to above.

The heights for counterfort walls were 20, 24, 28 and 32 feet and for each of these, counterfort spacings of 6, 8, 10, 12, 14 and 16 feet respectively were adopted. Designs for plain reinforced walls were worked out for heights of 16, 20, 24, 28 and 32 feet. The basic prices were: concrete \$12 per cubic yard; steel in place, 3½c. per lb.; forms, 15c. per square foot.

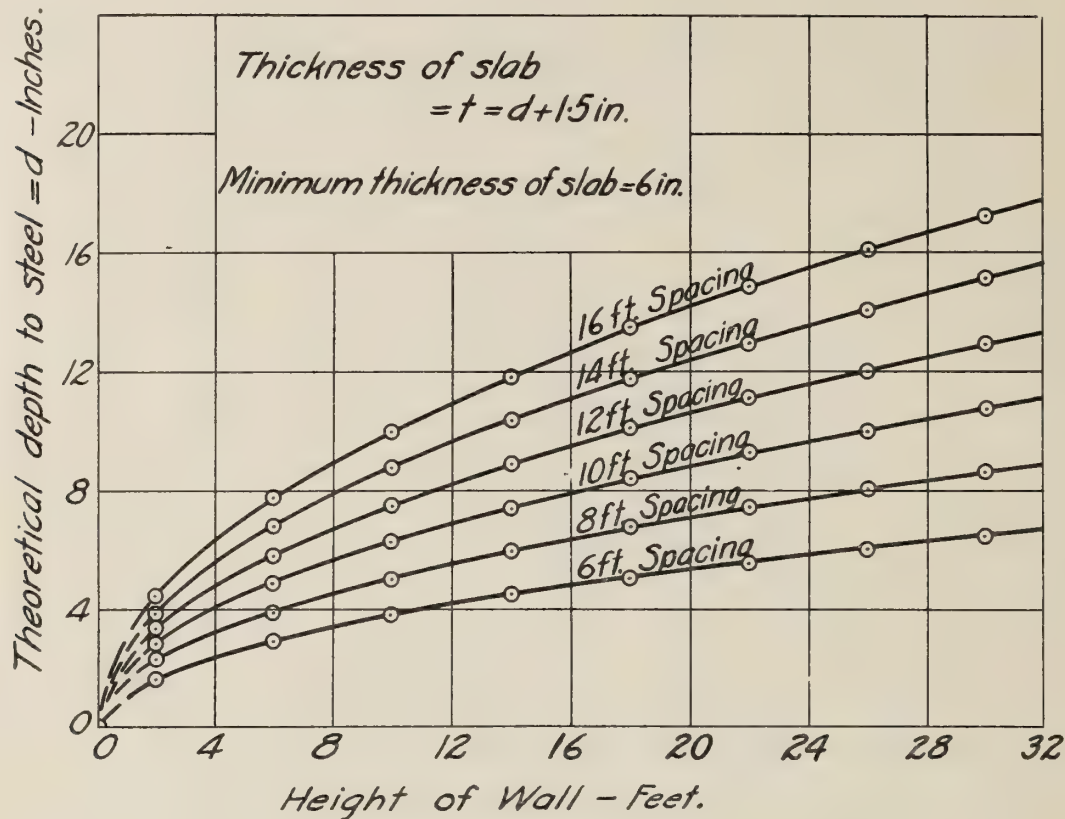


FIG. 2—Counterfort Wall—Slab Thicknesses

Frequently in the design of reservoir walls, the earth back-filling is relied upon in part as offsetting the internal water pressure. Failures of groined arch roofs have resulted from the removal of back-filling which left the unsupported wall to resist the combined roof thrust and water pressure. There are occasions when it is necessary to make extensive excavations on the outside of walls while the reservoir is still full. Shrinkage openings may form between back-fill and the concrete wall, especially if the fill be a clay. Any of these conditions will leave full water pressure on the inside with no opposing support from fill on the outside. At other times the reservoir must be emptied for cleaning, leaving the earth pressure on the outside without compensating water pressure from within. To meet these contingencies, all designs have provided against full water pressure without earth support and for full earth pressure without water support. There is, at the present time, a tendency to increase allowable stresses for the design of reinforced concrete. For structures intended to hold water, it is believed, however, that a conservative policy should be followed. In this study this has

been done by figuring loads with a reasonable precision and designing to stresses which on the whole are moderate.

The wall between counterforts is regarded as a continuous slab spanning from centre to centre of buttresses, any horizontal strip of which is uniformly loaded. For practical reasons a minimum top thickness for this slab of six inches was adopted. Adequate provision is made to bond the slab to both roof and floor.

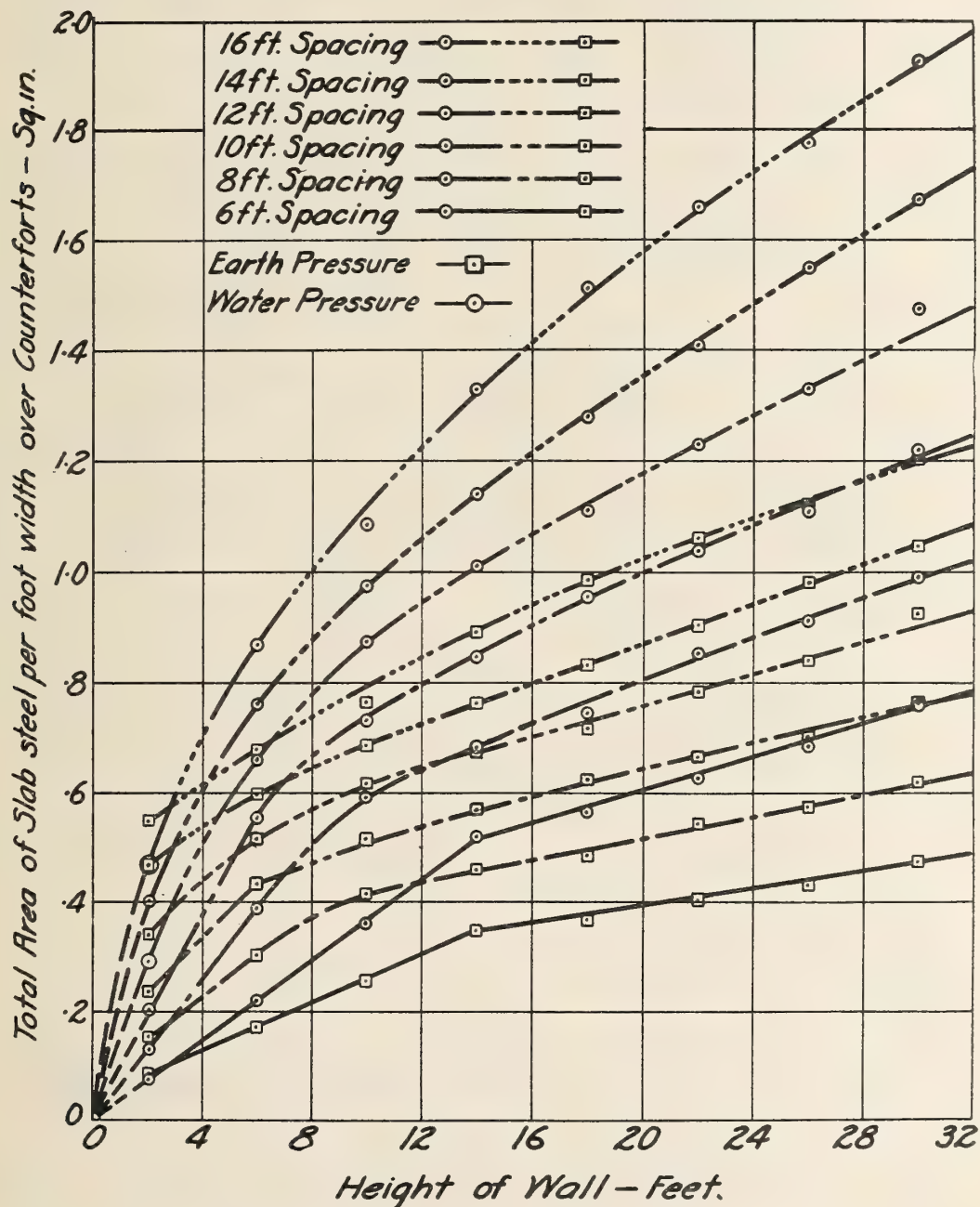


FIG. 3—Counterfort Wall—Slab Reinforcement over Supports

Counterforts can be placed either outside or inside the reservoir. Since the water exerts the greater pressure, the T-beam may with advantage be employed to resist water moment and so counterforts have been placed on the outside. In either position the necessity arises for providing adequate anchorage to both roof and footing. This is accomplished by carrying part of the main reinforcement into the roof

and by placing dowels in the footing. All such ties are carried sufficiently far to develop by bond the full reaction due to water pressure.

Sections of counterforts, top and bottom, were determined from requirements of shear. This gave an area at the bottom double that at

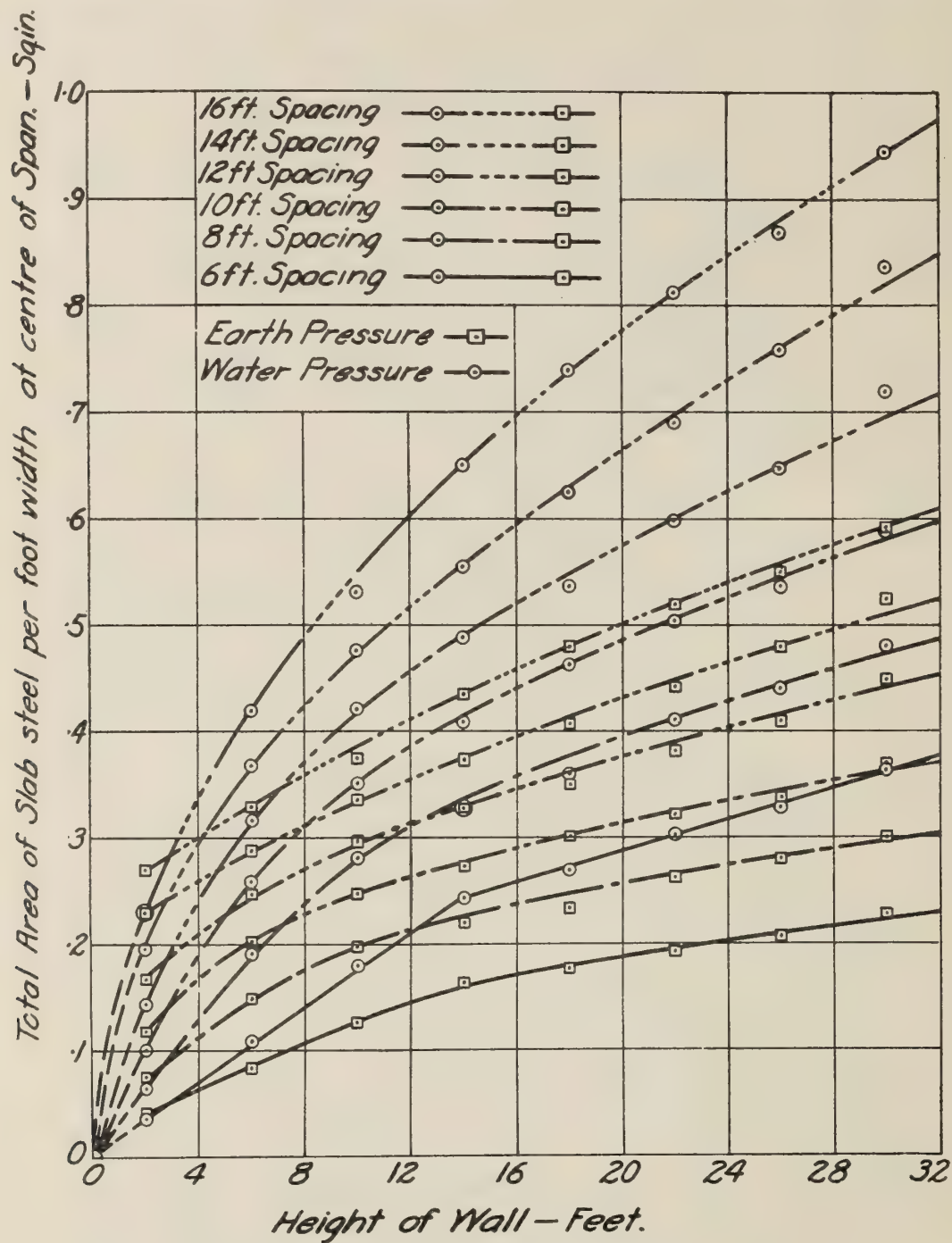


FIG. 4—Counterfort Wall—Slab Reinforcement at Midspan

the top, the middle area being the mean of the two. The determination of the width and depth at any section to some extent depends upon the caprice of the designer. A ratio of depth to width of unity at mid-height (giving corresponding ratios of 2/3 at top and 4/3 at bottom) while satisfying the requirements of shear might conceivably be quite inadequate as to moment-resisting capacity. In other words, the moment to be resisted might produce excessive flexural stress in the

concrete. On the other hand, a ratio of depth to width of 3 at mid-height (equivalent to 4 at bottom and 2 at top) while requiring sensibly the same volume of concrete would mean less steel, lower flexural stress

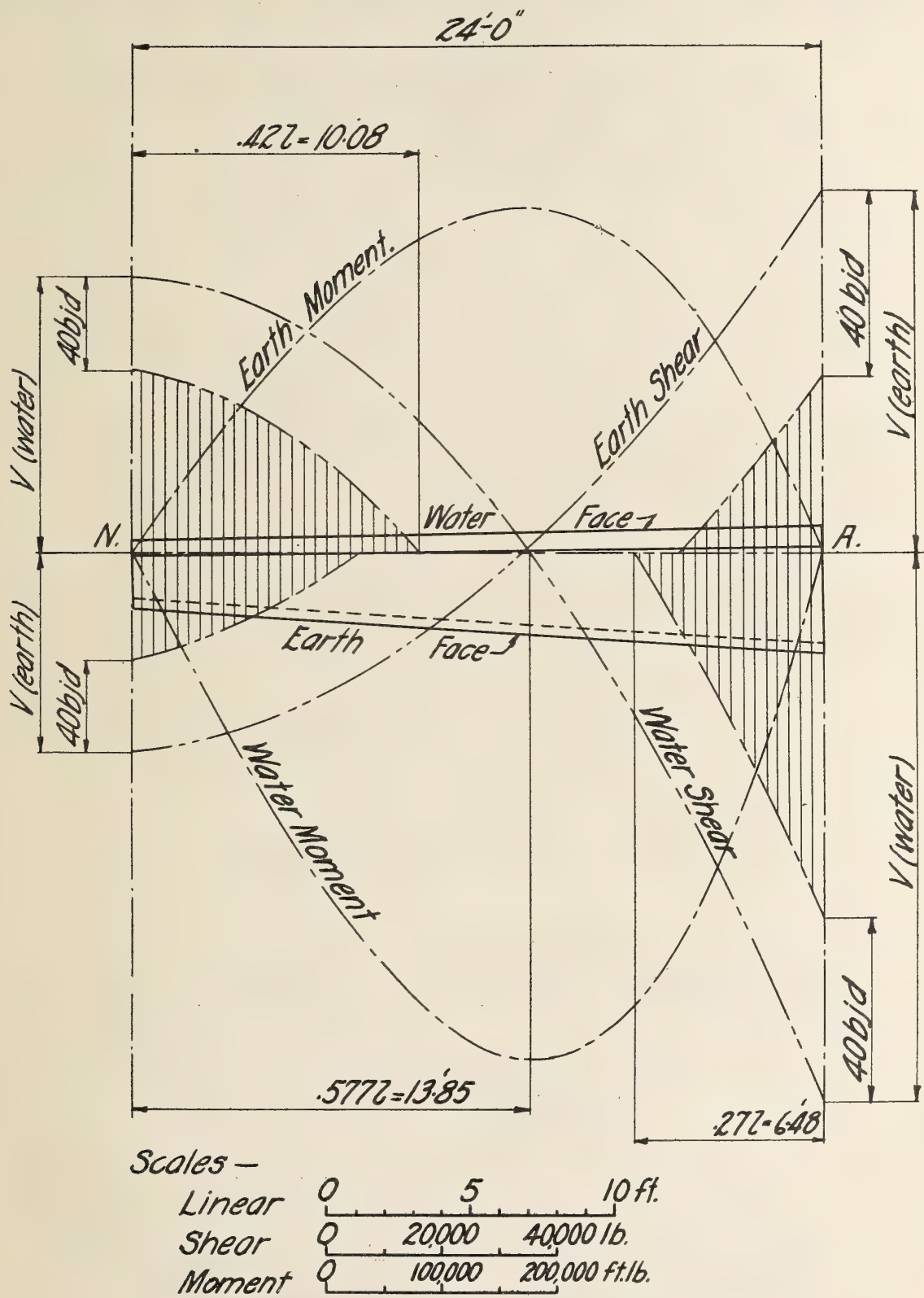
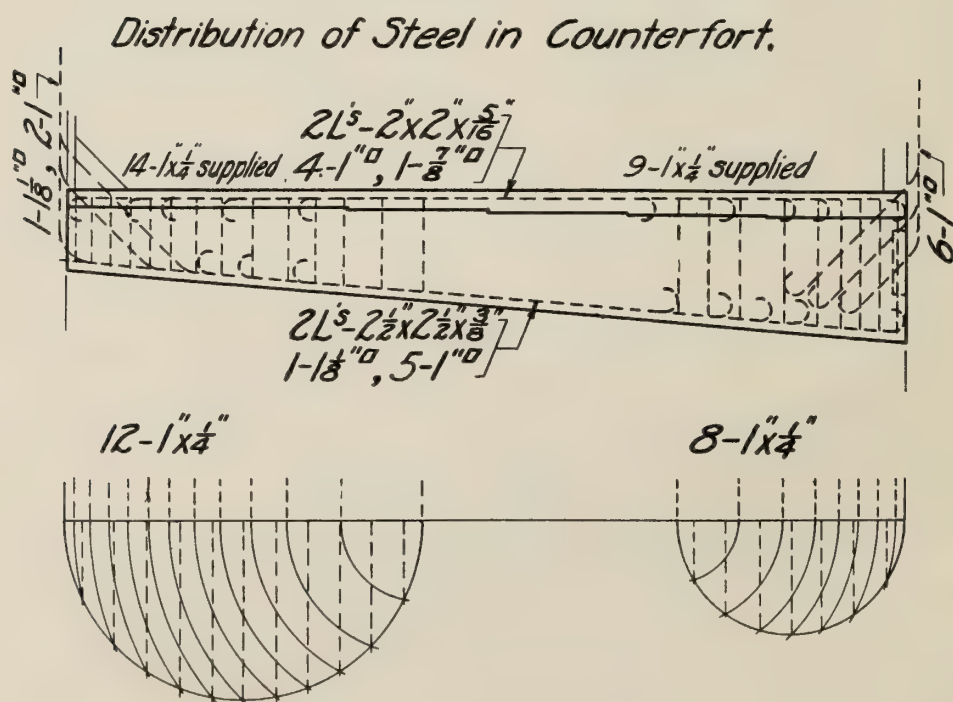
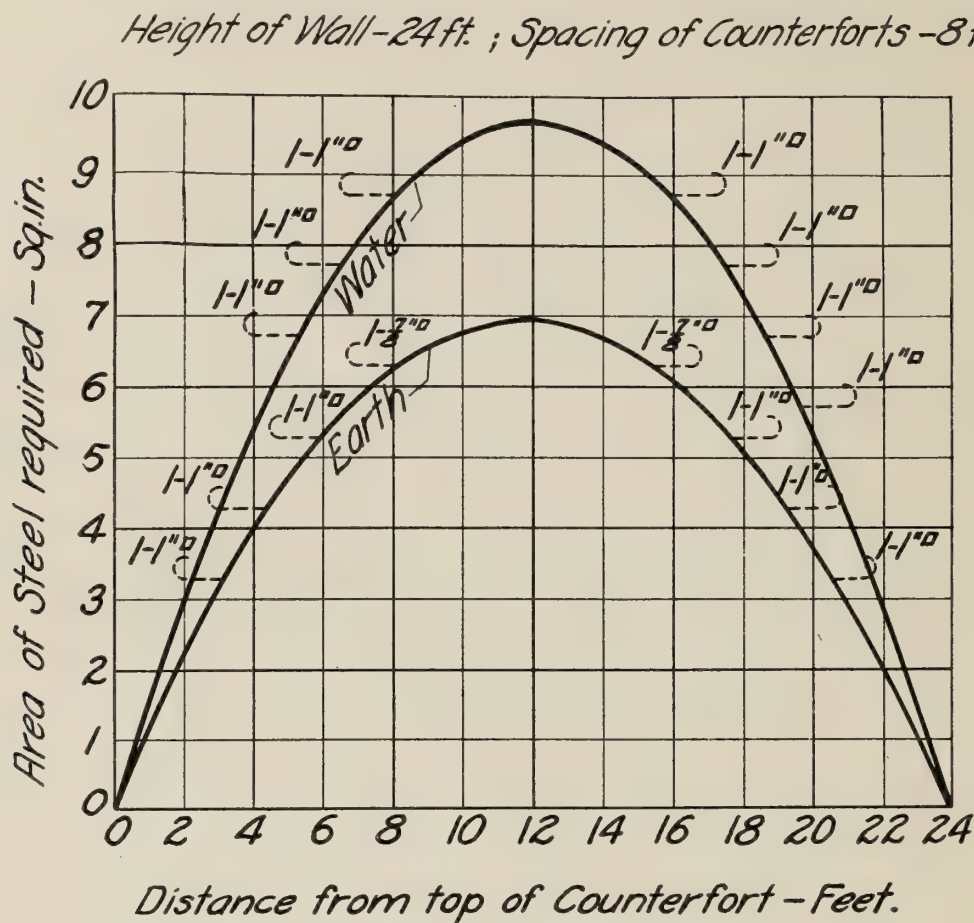


FIG. 5—Moments and Shears in Counterfort—Span 24 ft.

in concrete but more forms. Apparently, then, this increasing of the depth-to-width ratio could be extended more or less indefinitely, each increase representing less metal but greater difficulty in its disposition, lower flexural stress in concrete, increasing area for forms but practically

unchanging volume of concrete. It should be remembered, too, that these remarks apply to earth moment as well as to water moment and that excessive flexural stress in concrete due to employing too small a ratio of depth-to-width, will probably result from the former rather than



Method of spacing Stirrups

FIG. 6—Reinforcement for Counterfort—Span 24 ft.

from the latter. This, of course, is due to the fact that earth moment is resisted by a rectangular beam, not a T-beam. All beams are doubly reinforced and for these the accepted methods of design must be used.

Excessive depth relative to width is to be avoided. A constant ratio, say at midheight, for all counterforts permits quantity and cost comparisons to be made with a measure of fairness. A ratio of 2, which means 2.66 at the bottom, is moderate, does not violate the fashions of accepted practice and for the problem in hand, insured, with only one exception, flexural stresses in concrete less than 650 lb. per sq. in. This was therefore accepted as a standard, although it was recognized that it was not necessarily the best, all things considered. The selection of the most desirable ratio is a complex problem.

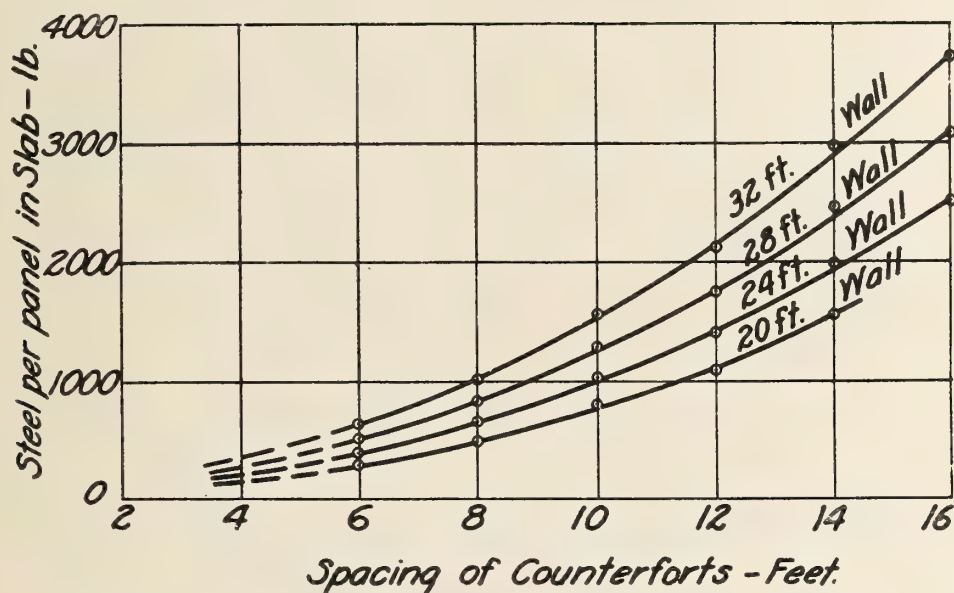


FIG. 7—Counterfort Wall—Weight of Steel in Slab

A feature in the design of the counterforts is that (assuming no restraint top or bottom) while the maximum bending moment due to water occurs at .577*l* from the top, the greatest area of metal occurs at midheight. The general expression for area of metal required at any distance *x* from the top, is $A_w = \frac{12slw_2x(l-x)}{6f_sj d_1}$, where *d*₁ is the depth of

counterfort at top, *s* is spacing centre to centre of counterforts, *l* is the span and *w*₂ the increment in loading per unit of depth. Equating to zero the first differential of *A*_{*w*} with respect to *x*, the position where area of metal is a maximum is found as stated. The surcharge of earth modifies this condition slightly so far as earth moment is concerned, but very slightly in the general case. In a counterfort 24 ft. high, for example, the maximum area of metal occurs 11.71 feet from the top when the surcharge is 2 feet.

Because of the difficulty in placing steel rods of the usual kind, the use of structural angles as part of the reinforcing was preferred. To the

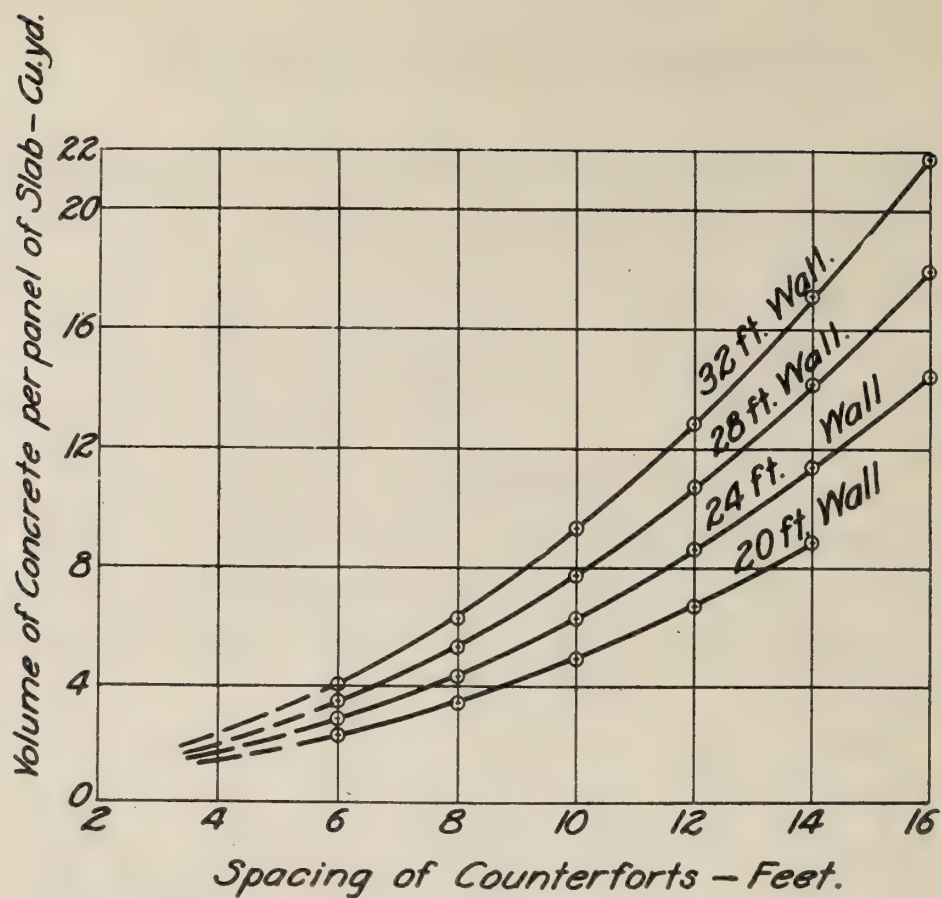


FIG. 8—Counterfort Wall—Volume of Concrete in Slab

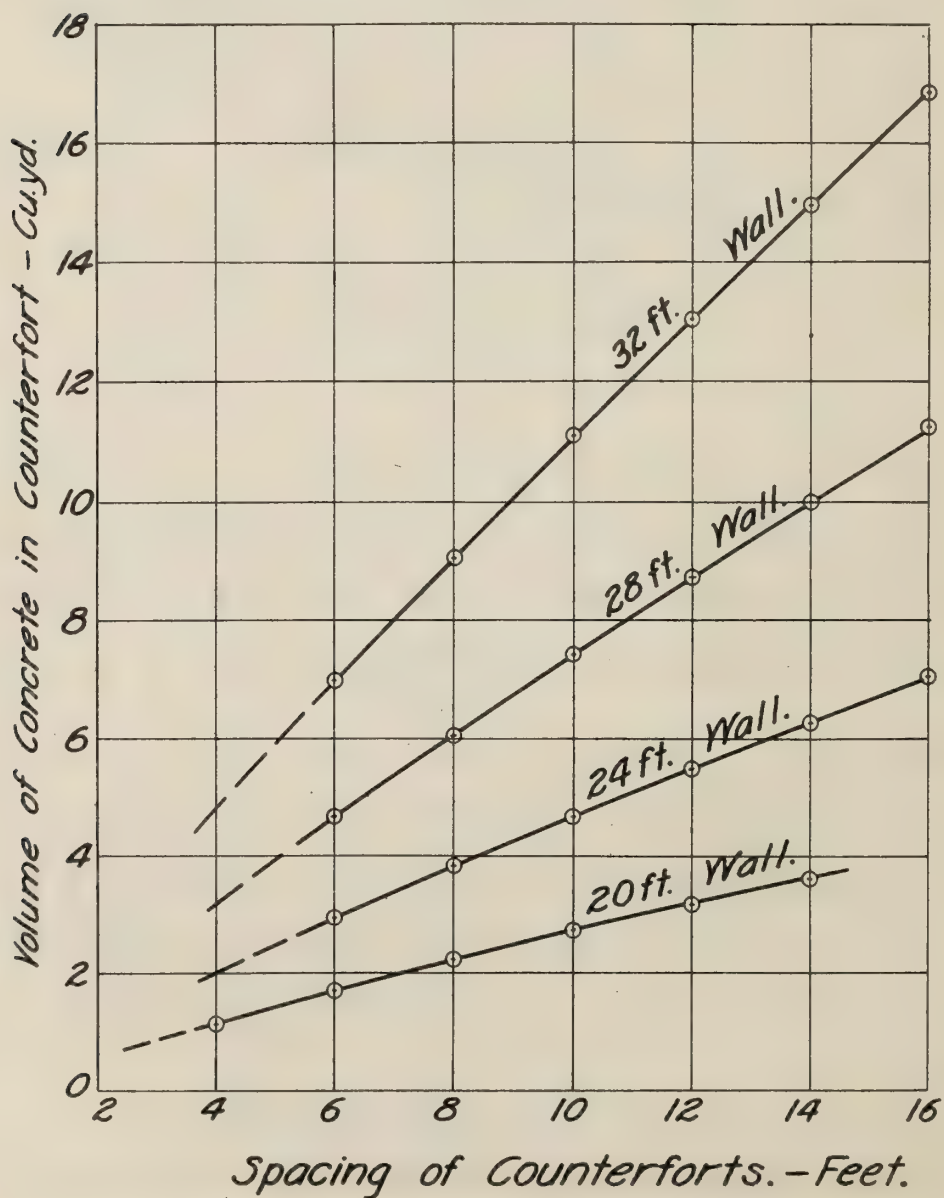


FIG. 9—Counterfort Wall—Volume of Concrete in Counterfort

legs of these, rectangular hoops for web reinforcing can be rigidly fastened. The remainder of the longitudinal steel being wired to the angles, the whole of the reinforcement for one counterfort will be entirely self-supporting and can be framed up and lifted into place as a unit.

Plain concrete footings have been employed and were designed for a soil bearing value of 2,000 lb. per square foot. The footings are assumed to carry the full weight of the wall slab, counterfort, concrete roof and two feet of earth fill. The ratio of depth to projection from the face of wall gives a 1:1 slope from the toe. Thus an area of critical shear is eliminated. Reinforcement is not required.

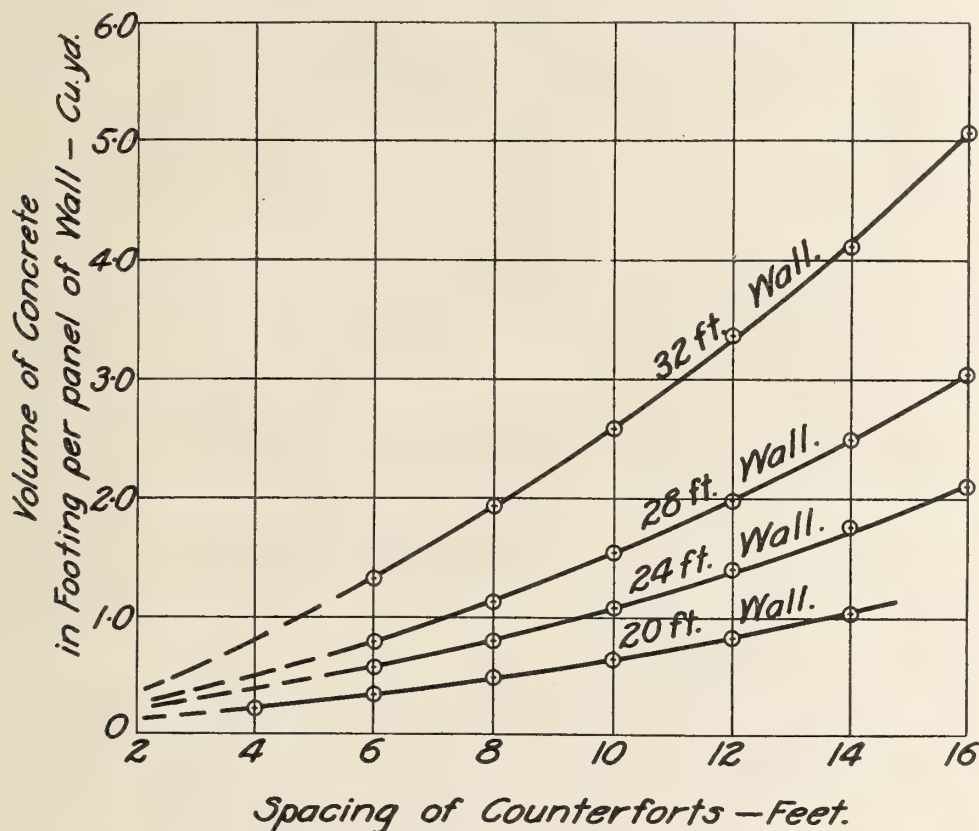


FIG. 10—Counterfort Wall—Volume of Concrete in Footings

Reservoirs, as a rule, are constructed at such an elevation that the excavation and fill will balance. That is, the footings are excavated to about half the depth of the wall below normal ground level. If the policy of sloping up the floor in the wall panel be adopted, the amount of excavation may favour the gravity wall, otherwise the counterfort wall will require less excavation.

It was expected that for a given condition there would be a determinate spacing of counterforts for which the cost per running foot of wall would be a minimum. This has been confirmed by the graphs showing the cost per foot of wall. The various points fall very close to a smooth curve and show marked inclination to rise toward a zero spacing of counterforts. This latter to a degree is confirmed by the rapid rise in

the cost curve for the plain reinforced wall which might be considered as a counterfort wall with counterforts spaced infinitely close. In addition to these cost graphs, a series of charts have been prepared from which the quantities of materials for any particular wall and spacing of counterforts can be easily determined.

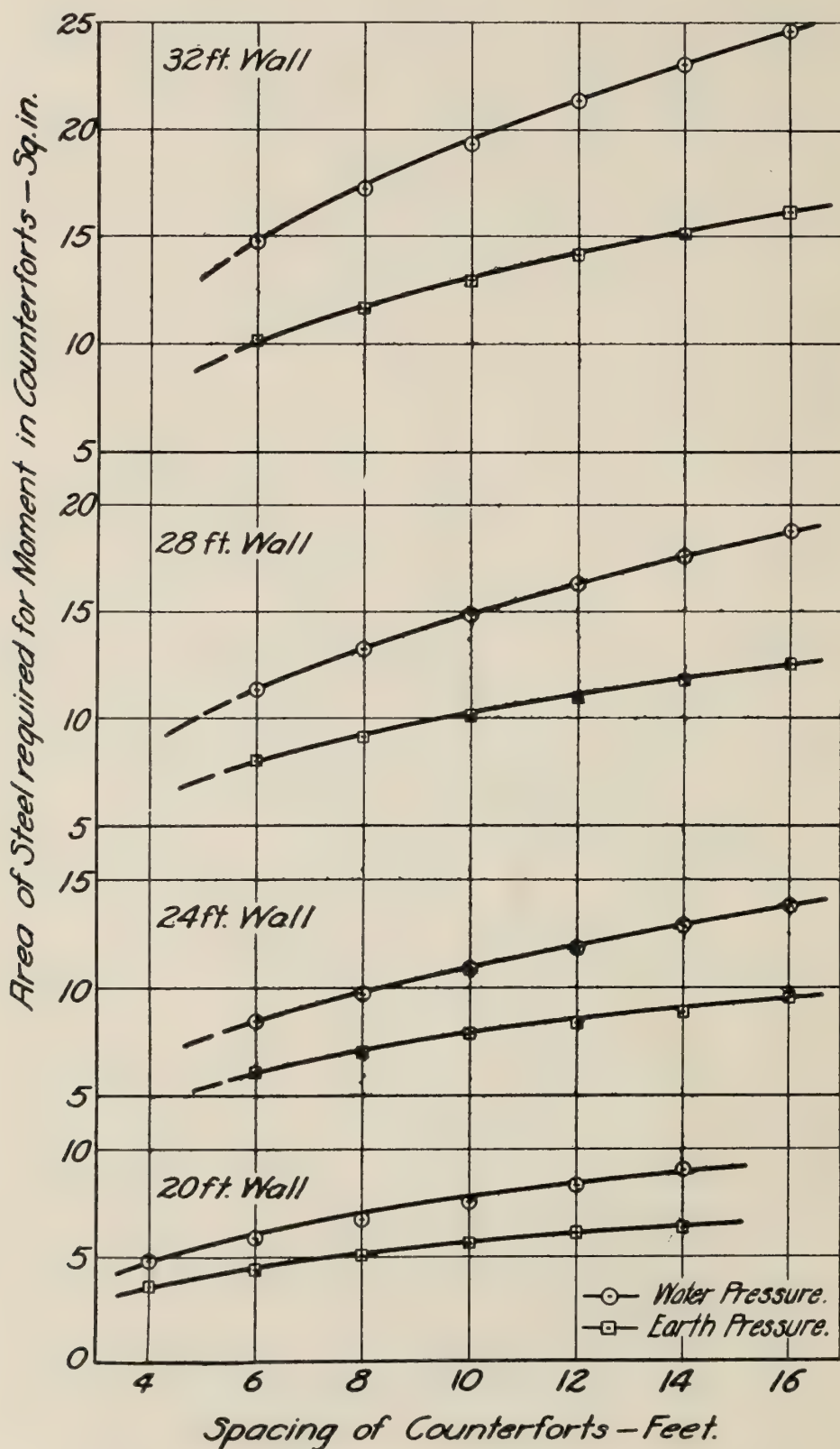


FIG. 11—Area of Steel in Counterforts

SUMMARY

- (1) For a given load and span the costs of groined arch and flat slab roofs are practically the same.
- (2) For a given height of wall the excavation is less for a counterfort wall than for a gravity wall.
- (3) For walls exceeding 16 ft. in height the cost is less for the counterfort wall than for the plain reinforced wall.
- (4) For low walls the advantage lies with the plain reinforced wall.
- (5) For the conditions assumed, minimum cost in counterfort walls occurs when counterforts are spaced about 8 feet centre to centre.
- (6) The combination of a flat slab or beam and slab roof with either a plain reinforced or counterfort wall, being self-supporting without the assistance of an earth back-fill, is one that should commend itself to engineers because of its safety and economy.

Typical computations and estimates are appended.

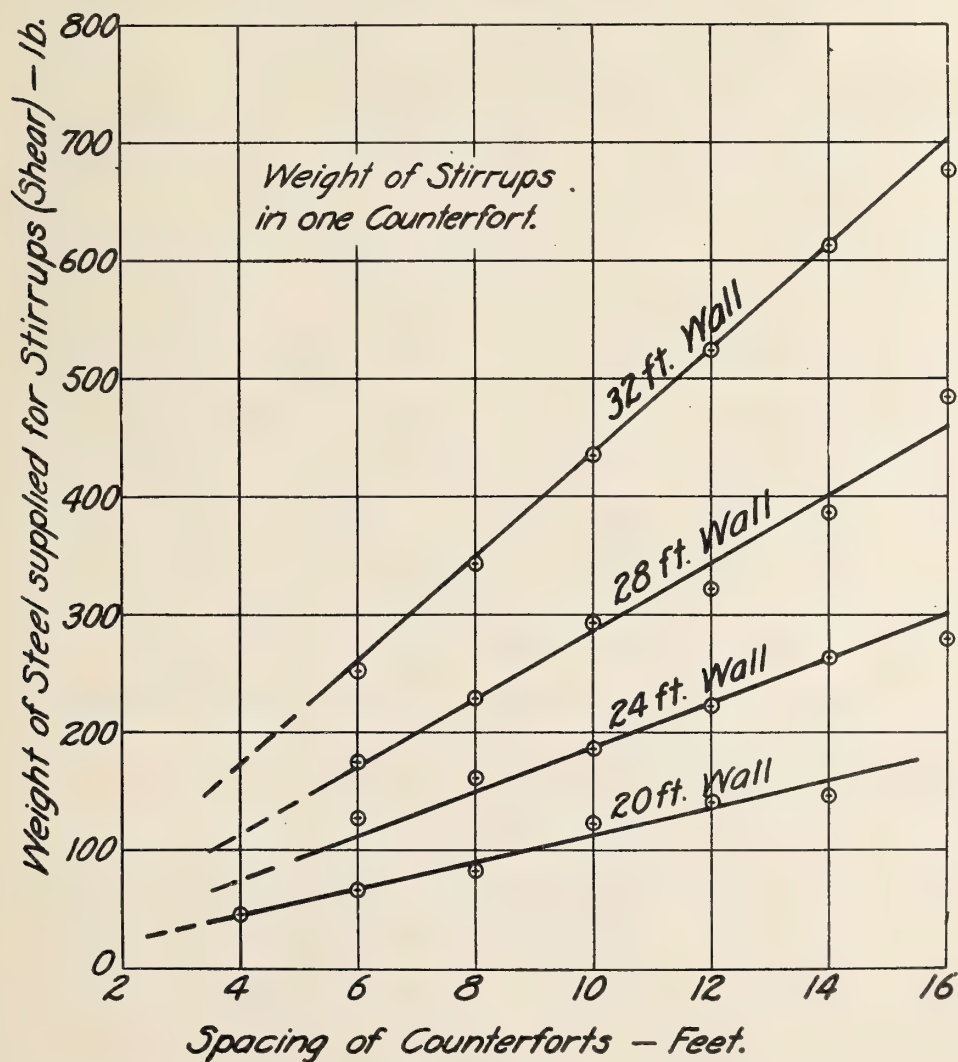


FIG. 12—Counterfort Wall—Weight of Stirrups in Counterforts

TYPICAL COMPUTATION FOR COUNTERFORT WALL

Assumptions

Height of wall, 24 ft.

Spacing of counterforts, 8 ft.

The following assumptions have been made:

1. The counterforts are unrestrained beams spanning from roof to footing.
2. The intervening wall is a continuous slab spanning from counterfort to counterfort, any horizontal strip of which is uniformly loaded.
3. The wall must support a full reservoir of water without assistance from earth backing.
4. The wall must support a full earth load without support from water on the inside.
5. (a) Weight of water = 62.5 lb. per cu. ft.
 (b) Weight of earth = 110 lb. per cu. ft.
 (c) Intensity of lateral earth pressure = $1/3$ of vertical pressure.

Symbols

- l , Span of beam.
- x , distance measured along counterfort from top support.
- d , depth of beam.
- d' , distance of compression steel from compression face of beam.
- t , overall thickness of slab.
- u , bond stress.
- w , uniformly distributed load, per unit.
- w_1 , increment in load per unit, due to earth.
- w_2 , increment in load per unit, due to water.
- q , uniform load per unit due to earth surcharge.
- A_w , area of tension metal necessitated by water moment.
- A_e , area of tension metal necessitated by earth moment.
- M_w , bending moment due to water.
- M_e , bending moment due to earth.
- R , ratio of moment to product of width and square of depth of beam.
- p , area of tension metal relative to beam section.
- p' , area of compression metal relative to beam section.
- V_w , shear due to water.
- V_e , shear due to earth.
- V , residual shear carried by steel.

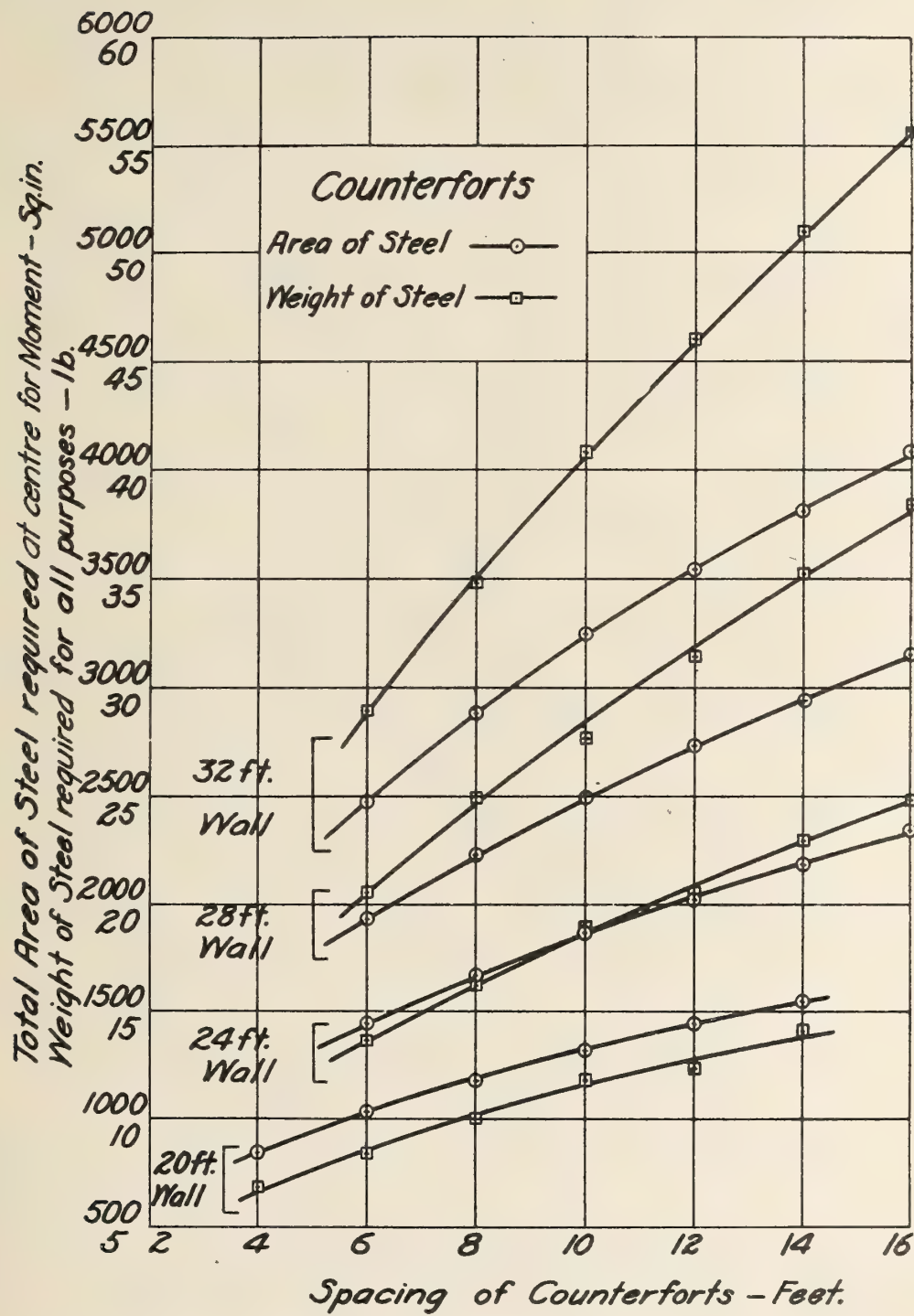


FIG. 13—Total Area and Total Weight of Steel per Counterfort

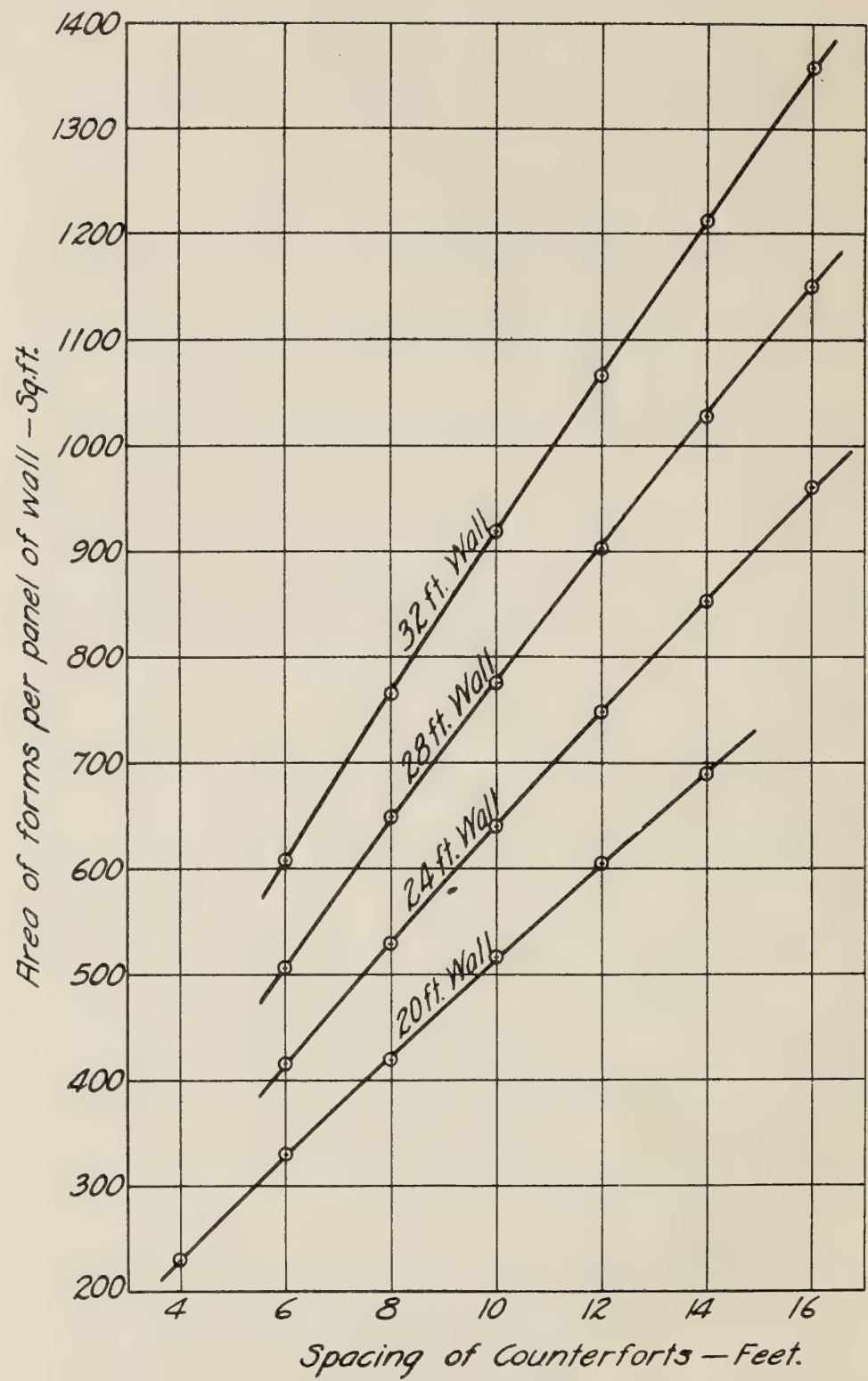


FIG. 14—Counterfort Wall—Area of Forms per Panel

Permissible Stresses

f_s , 16,000 lb. per sq. in.

f_c , 650 lb. per sq. in. increasing to 750 lb. per sq. in. at supports.

Shear, 120 lb. per sq. in.; taken by concrete 40 lb. per sq. in.

Bond, 80 lb. per sq. in.

Slab

The assumed bending moments are $1/24 wl^2$ at centre of span and $1/12 wl^2$ over supports.

Consider a section of slab 4 ft. deep at bottom of wall.

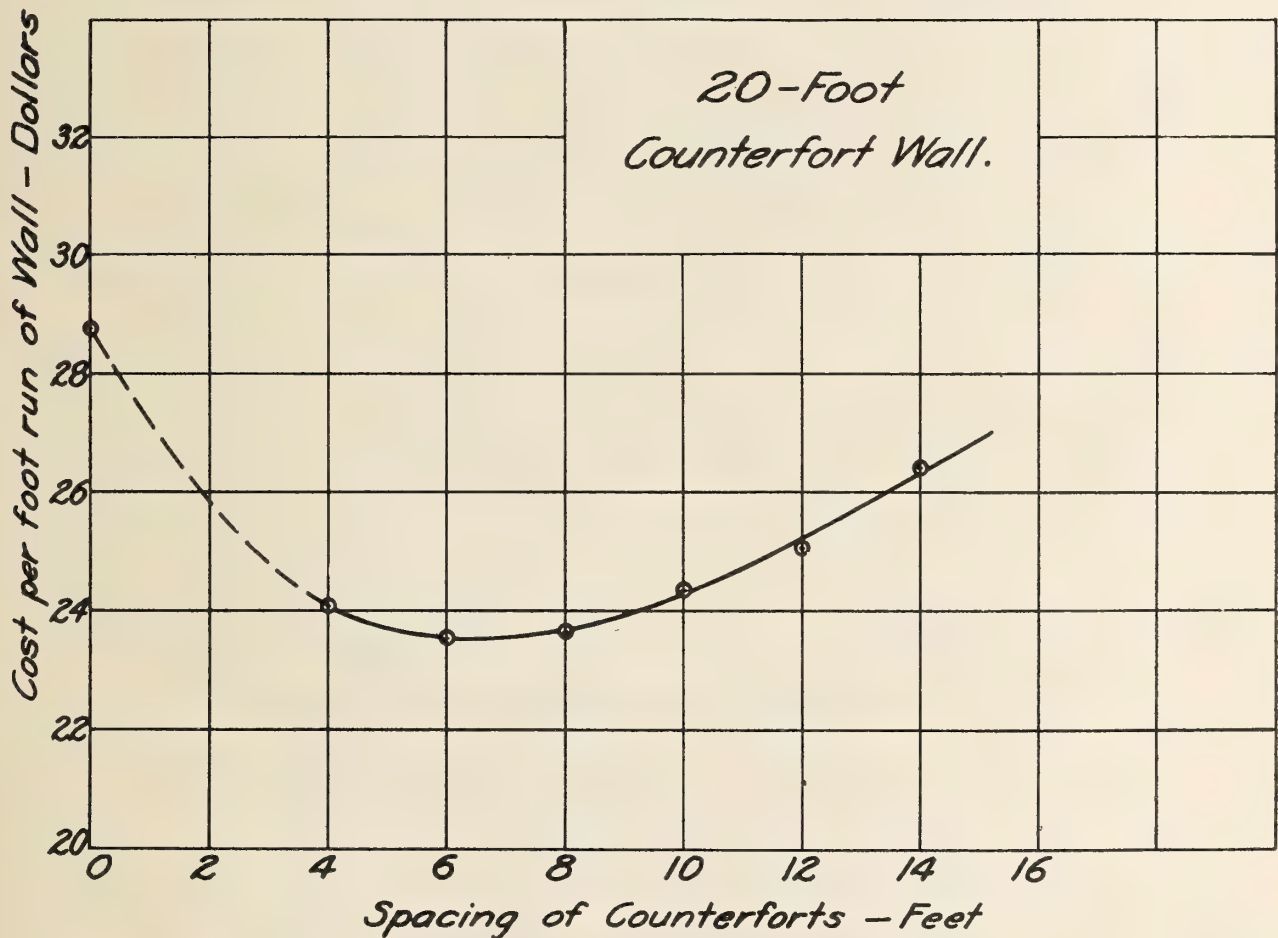


FIG. 15—Counterfort Walls—20 ft. high—Theoretic Cost per foot of Wall

Water Pressure

Average depth of water on this section = 22 ft.

$$M_w \text{ per foot width over support} = \frac{62.5 \times 22 \times 8 \times 8 \times 12}{12} = 88,000 \text{ in.-lb.}$$

Tentatively, $R = 133.5$; $j = .862$.

$$\text{Then } d^2 = \frac{88,000}{133.5 \times 12}$$

$$\text{and } d = \sqrt{\frac{88,000}{133.5 \times 12}} = 7.4 \text{ in., say } 7.5 \text{ in.}$$

$$t = 7.5 + 1.5 = 9.0 \text{ in. overall.}$$

$$A_w = \frac{88,000}{16,000 \times .862 \times 7.5} = .85 \text{ sq. in.}$$

$$p = \frac{.85}{12 \times 7.5} = .00945.$$

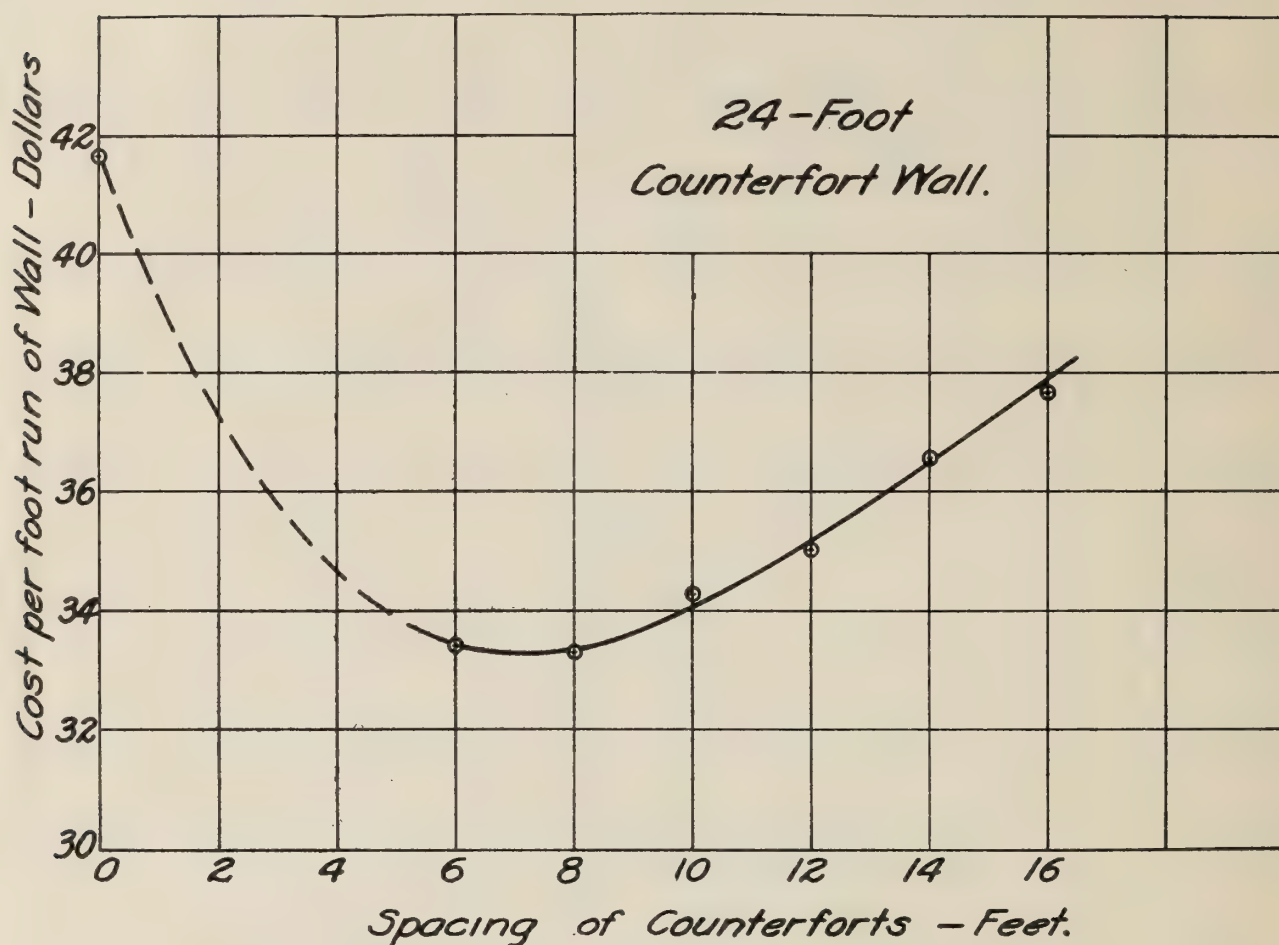


FIG. 16—Counterfort Walls 24 ft. high—Theoretic Cost per foot of Wall

Earth Pressure

Consider same section and moment at support due to earth.

$$M_e = \frac{110}{3} \times \frac{24 \times 8 \times 8 \times 12}{12} = 56,320 \text{ in.-lb.}$$

$$A_e = \frac{56,320}{16,000 \times .862 \times 7.5} = .544 \text{ sq. in.}$$

$$p = \frac{.544}{12 \times 7.5} = .00604.$$

For water
 $p' = .00604$
 $p = .00945$
 $d'/d = .2$
 Then $j = .86$

For earth
 $p' = .00945$
 $p = .00604$
 $d'/d = .2$
 Then $j = .87$

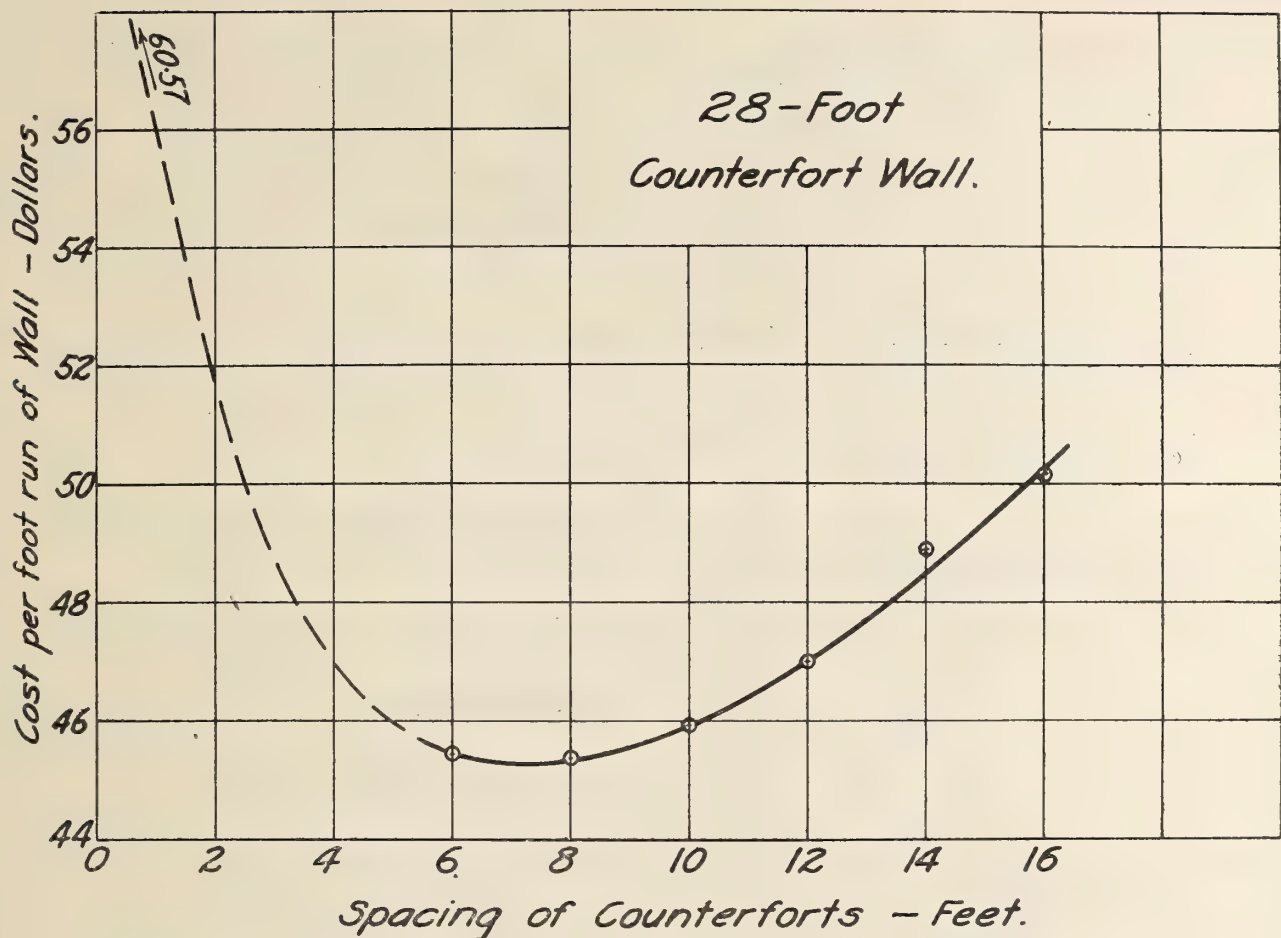


FIG. 17—Counterfort Walls 28 ft. high—Theoretic Cost per foot of Wall

Revising,

$$A_w = \frac{88,000}{16,000 \times .86 \times 7.5} = .853 \text{ sq. in.}; p = .00947.$$

$$A_e = \frac{56,320}{16,000 \times .87 \times 7.5} = .538 \text{ sq. in.}$$

Hence $R = pf_s j = 130.31$ for water

$$\text{and } d = \sqrt{\frac{M_w}{Rb}} = \sqrt{\frac{88,000}{130.31 \times 12}} = \sqrt{56.276} = 7.5 \text{ in.}$$

$$M_w \text{ per foot width at midspan} = \frac{62.5 \times 22 \times 8 \times 8 \times 12}{24} = 44,000 \text{ in.-lb.}$$

$$A_w = \frac{44,000}{16,000 \times .874 \times 7.5} = .42 \text{ sq. in.}$$

$$p = \frac{.42}{12 \times 7.5} = .00466.$$

$$M_e = \frac{110}{3} \times \frac{24 \times 8 \times 8 \times 12}{24} = 28,160 \text{ in.-lb.}$$

$$A_e = \frac{28,160}{16,000 \times .874 \times 7.5} = .268 \text{ sq. in.}$$

$$p = \frac{.268}{12 \times 7.5} = .00298.$$

For water

$$p' = .00298$$

$$p = .00466$$

$$d'/d = .2$$

$$\text{Then } j = .89$$

For earth

$$p' = .00466$$

$$p = .00298$$

$$d'/d = .2$$

$$\text{Then } j = .90$$

Revising,

$$A_w = \frac{44,000}{16,000 \times .89 \times 7.5} = .411 \text{ sq. in.}$$

$$A_e = \frac{28,160}{16,000 \times .90 \times 7.5} = .260 \text{ sq. in.}$$

Vertical reinforcement in the slab was provided to the extent of 25% of that required for bending.

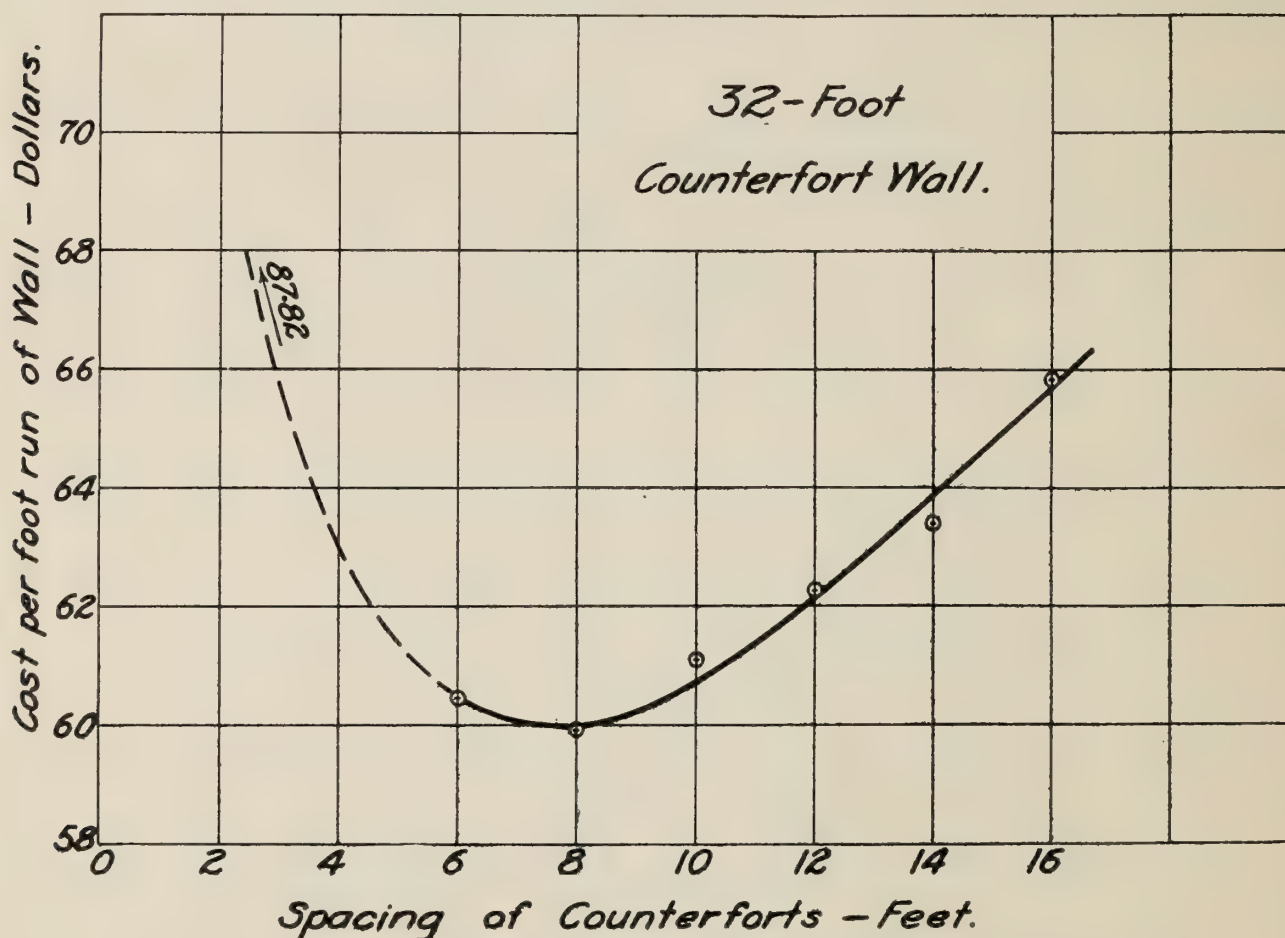


FIG. 18—Counterfort Walls 32 ft. high—Theoretic Cost per foot of Wall

Quantities of material in lowest 4 ft. of wall

Steel:

Vertical — 7 pieces $\frac{1}{2}'' \square \times 5' - 0'' = 35.0 \text{ ft.}$

Water face 6 " $\frac{1}{2}'' \square \times 8' - 0'' = 48.0 \text{ ft.}$

2 " $\frac{1}{2}'' \square \times 5' - 6'' = 11.0 \text{ ft.}$

6 " $\frac{1}{2}'' \square \times 4' - 0'' = 24.0 \text{ ft.}$

$$\begin{aligned} &118.0 \text{ ft. at } .85 \text{ lb. per ft.} \\ &= 100.30 \text{ lb.} \end{aligned}$$

Earth face—12 pieces $\frac{3}{8}'' \square \times 8'-0'' = 96.0$ ft.
 2 " $\frac{3}{8}'' \square \times 5'-6'' = 11.0$ ft.
 2 " $\frac{3}{8}'' \square \times 4'-0'' = 8.0$ ft.

115.0 ft. at .478 lb. per ft.

= 54.97 lb.

Total = 155.27 lb.

Concrete, .89 cu. yd.

Forms, 59.0 sq. ft.

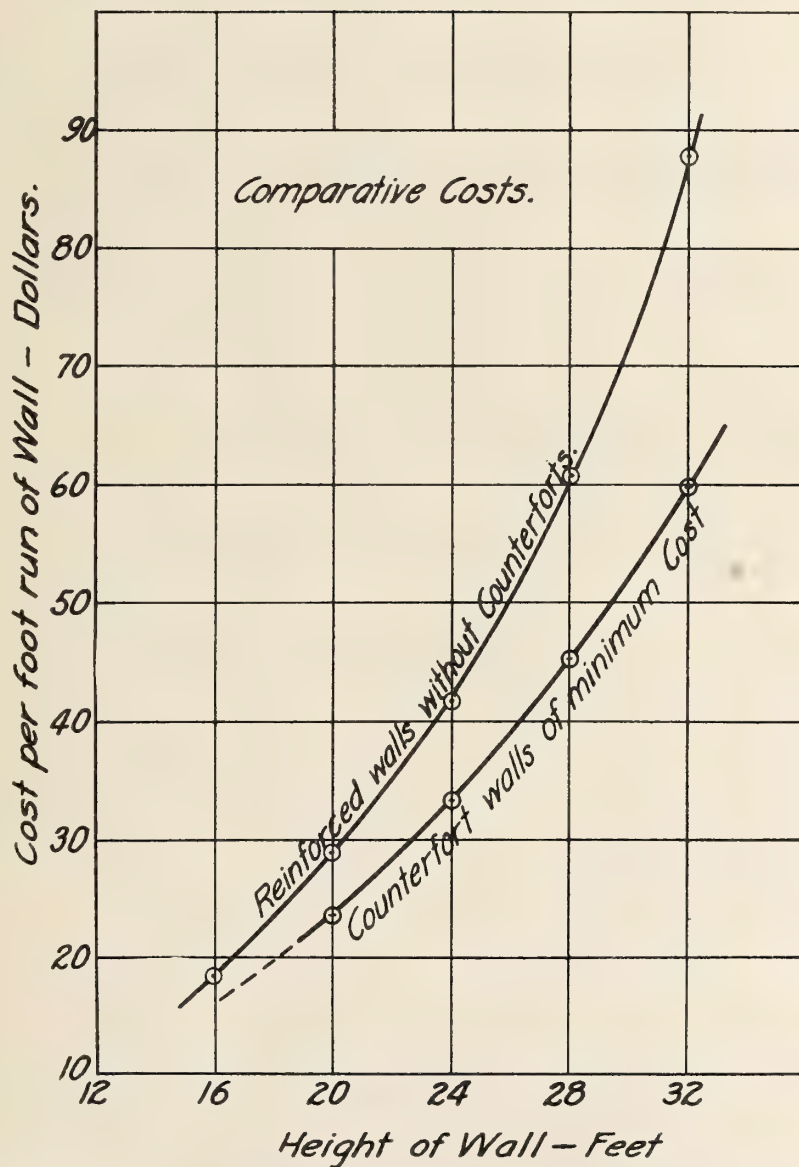


FIG. 19—Walls with and without Counterforts—Comparative Costs

Counterfort

Depth of water, 24 ft.; depth of earth, 26 ft.

Against water pressure the counterfort is a T-beam of varying depth spanning from roof to footing and resisting a lateral load from a section of wall 8 ft. $\times$ 24 ft. The load intensity varies from zero at the top to a maximum at the bottom.

Against earth pressure it is a rectangular beam. It resists, however, a uniformly distributed load due to the surcharge in addition to a varying load similar to the other.

Water Pressure

$$\begin{aligned}\text{Shear at top due to water} &= V_w = \frac{8w_2l^2}{6} \\ &= \frac{1}{6} \times 62.5 \times 8 \times 24 \times 24 = 48,000 \text{ lb.}\end{aligned}$$

$$\begin{aligned}\text{Shear at bottom due to water} &= V_w = \frac{8w_2l^2}{3} \\ &= \frac{1}{3} \times 62.5 \times 8 \times 24 \times 24 = 96,000 \text{ lb.}\end{aligned}$$

$$\text{Area of beam section required at top} = \frac{48,000}{120 \times .92} = 436.3 \text{ sq. in.}$$

$$\text{Area of beam section required at bottom} = \frac{96,000}{120 \times .92} = 872.7 \text{ sq. in.}$$

To maintain a depth-to-width ratio of 2 at midheight, use
 a top section = $18.15 \times 24.2 = 439.23$ sq. in.,
 a bottom section = $18.15 \times 48.4 = 878.46$ sq. in. and
 a middle section = $18.15 \times 36.3 = 658.84$ sq. in.

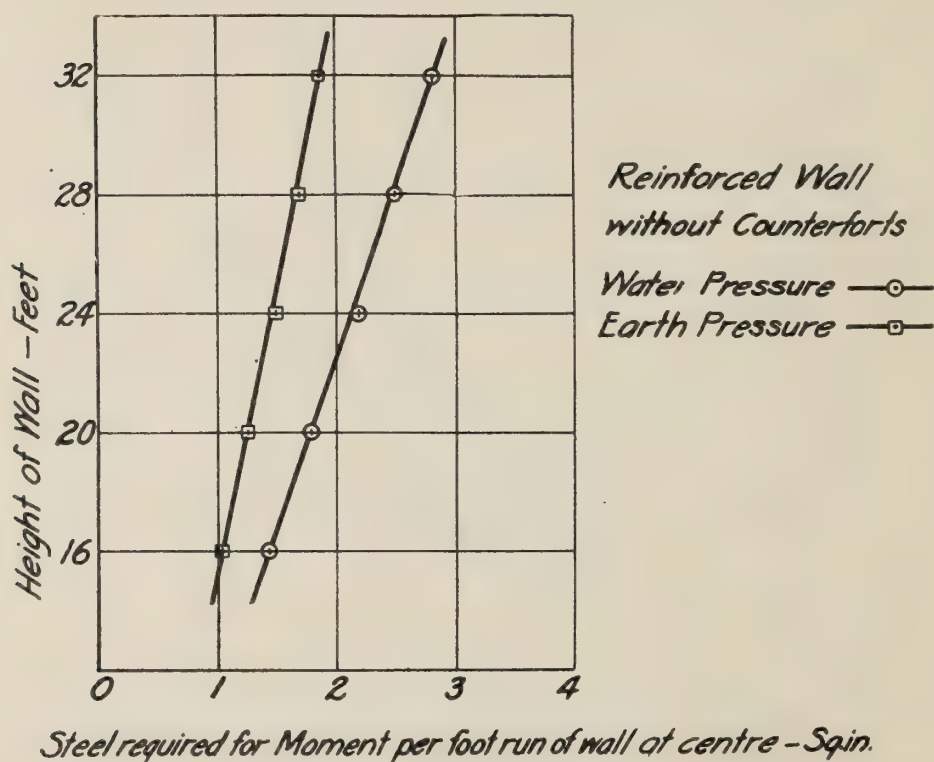


FIG. 20—Reinforced Walls without Counterforts—Steel required

$$\begin{aligned}\text{Moment at centre of counterfort due to water, } M_w &= \frac{8w_2l^3}{16} \\ &= \frac{62.5 \times 8 \times 24 \times 24 \times 24}{16} = 432,000 \text{ ft.-lb.} \\ &= 5,184,000 \text{ in.-lb.}\end{aligned}$$

$$\frac{t}{d} = \frac{6.5}{36.3} = .179.$$

Permissible width of flange = $\frac{1}{4}$ span = 72 in.

$$R = \frac{M_w}{bd^2} = \frac{5,184,000}{72 \times (36.3)^2} = 54.6.$$

For $\frac{t}{d} = .179$ and $R = 54.6$, $f_c = 465$ and $j = .92$.

$$A_w = \frac{5,184,000}{16,000 \times .92 \times 36.3} = 9.71 \text{ sq. in.} \quad p = .01475.$$

Area of Steel Supplied = 2 $\angle$'s. $2 \frac{1}{2}'' \times 2 \frac{1}{2}'' \times \frac{3}{8}'' = 3.46 \text{ sq. in.}$

$$1 - 1 \frac{1}{8}'' \square = 1.26 \text{ " "}$$

$$5 - 1'' \square = 5.00 \text{ " "}$$

$$\text{Total} = 9.72 \text{ " "}$$

$$\text{Perimeter required to develop bond stress} = \sum O = \frac{96,000}{80 \times .92 \times 48.4} = 27 \text{ in.}$$

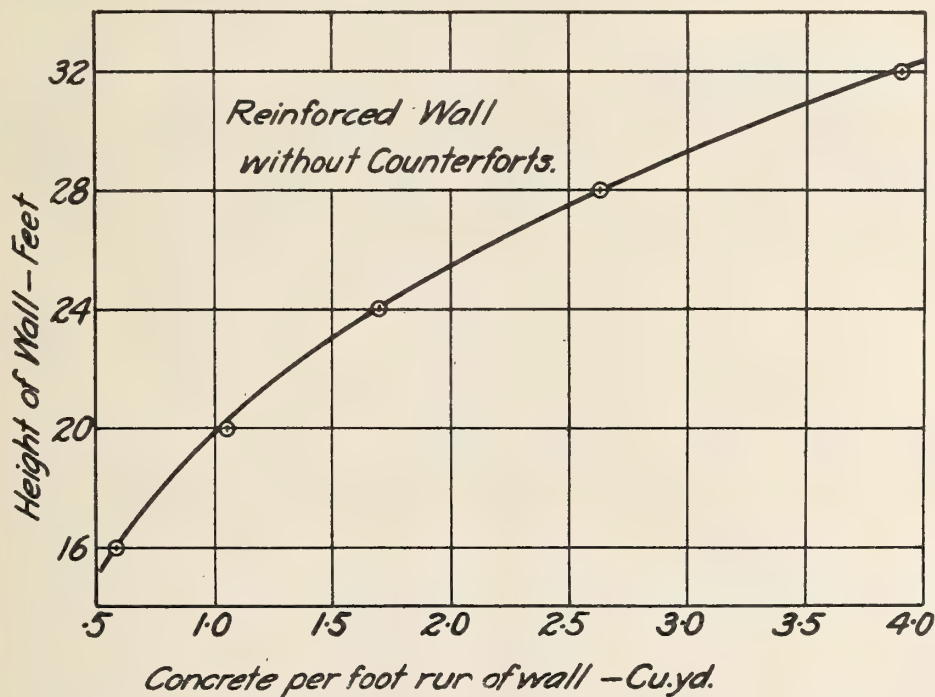


FIG. 21—Reinforced Walls without Counterforts—Concrete in Walls

Earth Pressure

$$\text{Shear at top, } V_e = 8 \left(\frac{ql}{2} + \frac{w_1 l^2}{6} \right) = 8 \left(\frac{73.3 \times 24}{2} + \frac{880 \times 24}{6} \right) = 35,200 \text{ lb.}$$

$$\text{Shear at bottom, } V_e = 8 \left(\frac{ql}{2} + \frac{w_1 l^2}{3} \right) = 8 \left(\frac{73.3 \times 24}{2} + \frac{880 \times 24}{3} \right) = 63,360 \text{ lb.}$$

Bending moment at distance x from top

$$M_e = 8 \left(\frac{qlx}{2} + \frac{w_1 l^2 x}{6} - \frac{qx^2}{2} - \frac{w_1 x^3}{6} \right)$$

For $x = l/2$,

$$M_e = 8 \left(\frac{ql^2}{4} + \frac{w_1 l^3}{12} - \frac{ql^2}{8} - \frac{w_1 l^3}{48} \right)$$

$$= 8 \left(\frac{73.3 \times 24 \times 24}{8} + \frac{110}{3} \times \frac{24 \times 24 \times 24}{16} \right) = 295,660 + 1.16.$$

$$= 3,547,900 \text{ in.-lb.}$$

$$A_e = \frac{3,547,900}{16,000 \times .874 \times 36.3} = 7.0 \text{ sq. in.}$$

$$p = .0106$$

For earth

$$p' = .01475$$

$$p = .0106$$

$$d'/d = .124$$

$$\text{Then } j = .882$$

Revising,

$$A_e = \frac{3,547,900}{16,000 \times .882 \times 36.3} = 6.93 \text{ sq. in.}$$

$$p = .0105$$

Finally for earth

$$p' = .01475$$

$$p = .0105$$

$$d'/d = .124$$

$$\text{Then } j = .882$$

$$k = .34$$

$$f_e = 550$$

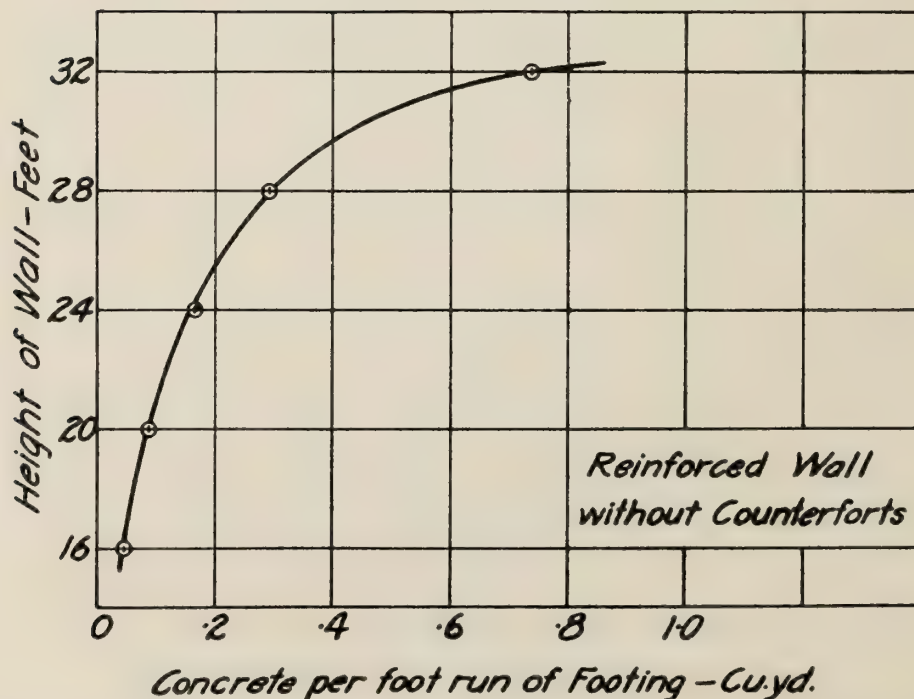


FIG. 22—Reinforced Walls without Counterforts—Concrete in Footings

Area of steel supplied:

$$\begin{array}{rcl}
 2 \angle's - 2'' \times 2'' \times 5/16'' & = & 2.30 \text{ sq. in.} \\
 4 - 1''\square & = & 4.00 \text{ " " } \\
 1 - \frac{7}{8}''\square & = & .76 \text{ " " } \\
 \hline
 & = & 7.06 \text{ " " }
 \end{array}$$

Bond Stress

$$\text{Perimeter required for bond} = \sum O = \frac{V_e}{u_j d} = \frac{63,360}{80 \times .882 \times 48.4} = 18.5 \text{ in.}$$

Perimeter supplied, water:

$$\begin{array}{rcl}
 2 \angle's - 2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{3}{8}'' & = & 20.0 \text{ in.} \\
 1 - 1\frac{1}{8}''\square + 1 - 1''\square & = & 8.5 \text{ in.} \\
 \hline
 \text{Total} & = & 28.5 \text{ in.}
 \end{array}$$

Perimeter supplied, earth:

$$\begin{array}{rcl}
 2 \angle's - 2'' \times 2'' \times 5/16'' & = & 16.0 \text{ in.} \\
 1 - 1''\square & = & 4.0 \text{ in.} \\
 \hline
 \text{Total} & = & 20.0 \text{ in.}
 \end{array}$$

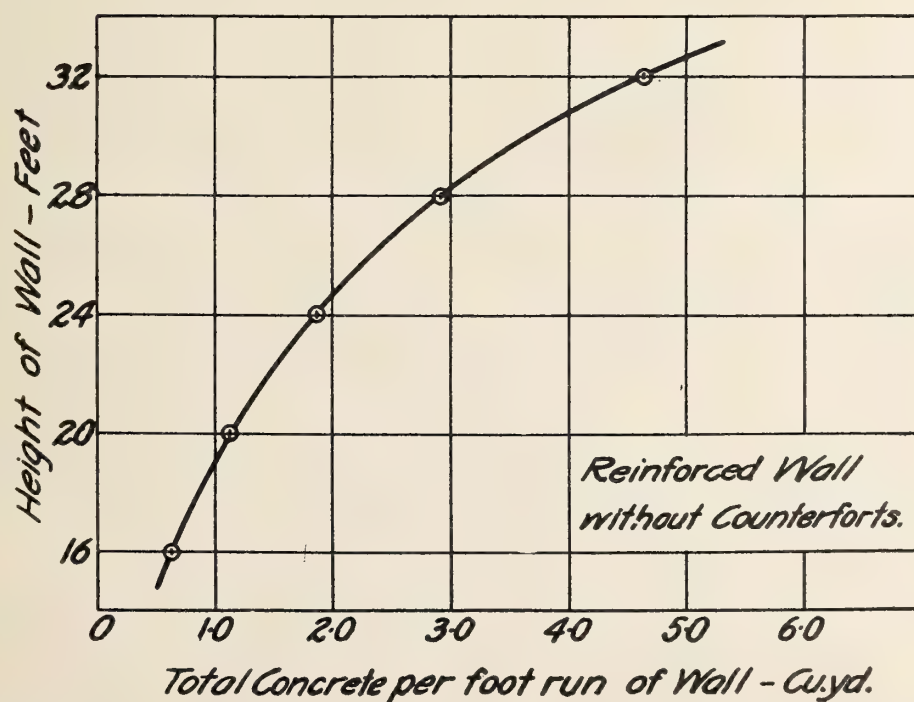


FIG. 23—Reinforced Walls without Counterforts—Total Concrete

Shear

Shear to be taken by steel at top, $V = V_w - 40bjd = 48,000 - 16,160 = 31,840$ lb.

Web reinforcement is required in upper .42 of counterfort (10.08 ft.)

Shear per in. of length taken by steel = $\frac{V}{jd} = \frac{31,840}{.92 \times 24.2} = 1430$ lb.

Total Shear = $\frac{1,430 + 0}{2} \times 10.08 \times 12 = 86,468$ lb.

No. of $1'' \times \frac{1}{4}''$ stirrups = $\frac{86,486}{2 \times .25 \times 16,000} = 10.8$

Fourteen were provided at top.
Similarly, nine were provided at bottom.

Footing

Roof load (assumed) = 24,800 lb.
Wall and counterfort = 36,000 lb.
Footing = 3,200 lb.

Total = 64,000 lb.

This is provided by an offset footing $10\frac{1}{2}$ in. wide and 8 in. deep.

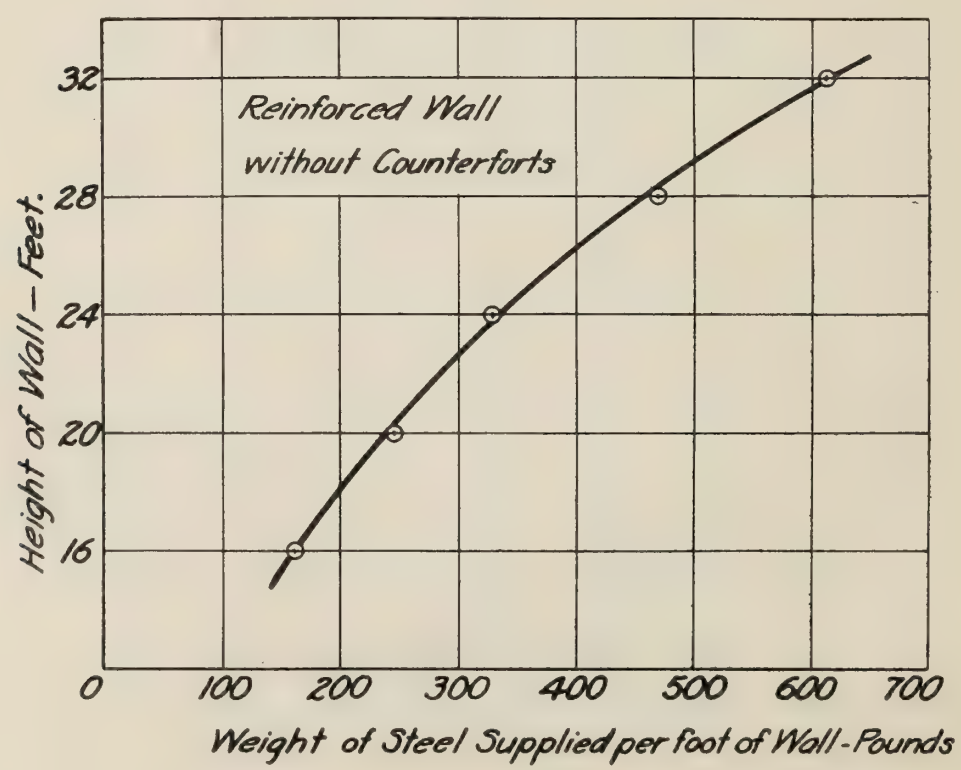


FIG. 24—Reinforced Walls without Counterforts—Steel per foot of Wall

Estimate of Quantities per Panel

(a) Counterfort.

Steel

Water Pressure = 2 ∠'s $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{3}{8}'' \times 24' - 3'' = 48.5$ ft.
at 5.9 lb. per ft..... = 286.15 lb.
2 pieces $1'' \square \times 15' - 9'' = 31.5$ ft.
1 piece $1'' \square \times 16' - 0'' = 16.0$ "
1 " $1'' \square \times 28' - 6'' = 28.5$ "
1 " $1'' \square \times 31' - 3'' = 31.25$ "

107.25 ft. at
3.4 lb. per ft.
= 364.65 lb.

1 " $1\frac{1}{8}'' \square \times 31' - 3'' = 31.25$ ft. at
4.303 lb. per ft
= 134.47 lb.

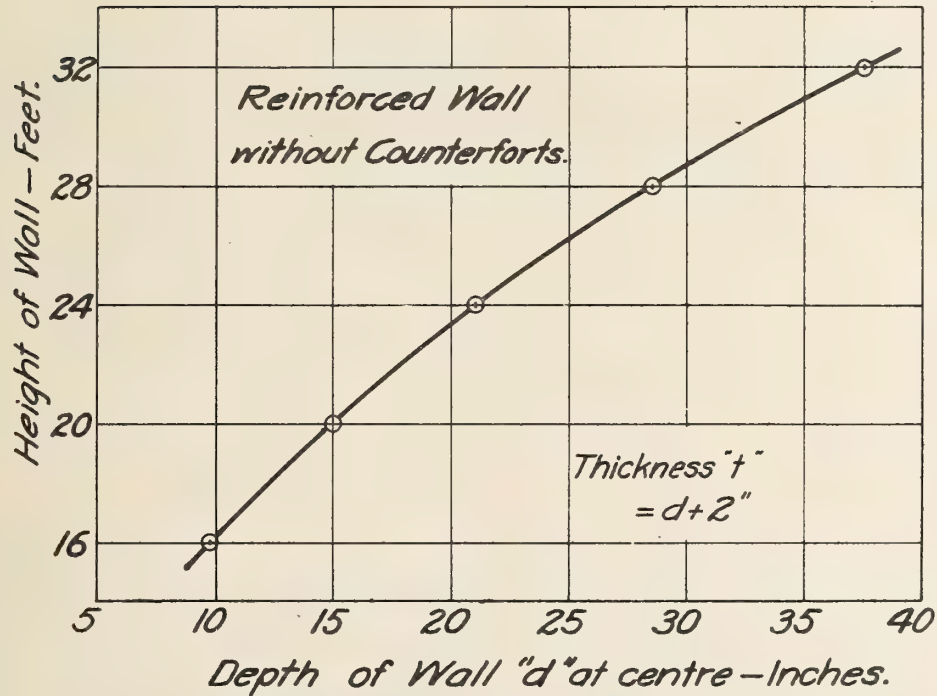


FIG. 25—Reinforced Walls without Counterforts—Depth at midheight

Earth Pressure – 2 ∠'s $2'' \times 2'' \times 5/16'' \times 24' - 3'' = 48.5$ ft.
at 3.92 lb. per ft..... = 190.12 lb.
2 pieces $1'' \square \times 21' - 0'' = 42.0$ ft.
1 piece $1'' \square \times 16' - 6'' = 16.5$ ft.
1 " $1'' \square \times 26' - 0'' = 26.0$ ft.

84.5 ft. at 3.4
lb. per ft..... = 287.30 lb.

1 " $\frac{7}{8}'' \square \times 12' - 0'' = 12.0$ ft. at
2.603 lb. per
ft..... = 31.23 lb.

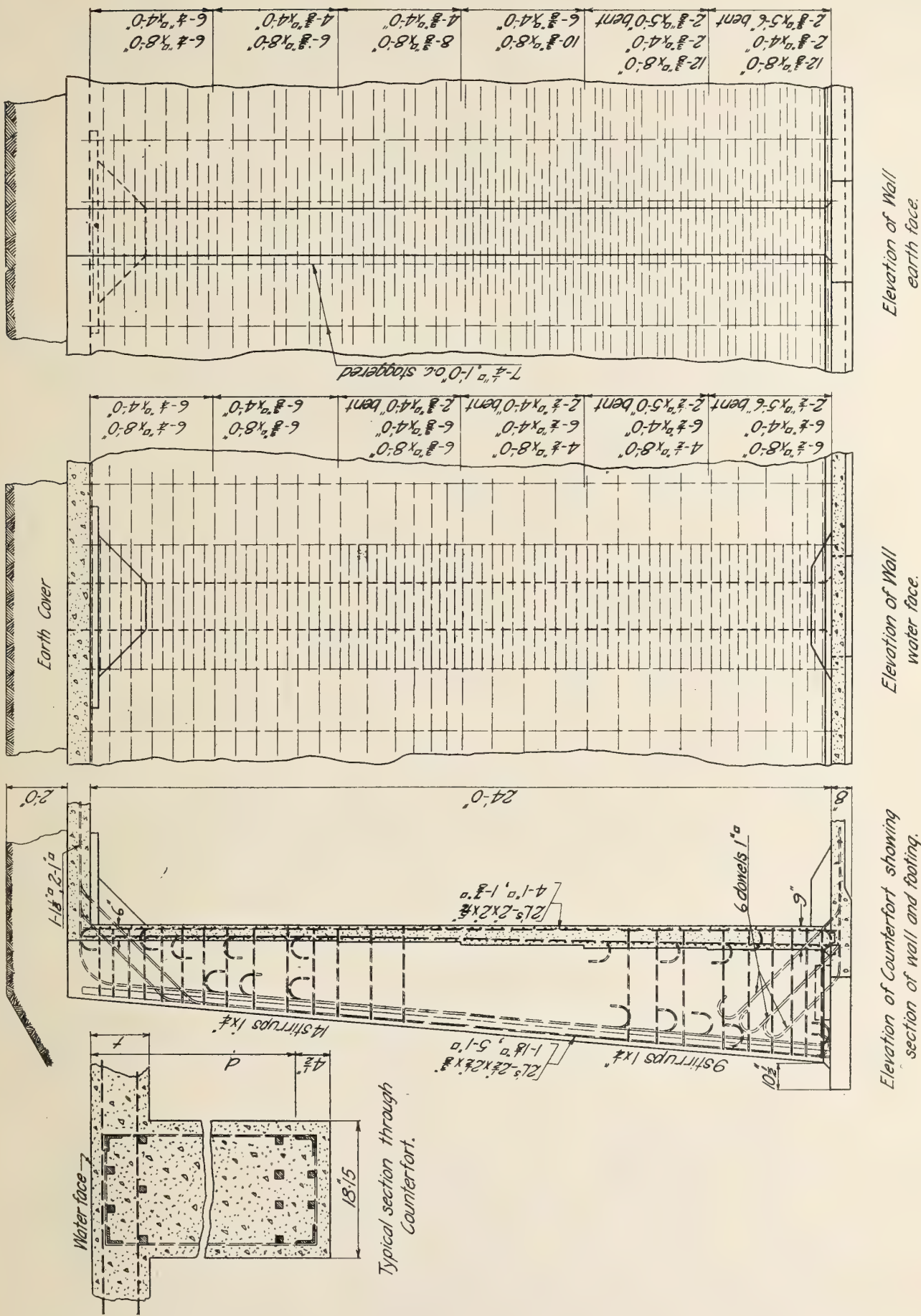


FIG. 26—Counterfort Wall—Details for Counterfort 24 ft. high

WALLS OF COVERED RESERVOIRS
Data per foot of wall. Height = distance from floor level inside at wall to half distance between water level and crown.

Reservoir	Type	Height c.y.	Forms sq. ft.	Steel lb.	Theoretic Cost	Remarks
Baldwin, Cleveland, O.	Gravity with Reinforcement	36'	75	65	\$185.13	Roof groined arch with some re- inforcement
Jerome Park, New York, Filtered Water Reservoir	Gravity	20'	46		41.70	Groined arch roof
Jerome Park, New York, Settling Basin	Gravity	24'.5	50		62.70	Groined arch roof
Pittsburg, Pa. Filtered Water Reservoir	Gravity	23'.5	51		53.25	Groined arch roof.
Newton, Mass. Waban Hill Reservoir	Gravity with Reinforcement	16'	34	30	28.95	Groined arch roof
Columbus, O. Filtered Water Reservoir	Gravity	14.5	34		21.90	Groined arch roof
Dayton, O., Reservoir	Gravity	23.5	53		97.95	Flat slab roof
Washington, D.C. Reservoir	Gravity	25'	51		85.65	Groined arch roof
Toronto, Ont. Filtered Water Reservoir	Gravity	13'	25		19.35	Groined arch roof
St. Paul, Minn. Water Reservoir	Buttressed	24'	97	508	85.13	Groined arch roof
New Orleans, La. Buttressed, Pile Foundation	Buttressed, Pile Foundation	15'	61	164	42.49	Groined arch roof
Belmont, Philadelphia Filtered Water Reservoir	Gravity	15'	30		23.70	Groined arch roof

DATA RE WALL SLAB—SPAN 8 FT.

The following table gives complete data for slab, as computed by method preceding, for all four-foot intervals in wall 24 ft.. high.

Interval from top ft.	Depth to Steel in.	Thickness of slab in.	Moment per foot over Support		Moment per foot at Centre		Required Area of Steel per foot over Support		Required Area of Steel per foot at Centre	
			Water	Earth	Water	Earth	Water	Earth	Water	Earth
24-20	7.5	9	88000	56320	44000	28160	.853	.538	.411	.260
20-16	7.0	8.5	72000	46930	36000	23460	.746	.486	.359	.234
16-12	6.0	7.5	56000	37540	28000	18770	.684	.459	.328	.220
12-8	5.0	6.5	40000	28160	20000	14080	.592	.416	.281	.198
8-4	4.5	6.0	24000	18770	12000	9380	.387	.302	.191	.149
4-0	4.5	6.0	8000	9380	4000	4690	.129	.151	.064	.076

DATA RE WALL SLAB (Continued).

Interval from top	Required Area of Vertical Steel per foot, sq. in.	Weight of Steel Supplied lb.	Volume of Con- crete Supplied cu. yd.	Area of Forms Supplied sq. ft.	Intensity of Shear at Support lb. per sq. in.		Required Area of Steel for shear, sq. in.	
					Water	Earth	Water	Earth
24-20	.214	155.3	.89	59.0	71	45	.089	..
20-16	.187	132.1	.84	58.0	62	40	.051	
16-12	.171	125.6	.74	58.0	57	38	.025	
12-8	.15	93.2	.64	58.0	49	34		
8-4	.097	81.7	.59	58.0	32	25		
4-0	.032	38.1	.59	58.0	10	12		
Total		626.0	4.29	349.0				

DESIGN OF COUNTERFORT

Height, 24 ft. Spacing of Counterforts, 8 ft. Distance from top of counterfort to section = x .

For Water Pressure $V = 8\left(\frac{w_2 l^2}{6} - \frac{w_2 x^2}{2}\right)$; $M_w = 8\left(\frac{w_2 l^2 x}{6} - \frac{w_2 x^3}{6}\right)$; $A_w = \frac{12 M_w}{f_s j d}$.

For Earth Pressure $V_e = 8\left(\frac{q l}{2} + \frac{w_1 l^2}{6} - qx - \frac{w_1 x^2}{2}\right)$; $M_e = 8\left(\frac{q l x}{2} + \frac{w_1 l^2 x}{6} - \frac{q x^2}{2} - \frac{w_1 x^3}{6}\right)$; $A_e = \frac{12 M_e}{f_s j d}$.

In the tabulations below, signs have been ignored.

x	Water Shear, lb.	Earth Shear, lb.	Water Moment, ft.-lb.	Earth Moment, ft.-lb.	Depth "d", in	A_w , sq. in.	A_e , sq. in.
0	48000	35200	0	0	24.2	0	0
2	47000	33440	95300	68800	26.2	2.96	2.23
4	44000	30510	187300	133000	28.2	5.41	4.00
6	39000	26400	270000	190100	30.2	7.27	5.33
8	32000	21120	341300	237800	32.3	8.65	6.25
10	23000	14670	393300	273800	34.3	9.43	6.78
11.71	13840	8210	428280	293500	36.0	9.70	6.94
12	12000	7040	432000	295670	36.3	9.71	6.93
14	1000	1760	443300	301200	38.3	9.43	6.68
16	16000	11730	426600	287900	40.3	8.65	6.07
18	33000	22880	378000	253400	42.4	7.27	5.08
20	52000	35200	293300	195600	44.4	5.41	3.75
22	73000	48690	168600	111900	46.4	2.96	2.05
24	96000	63360	0	0	48.4	0	0

THE EFFECT OF HEAT UPON CELLULOSE

By J. W. BAIN, *Professor of Chemical Engineering*,
and G. F. KAY, *Research Assistant*

The study of the behaviour of cellulose on heating has assumed a fresh interest since the discovery of Pictet and Sarasin¹ of a well defined crystalline compound among the volatile products resulting from the distillation of starch or cellulose under diminished pressure. This compound was identified with a substance obtained by Tanret in 1894 from certain natural glucosides and named by him laevo-glucosan with the formula $C_6H_{10}O_5$; it is preferable to designate it as β -glucosan. The former investigators showed further that β -glucosan on heating to 270° decomposed giving off formic and acetic acids, acetone, tar, etc., the familiar products of the destructive distillation of cellulose under ordinary conditions. Pictet and Sarasin express the belief that the atomic grouping of the β -glucosan exists unchanged in starch and cellulose, or in other words that β -glucosan is a primary product of the decomposition of the two latter substances.

Irvine and Oldham² by a comparison of the trimethyl glucoses prepared from β -glucosan and trimethyl cellulose, conclude that in the dry distillation of cellulose "it is highly probable that hydrolysis to β -glucose is an essential factor and that the sugar is thereafter converted into the corresponding anhydride".

Pictet and Sarasin¹ attempted to prepare β -glucosan from glucose and obtained only a very small quantity. Karrer³ pointed out that the ordinary grape sugar which they used was α -glucose, and found on distilling β -glucose in a vacuum that large quantities of β -glucosan were obtained. This confirms the views of Irvine and Oldham and leads to the conclusion that β -glucosan is not a primary product.

It occurred to us that a definite conclusion might be reached by making a direct experiment with cellulose. If cellulose decomposes with β -glucose as one of the products, the latter then yielding β -glucosan, which distils over, it should be possible to isolate β -glucose from cellulose which has been heated until it commences to decompose.

Absorbent cotton or cotton wool of the best quality was used. After some preliminary experiments, it was decided to carry out the heating in a rotating sheet metal drum about 21 cm. long and 11 cm.

¹Helv. Chim. Acta, 1918, 1, 87.

²J. Chem. Soc., 1921, 119, 1744.

³Karrer, Helv. Chim. Acta, 1920, 3, 258.

diam.; this held conveniently about 30 grams cotton. The drum was rotated at 120 revs. per min., which ensured a much more uniform heating than was possible without agitation. The drum was enclosed in a hot air oven heated from below, and temperatures above and below the drum were measured. The differences between the readings on the upper and lower thermometers amounted to 30° and even 40° on occasions; we have assumed that the temperature in the drum was the mean of these two readings, although this may be too low. We hope to be able to make more definite measurements of the temperatures later.

After heating, the cotton varied in colour from light yellow to dark brown according to the temperature and length of heating. It was boiled with distilled water, and the extract was evaporated to a thick syrup on the water bath. On diluting with water an insoluble residue remained and on re-evaporation a very dark brown syrupy liquid with a sweetish odor resulted; it was faintly acid. On treatment with phenylhydrazine in the usual way, dark brown resinous precipitates were formed and no osazone could be detected. Tanret's⁴ method for the purification of the osazones was tried but without success. Paranitrophenylhydrazine⁵ was prepared and tried on an alcoholic solution of the extract, but no definite osazone could be identified. After numerous attempts an osazone was isolated by the following method: To 2 gm. of the extract dissolved in 20 cc. water were added 4 gm. phenylhydrazine hydrochloride and 6 gm. cryst. sodium acetate. After heating for 10 mins. the resinous precipitate was filtered off and the filtrate heated again on a boiling water bath. After another 10 mins. a brown precipitate of very small particles had formed; on filtering and extracting with alcohol the residue had a melting point of 190° , but was obviously impure. By heating the original solution a third time (20 mins.) a yellow crystalline precipitate, together with some resinous matter separated; this was filtered off and purified by washing with cold absolute alcohol which removed the resins.

The remaining yellow precipitate showed clusters of very fine needles under the microscope and appeared to be identical with pure glucosazone. Further heating of the original solution for hours gave no precipitate. The osazone showed a melting point when rapidly heated, of 202° - 203° , which was compared with that of pure glucosazone which melted at 204° . Our sample was slightly impure but was so small in amount that it was unwise to attempt further purification.

⁴Tanret, *Bull. Soc. chim.* (3), 27, 392 (1902).

⁵van Ekenstein and Blanksma, *Rec. trav. Pays-Bas* (2), 7, 436 (1903).

The extract reduced Fehling's solution. The determination of its optical activity was rendered difficult by the dark brown colour which could not be removed by boiling with animal charcoal. By using a very dilute solution of the extract in a 100 mm. tube with a projection lantern as a source of light, a dextro rotation of about 2° was definitely ascertained with a saccharimeter.

A short investigation was made regarding the influence of temperature and duration of heating upon the quantity of the extract from the cotton.

After extracting the heated cotton with distilled water, the resulting solution was evaporated to constant weight on the water bath, but the product always remained liquid.

Grm. Extract per 100 grm. air dry cotton—

Hours	Temp.					
	190°	210°	230°	250°	265°	280° C.
2	0.88	0.97	1.73	1.78	1.85	0.93
4	0.91	1.88	1.58	1.50	charred	charred

The cotton contained on an average 6% water.

The temperatures given above are, as has been stated previously, the average of the upper and lower thermometers.

The decomposition of the cellulose is progressive and after a sample of cotton has been heated and extracted, it can be returned to the oven, reheated and a fresh amount of extract obtained. This will be apparent from the following figures:

30 grm. cotton heated for 3 hours at 230° C. extract 1.31%; reheated for 1 hour at 230° C. extract 1.8%.

Another sample similarly treated gave

1st extract 2.08; 2nd extract 0.88%.

We have not been able to determine for how long this heating may be carried on, so that the result of maintaining cellulose at 230° for a long period is unknown.

The fact that cellulose on heating yielded to water an appreciable amount of some substances whose exact nature is not yet determined, led us to consider the behaviour of cellulose when heated with water under pressure. On searching the literature we have been unable to discover a reference to more than one investigation on this point; that of Fischer and Schrader⁶ which is only available as an abstract. These authors found that cellulose decomposes to some extent when heated with water to 200° C. giving acids and other substances soluble in water, gases and a solid residue. The decomposition is stated to increase with the temperature.

⁶Fischer and Schrader, Jour. Soc. Chem. Ind. 42, 140A (1923).

This report escaped our notice until our first experiments had been made, and in addition did not deal with the points which interested us. Since our results are somewhat different from those reported above we decided to pursue our experiments further.

Thirty-five grams of absorbent cotton were heated in an autoclave with about 500 cc. of distilled water to a temperature of 190° to 200° C. for six hours or less. After heating the cotton was squeezed and all the water was evaporated to dryness or to a syrupy consistency. The cotton was slightly brownish, and was returned to the autoclave for a second digestion. Six successive treatments in the autoclave were carried out on the same sample of cotton, which at the conclusion had a dark grey colour, with very brittle fibres; it weighed 20 grams.

The extracts were all blackish when concentrated and some solid separated during the evaporation. To facilitate the search for sugars, the extracts were poured into boiling 95% alcohol, the precipitate was filtered off and the solution was evaporated till nearly dry. An excess of $\text{Ba}(\text{OH})_2$ was now added with a little water and after standing for a short time the solution was poured into an excess of 95% alcohol. The precipitate was washed into a cylinder, treated with CO_2 and the BaCO_3 filtered off. The filtrate on concentrating yielded a clear brownish syrup which was tested with phenylhydrazine, but no osazone could be isolated; a yellow precipitate formed but it was mixed with much resinous matter and on account of the small quantity available, no method of purification could be worked out, although several were tried. The syrup showed strong reducing action on Fehling's solution, and the presence of a sugar or sugars is suspected, although we have been unable to obtain definite proof.

The preparation of both of the extracts referred to in larger quantities and an investigation as to their composition will be undertaken shortly.

THE DOUBLE SALTS FORMED BY SODIUM AND POTASSIUM CARBONATES

By J. W. BAIN, *Professor of Chemical Engineering*

In a former communication (Trans. R.S.C. 1916, III, 65) it was reported that the double salt $\text{K}_2\text{CO}_3 \cdot \text{Na}_2\text{CO}_3 \cdot 12\text{H}_2\text{O}$ had a transition point at 35°C ., and that at and above this temperature the double salt $\text{K}_2\text{CO}_3 \cdot 3\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$ was formed by the decomposition of the former. When the investigation was renewed three years ago, efforts were made to prepare the latter salt, but without success. In 1916 crystals had been obtained which showed on analysis a composition very close to $\text{K}_2\text{CO}_3 \cdot 3\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$, and it was assumed that the discrepancy was due to the difficulty in completely removing the mother liquor. For the past three years numerous efforts have been made to secure crystals of this double salt, by the evaporation of solutions of varying concentration at temperatures higher than 35° . The crystals which were obtained have been found to vary in composition to such an extent that it was difficult to ascribe the variation to pollution with mother liquor.

Some of these crystals were difficult to grow and were found to be very hygroscopic, which has made the experimental work long and tedious, as well as unsatisfactory.

An isothermal diagram of the solubilities of K_2CO_3 and Na_2CO_3 at 40° has been obtained, but will not be presented with this paper owing to some individual determinations which are apparently in error through unsaturation. This diagram shows no curve for a double salt unless the intersections are very obtuse.

By analysing two saturated solutions with the supposed double salt as the solid phase, and the solid phases also, Schreinemakers¹ method was applied to this case and led to the conclusion that the composition of the solid phases was $14\text{Na}_2\text{CO}_3$ to $1\text{K}_2\text{CO}_3$. From our experience with the difficulty in securing equilibrium between the solid phase and the solution we are inclined to regard this as evidence that a double salt does not exist.

When $\text{K}_2\text{CO}_3 \cdot \text{Na}_2\text{CO}_3 \cdot 12\text{H}_2\text{O}$ is heated under the microscope, small needles are observed to form above 35° . Professors T. L. Walker and A. L. Parsons of the University of Toronto have kindly studied these crystals for us and believe them to be identical with

¹Schreinemakers, Zeit. physik. Chem. 11, 76, (1893)

those of $\text{Na}_2\text{CO}_3 \cdot 2\text{H}_2\text{O}$. Their conclusions are based on crystallographic measurements and also on determinations of the refractive index.

Further work is in progress.

I should like to acknowledge the assistance of Messrs. F. P. Downey, R. A. Parker, J. D. Relyea, H. C. Maedel, R. A. Gordon, and A. G. Campbell.

AN INVESTIGATION INTO THE EFFECT OF CONSTITUTION ON THE MALLEABILITY OF STEEL AT HIGH TEMPERATURES¹

By OWEN W. ELLIS,
Assistant Professor of Metallurgical Engineering

DEFINITIONS OF CERTAIN TERMS EMPLOYED

(a) *Normal Sample*.—A cylindrical metal sample whose height is equal to its diameter.

(b) *Percentage Reduction in Height of Sample*.—The reduction in the height of a cylindrical sample of metal occasioned by drop-forging expressed in terms of its initial height. Let H be the initial height of a sample, before compression, and h the final height of the sample, after compression; then $\frac{H-h}{H} \times 100$ is the percentage reduction in the height of the sample.

(c) *Percentage Increase in Diameter of Sample*.—The increase in the diameter of a cylindrical sample of metal occasioned by drop-forging expressed in terms of its initial diameter. Let d be the initial diameter of a given sample, before compression, and D the final diameter of the sample, after compression; then $\frac{D-d}{d} \times 100$ is the percentage increase in the diameter of the sample.

(d) *Percentage Deformation*.—The deformation of a metal in terms of its initial dimensions; for example, the percentage reduction in the height of a sample (occasioned by drop-forging), the percentage increase in diameter of a sample (occasioned by drop-forging), the percentage reduction in the thickness of strip (occasioned by rolling), the percentage reduction in the diameter of wire (occasioned by drawing), etc.

(e) *Critical Range of Progressive Deformation*.—It has been found that at certain stages in the progressive deformation of metals and alloys by rolling or drawing the amount of energy required to effect further deformation increases so slightly as almost to remain constant. The energy-percentage deformation relationship in this case is characterised by an inversion, such as is clearly shown in Fig. 1, which represents this relationship for 30 per cent. zinc-brass. The range of deformation wherein the amount of energy required to effect further defor-

¹Reprinted by permission of the Council of the Iron and Steel Institute.

mation tends to remain constant is referred to below as the *critical range of progressive deformation*.

(f) *Critical Range of Completed¹ Deformation*.—No one, as far as the author knows, has yet attempted to measure the energy required to deform a series of fully annealed metal or alloy strips of uniform thickness consecutively by increasing amounts; in other words, no one has taken samples of fully annealed metal strip of uniform thickness and subjected first one or more samples to percentage reductions in thickness of 10, then one or more samples to percentage reductions in thickness of 20, then one or more samples to percentage reductions in thickness of 30, and so on, measuring the amount of energy required to deform the metal in each case.

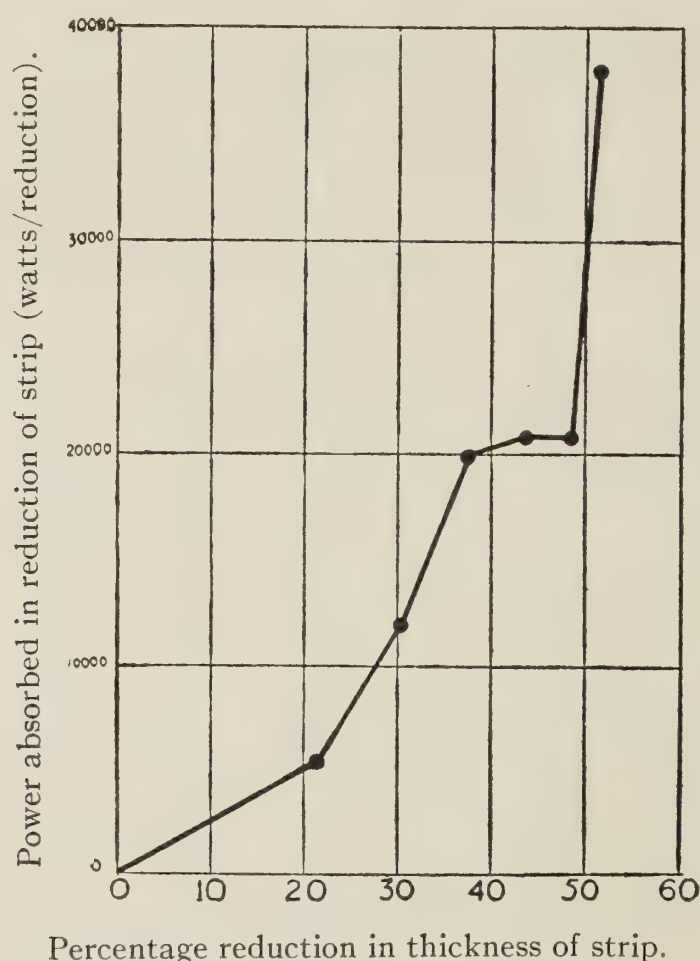


Fig. 1.—Cast 70 : 30 brass.

Whether the energy required to reduce a strip of 30 per cent. zinc-brass 47 per cent. in thickness by one pass would be equal (or almost equal) to that required to deform a similar sample 37 per cent. in thickness by one pass, as is the case where the alloy is reduced by these amounts progressively (see Fig. 1), is quite unknown. Were the energy-deformation relationship in the former case (complete reduction in one pass) to suffer an inversion such as is observed in the latter case (re-

¹The desired deformations (10 per cent., 20 per cent., 30 per cent., and so on) being completed in one reduction.

duction in a number of passes), it might be expected that the relationship between the height of drop of the hammer in drop-forging (energy) and the percentage reduction in height (deformation) of a series of samples of similar metal and subjected to single blows would also show an inversion.

The range of deformation wherein the amount of energy required to effect *an increase in completed deformation* tends, if it ever does, to remain constant is referred to below as the *critical range of completed deformation*.

(g) *Hardness*.—Resistance to permanent deformation under slowly applied stress. Hardness has been measured (i) by means of the Rockwell direct reading hardness tester, using a $\frac{1}{16}$ -inch ball and a 100-kilogramme load, and (ii) by means of a Tinius-Olsen Brinell hardness tester, using a 10-millimetre ball and a 3000 kilogramme load.

OBJECTS OF THE INVESTIGATION

1. *To Determine the Influence of the Constitution of Steel upon its Malleability at High Temperatures.*

The relation between the temperature of forging (under a drop-hammer of known weight) and the percentage reduction in height of a number of normal $\frac{1}{2}$ -inch samples of four steels of different carbon content has been investigated. By application of this relation it has been possible to determine the energy required to effect the deformation of normal $\frac{1}{2}$ -inch samples of steel of varying carbon content at different temperatures.

The temperatures employed, ranging from 540° C. to 1040° C. in these experiments, have completely covered the transformation ranges of the steels investigated.

2. *To Determine the Influence of Forging Temperature on the Hardness of Forged Steel.*

The influence of the temperature of drop-forging upon the hardness of the forgings formed from the normal $\frac{1}{2}$ -inch samples of the four steels investigated has been determined. While the results that have been obtained with steels containing 0.08 per cent. and 0.23 per cent. of carbon respectively appear to have been quite satisfactory, those obtained with the higher carbon steels have been vitiated by the chilling effect of the tup and die for temperatures slightly in excess of A1.

3. *To Determine whether a Critical Range of Completed Deformation exists in the Energy-Deformation Relationship for Metals subjected to Single Blows under the Drop-Hammer.*

Energy-deformation curves (1) for copper at room temperature, and (2) for a series of steels at high temperatures, have been drafted. No inversion corresponding with a critical range of deformation has been noted in any of the curves obtained.

DESCRIPTION OF THE EXPERIMENTS

The majority of these experiments were made on normal samples of metal $\frac{1}{2}$ inch in height (*vide* Tables I to V). A few experiments were made on normal samples of mild steel 1 inch in height (*vide* Table VI).

The malleability of the metal tested was measured by observing the percentage reduction in height of the samples at various temperatures under single blows of varying force.

The blows were applied to the samples by means of a hammer weighing 113 lbs. This hammer formed part of a standard drop-forging machine supplied by the A. R. Williams Co., of Toronto, Ont. When not in use one corner of the hammer rested upon a small pin, which was inserted in one of a series of holes specially drilled in one of the vertical guides of the machine. These holes were placed at equal distances apart, such that when the pin was withdrawn from beneath the hammer the latter fell freely between the guides, which were carefully lubricated, from heights of 6, 12, 18, 24, 30, 36, 42, 48, or 54 inches respectively. Blows of $56\frac{1}{2}$, 113, $169\frac{1}{2}$, 226, $282\frac{1}{2}$, 339, $395\frac{1}{2}$, 452, or $508\frac{1}{2}$ ft.-lb. energy could thus be delivered to the samples. The faces of the hammer and the die were parallel with one another, the distance between the two parallel faces when the hammer was in position for delivering blows to the normal $\frac{1}{2}$ -inch sample being $6\frac{1}{2}$, $12\frac{1}{2}$, $18\frac{1}{2}$, etc., and 7, 13, 19, etc., inches when the hammer was in position for forging the 1-inch samples, a shallower die being used in the latter case.

The samples were heated to the forging temperatures in a small electric crucible furnace. The samples prior to their introduction into this furnace were pre-heated in a gas-fired muffle furnace. Their introduction into the electric furnace did not, therefore, materially affect its temperature, which was held as nearly constant as possible throughout each series of tests. Measurement of the temperature of the samples was made by means of a platinum-platinum-rhodium couple, which was calibrated against the pure metals copper, zinc, and tin prior to and during the experiments, and against a couple standardized by the American Bureau of Standards at the conclusion of the experiments. The method of heating the samples and of making the tests was as follows:

A sample was removed from the pre-heating furnace to the electric furnace, which was momentarily opened for its receipt and as quickly closed. The hot-junction of the couple was then brought into close proximity with the sample and the electric furnace allowed to return

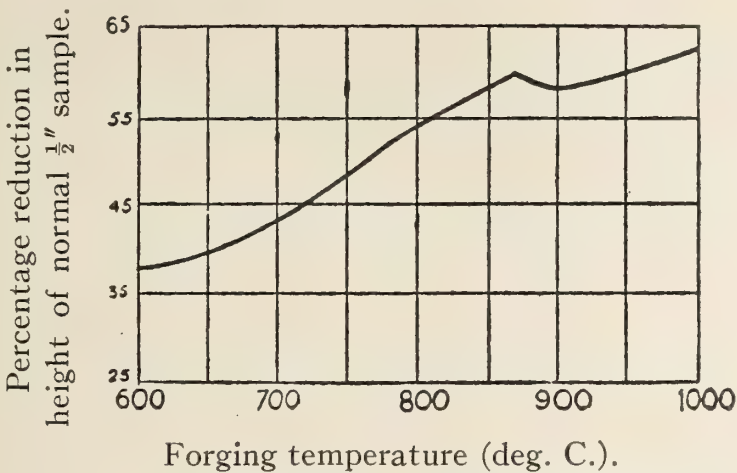


Fig. 2.—Steel No. 1. Carbon, 0.08 per Cent.

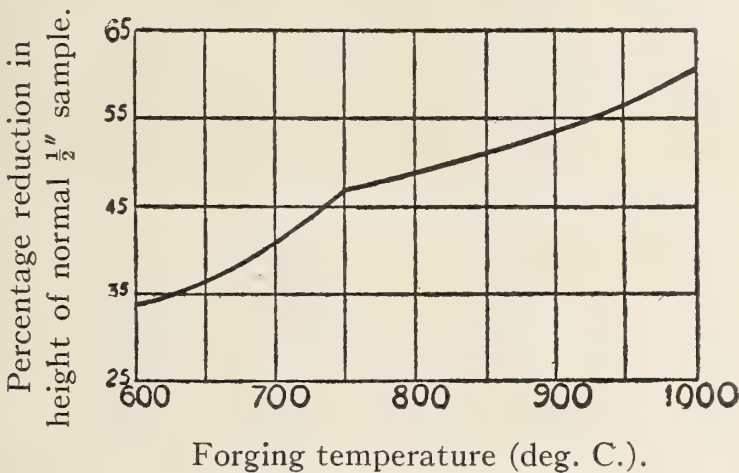


Fig. 3.—Steel No. 2. Carbon, 0.23 per Cent.

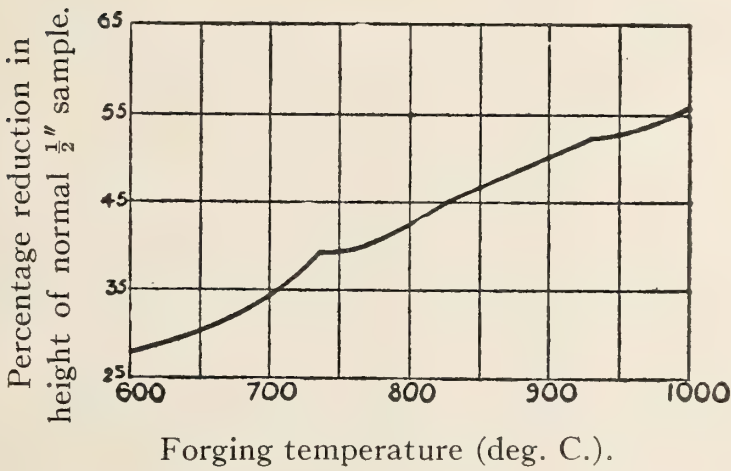


Fig. 4.—Steel No. 3. Carbon, 0.40 per Cent.

to its original temperature. When the furnace had stood at this temperature for a few minutes the sample, which was assumed to be at the temperature of the furnace, was transferred to the hammer and immediately subjected to deformation. The time occupied in moving

the sample to the die and in making the blow was never in excess of two seconds. The temperature of the sample, though small, underwent, therefore, but slight change during its passage from the furnace to the hammer. Some allowance must, however, be made for the cooling effect. That the amount of cooling was small has, however, been confirmed by the results of the tests, the critical points noted in the forging experiments (see Figs. 2 to 5) having been found to agree closely with those which might be expected for steels of the types tested.

The heights of the samples were measured before and after deformation, and the percentage reductions in height were calculated. The maximum diameters of the samples subsequent to forging were also recorded and the percentage increases in diameter estimated (see Appendix I, p. 192). It was decided that only in cases where results were not duplicated within 5 per cent. of the mean of two readings would more than two samples be tested. In a great majority of cases, however, results were identically duplicated. It was only in the case of the harder steel samples forged under blows of low force at low temperatures that differences approaching 5 per cent. were observed, as, for example, in the case of a sample of the 0.56 per cent. carbon steel, which was reduced

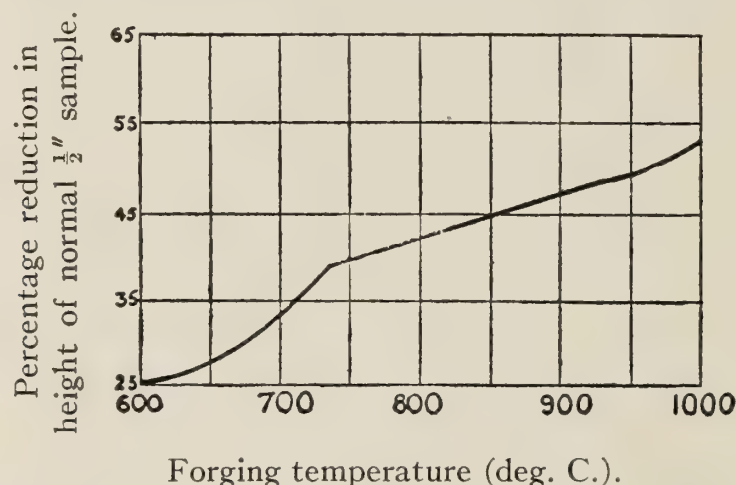


Fig. 5.—Steel No. 4. Carbon, 0.56 per Cent.

in height 12 per cent. in one case and 13 per cent. in another case when forged at 540° C. under a blow of 169½ ft.-lb., a difference from the mean

of the two readings of $\frac{0.5 \times 100}{12.5} = 4$ per cent. being thus observed. This,

it may be noted, was the widest divergence from the mean of two readings that was observed.

Upwards of nine hundred separate tests were made—204 on a 0.08 per cent. carbon steel, 187 on a 0.23 per cent. carbon steel, 279 on a 0.40 per cent. carbon steel, 268 on a 0.56 per cent. carbon steel, 64 on a 0.11 per cent. carbon steel, and a number (about 30) on pure copper.

DESCRIPTION OF THE MATERIALS USED

A. Normal $\frac{1}{2}$ -inch Steel Samples (Steels Nos. 1 to 4).

In the preparation of the normal $\frac{1}{2}$ -inch steel samples four steels of basic open-hearth origin obtained from the Steel Company of Canada through the courtesy of Messrs. R. Hobson (President of the company) and J. G. Morrow (Inspecting Engineer of the company) were used. The steels were originally in the form of $\frac{1}{2}$ -inch diameter rods, from which the normal $\frac{1}{2}$ -inch steel samples were machined. The careful machining of these samples was one of the most arduous tasks of the whole investigation.

The full analyses of the steels are tabulated below.

Macroscopic examination of the two higher carbon steels indicated that slight segregation existed in the rods. Unfortunately, this segregation was not noted until all the samples had been machined and practically all had been forged. In applying the results obtained by the author with these steels to other practice it is felt, therefore, that some consideration should be taken of the fact that segregation did exist in

Steel	Carbon per Cent.	Silicon per Cent.	Manganese per Cent.	Sulphur per Cent.	Phosphorus per Cent.
1	0.08	0.03	0.40	Not determined	Not determined
2	0.23	0.04	0.45	0.034	0.012
3	0.40	0.06	0.60	0.028	0.012
4	0.56	0.15	0.50	0.035	0.009

these rods, though, doubtless, its effect has been but slight.

B. Normal 1-inch Steel Samples (Steel No. 5).

The 1-inch diameter steel rods from which these samples were machined was of the same origin as the $\frac{1}{2}$ -inch diameter rods referred to above. The steel of which they were made was of about the same carbon content as steel No. 1; it contained 0.11 per cent. of this element.

C. Normal $\frac{1}{2}$ -inch Copper Samples.

The normal $\frac{1}{2}$ -inch copper samples were machined from $\frac{1}{2}$ -inch rolled 99.95 per cent. copper bar, the samples being annealed at 650° C. for half an hour before forging.

RESULTS OF THE INVESTIGATION

SERIES 1.—ON THE INFLUENCE OF THE CONSTITUTION OF STEELS UPON THEIR MALLEABILITY AT HIGH TEMPERATURES

A. The Malleability-Temperature Relationship

In Tables I, II, III, and IV are given the percentage reductions in height of normal $\frac{1}{2}$ -inch samples of the four steels investigated occasioned by blows of energy varying from 169.5 to 508.5 ft.-lb., applied at temperatures ranging from 540° C. to 1040° C.

That the samples might be conserved blows of 339 ft.-lb. energy content only were delivered at certain temperatures. At other temperatures blows of 169.5, of 339.0, and of 508.5 ft.-lb. energy content were applied; while at yet other temperatures, fewer in number, complete series of blows of energy content varying from 169.5 to 508.5 ft.-lb. were given to the steels.

There can be no doubt that the influence of the constitution of a given steel upon its malleability at high temperatures could be more satisfactorily represented by a curve based upon the averages of the deformations occasioned by seven blows of different energy content than by one based either upon the averages of the deformations resulting from single blows, or upon the averages of the deformations occasioned by three blows of different energy content. But, as stated above, it was necessary to conserve the samples. Hence it was impossible to apply more than a few complete series of seven blows.

Now, the average energy content of the seven blows (339.0 ft.-lb.) which were employed in the complete series is equal to the average energy content of the three blows which were used in the three-blow experiments, and to the energy content of the blow used in the one-blow tests. It happens, however, that the average percentage deformations occasioned by the seven blows exceed in all cases both the average percentage deformations produced by three blows and the average percentage deformations caused by one blow. It became necessary, therefore, to determine the relation between the average percentage reductions in height occasioned by the complete set of seven blows at the various forging temperatures and (1) the average percentage reductions in height occasioned by the three blows of 169.5, 339.0, and 508.5 ft.-lb. energy content at each of these temperatures, and (2) the percentage reductions in height caused by the single blows of 339.0 ft.-lb. energy content at each of these temperatures. The relations between these values were found to be linear in all cases.

Let X = the average percentage reductions in height occasioned by the complete sets of seven blows (169.5 to 508.5 ft.-lb.).

TABLE I—RELATION BETWEEN ENERGY OF BLOW AND PERCENTAGE REDUCTION IN
HEIGHT OF NORMAL $\frac{1}{2}$ -INCH SAMPLES
Steel No. 1 (Carbon, 0.08 per Cent.)

Tempera- ture Degrees C.	Energy of Blow. Ft.-lb.							Estimated Average Values Per Cent.
	169.5	226.0	282.5	339.0	395.5	452.0	508.5	
	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	
548	...	...	...	39.0	...	...	...	37.9
566	...	...	...	40.0	...	...	...	38.8
584	...	...	...	39.0	...	...	...	37.9
602	...	...	...	38.0	...	...	...	37.0
620	...	...	...	40.0	...	...	...	38.8
639	...	...	...	40.0	...	...	...	38.8
656	...	...	...	41.0	...	...	...	39.8
674	26.0	...	...	43.0	...	...	54.5	42.0
691	...	...	...	45.0	...	...	...	43.6
693	26.5	...	...	42.0	...	...	56.5	42.5
698	26.0	34.0	41.0	44.0	48.0	52.0	56.0	43.1
708	...	...	...	44.0	...	...	...	42.5
714	28.0	...	...	46.0	...	...	56.5	44.4°
734	28.5	...	...	47.5	...	...	59.5	46.1
739	...	...	...	45.0	...	...	...	43.6
740	32.0	39.0	45.0	48.5	54.5	58.0	60.8	48.4
754	32.0	...	...	48.3	...	...	61.0	48.0
773	...	...	...	50.0	...	...	...	48.3
777	35.5	...	...	53.0	...	...	63.0	51.5
792	...	...	...	54.0	...	...	...	52.0
793	37.0	43.0	50.0	55.0	59.0	63.0	65.0	53.7
808	...	...	...	55.0	...	...	...	53.0
827	...	...	...	60.0	...	...	...	57.6
847	40.5	...	...	61.0	...	...	70.0	58.4
854	...	...	...	62.0	...	...	...	59.6
858	42.5	...	...	61.0	...	...	71.0	59.4
861	...	...	...	62.0	...	...	...	59.6
866	43.5	...	...	61.0	...	...	70.0	59.4
877	...	...	...	62.0	...	...	...	60.4
878	43.0	50.5	57.0	61.5	66.0	68.0	70.0	59.3
886	...	...	...	60.0	...	...	...	57.6
887	43.0	...	...	59.3	...	...	69.0	58.3
895	44.0	...	...	60.0	...	...	...	59.4
902	...	...	...	62.0	...	...	...	59.6
913	42.5	...	...	60.0	...	...	70.5	58.8
917	42.0	48.5	55.0	61.0	64.5	67.5	70.0	58.6
930	43.5	...	...	61.0	...	...	70.0	59.4
952	43.0	...	...	61.5	...	...	71.0	59.7
963	44.0	52.0	58.0	62.0	66.0	70.0	71.0	60.5
969	43.5	...	...	62.5	...	...	73.0	60.8
993	44.5	...	...	63.5	...	...	70.0	60.5

TABLE II—RELATION BETWEEN ENERGY OF BLOW AND PERCENTAGE REDUCTION IN
HEIGHT OF NORMAL $\frac{1}{2}$ -INCH SAMPLES
Steel No. 2 (Carbon, 0.23 per Cent.)

Tempera- ture Degrees C.	Energy of Blow. Ft.-lb.							Estimated Average Values Per Cent.
	169.5	226.0	282.5	339.0	395.5	452.0	508.5	
	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	
548	...	...	...	33.0	...	...	...	30.5
566	...	...	...	34.0	...	...	...	31.6
584	...	...	...	33.0	...	...	...	30.5
602	...	...	...	35.0	...	...	...	32.6
620	...	...	...	35.0	...	...	...	32.6
639	...	...	...	36.0	...	...	...	33.6
656	...	...	...	36.5	...	...	...	34.1
674	...	...	...	41.0	...	...	...	38.7
691	...	...	...	40.0	...	...	...	37.7
698	23.0	29.0	35.0	41.0	46.0	51.0	54.0	40.6
708	...	...	...	41.0	...	...	...	38.7
722	...	...	...	45.0	...	...	...	42.7
725	28.5	34.5	40.0	45.0	49.0	53.0	57.0	44.1
739	...	...	...	48.5	...	...	...	46.4
750	30.0	38.0	43.0	48.0	52.5	56.0	58.5	46.7
754	...	...	...	46.0	...	...	...	43.8
773	...	...	...	48.0	...	...	...	45.8
775	30.5	38.5	44.5	49.0	55.0	58.0	60.0	48.1
792	...	...	...	49.0	...	...	...	46.9
800	33.0	39.5	44.5	51.0	54.5	58.0	60.5	48.4
800	31.0	...	...	49.0	...	...	54.0	45.4
808	...	...	...	48.0	...	...	...	45.8
822	30.5	...	...	50.0	...	...	60.0	47.5
825	34.5	41.0	46.0	52.0	54.5	58.0	62.5	50.0
827	...	...	...	50.0	...	...	...	47.9
847	...	...	...	50.0	...	...	...	47.9
850	34.5	42.0	47.5	53.0	56.5	60.5	63.0	51.0
861	...	...	...	52.0	...	...	...	49.9
875	34.5	42.0	49.5	54.5	57.5	60.5	64.0	51.9
879	...	...	...	52.0	...	...	...	49.9
895	...	...	...	52.0	...	...	...	49.9
900	36.5	44.0	51.5	55.5	60.5	63.5	67.0	54.3
913	...	...	...	55.0	...	...	...	53.0
947	...	...	...	58.0	...	...	...	56.1
950	43.0	...	...	59.5	...	...	70.0	58.5
961	...	...	...	58.0	...	...	...	56.1
978	...	...	...	60.0	...	...	...	58.1
1000	43.5	51.0	57.5	62.0	65.0	70.0	71.5	60.3

TABLE III—RELATION BETWEEN ENERGY OF BLOW AND PERCENTAGE REDUCTION IN HEIGHT OF NORMAL $\frac{1}{2}$ -INCH SAMPLES*Steel No. 3 (Carbon, 0.40 per Cent.)*

Temperature Degrees C.	Energy of Blow. Ft.-lb.							Estimated Average Values Per Cent.
	169.5	226.0	282.5	339.0	395.5	452.0	508.5	
	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	
540	15.0	19.0	23.0	27.0	32.5	35.0	38.0	27.1
548	...	...	...	26.0	...	...	...	26.9
566	...	...	...	26.0	...	...	...	26.9
584	13.5	...	...	28.2	...	...	40.0	26.9
602	...	...	...	29.0	...	...	...	29.6
620	...	...	...	29.0	...	...	...	29.6
630	17.0	...	...	30.5	...	...	42.5	29.9
639	...	...	...	30.0	...	...	...	30.5
656	...	...	...	30.0	...	...	...	30.5
674	18.2	23.0	26.0	33.0	36.5	41.5	47.0	32.2
691	...	...	...	34.0	...	...	...	34.1
693	19.5	...	...	34.0	...	...	50.0	34.5
708	...	...	...	38.0	...	...	...	37.8
714	22.0	25.5	29.0	39.5	42.3	43.0	50.0	36.4
722	24.0	...	...	40.0	...	...	50.0	38.1
734	24.0	...	...	41.5	...	...	52.5	39.4
739	22.5	...	...	40.0	...	...	53.5	39.0
747	26.5	...	...	42.0	...	...	50.5	39.8
754	25.0	29.5	35.0	39.5	42.0	48.0	53.5	39.1
773	...	...	...	43.0	...	...	...	42.3
777	27.0	...	...	41.0	...	...	52.0	40.1
792	...	...	...	44.0	...	...	...	43.2
800	27.5	34.0	40.0	44.0	46.0	51.0	56.0	42.7
808	...	...	...	44.0	...	...	...	43.2
822	29.5	...	...	46.0	...	...	56.0	44.0
827	29.0	...	...	46.5	...	...	56.0	44.0
838	30.5	...	...	47.5	...	...	58.0	45.5
847	31.0	37.0	42.5	47.0	52.5	54.5	60.0	46.4
854	31.5	...	...	48.0	...	...	60.0	46.7
861	32.0	...	...	49.5	...	...	60.0	47.2
887	34.0	38.3	45.0	51.3	55.0	58.0	62.0	49.1
913	33.5	42.0	47.5	50.5	55.5	60.0	62.3	50.6
930	34.7	42.0	49.0	54.0	58.0	61.5	64.5	51.9
952	36.5	...	...	54.0	...	...	65.5	52.3
969	36.5	44.5	50.0	56.5	60.5	64.0	65.3	53.8
993	39.0	...	...	59.0	...	...	68.5	56.0
1010	39.0	48.0	52.0	62.5	63.5	64.5	68.0	56.1
1040	41.0	...	...	60.0	...	...	70.0	57.5

TABLE IV—RELATION BETWEEN ENERGY OF BLOW AND PERCENTAGE REDUCTION IN
HEIGHT OF NORMAL $\frac{1}{2}$ -INCH SAMPLE
Steel No. 4 (Carbon 0.56 per Cent.)

Tempera- ture Degrees C.	Energy of Blow. Ft.-lb.							Estimated Average Values Per Cent.
	169.5	226.0	282.5	339.0	395.5	452.0	508.5	
	Per Cent	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	Per Cent.	
540	11.3	15.6	22.0	24.0	27.0	31.0	34.5	23.1
548	...	...	...	25.0	...	...	...	24.9
566	...	...	...	24.0	...	...	...	23.9
584	12.3	17.0	20.7	24.3	27.7	33.0	36.0	24.4
602	...	...	...	26.0	...	...	...	25.8
608	14.0	...	...	25.5	...	...	38.0	25.8
620	...	...	...	26.0	...	...	...	25.8
630	15.5	19.0	23.0	26.5	30.0	34.0	38.0	26.6
639	...	...	...	27.0	...	...	...	26.8
653	15.5	...	...	30.0	...	...	40.0	28.6
656	...	...	...	28.0	...	...	...	27.7
674	16.0	20.7	26.8	33.2	34.0	37.5	42.0	29.2
691	...	...	...	30.0	...	...	...	29.6
693	16.5	...	...	32.3	...	...	45.3	31.5
708	...	...	...	35.0	...	...	...	34.3
714	18.0	24.0	30.7	33.5	37.0	41.0	44.0	32.5
722	...	...	...	38.0	...	...	...	37.1
734	23.5	...	...	39.5	...	...	49.0	37.6
739	...	...	...	40.0	...	...	...	39.0
754	24.7	31.0	36.0	39.0	44.0	47.0	50.0	38.9
773	...	...	...	43.0	...	...	...	41.8
777	24.5	...	...	41.0	...	...	52.5	39.7
792	...	...	...	42.0	...	...	...	40.9
800	25.5	32.5	40.0	42.0	48.0	51.5	54.0	42.4
808	...	...	...	42.0	...	...	...	40.9
822	28.0	...	...	44.0	...	...	54.0	42.5
827	...	...	...	45.0	...	...	...	43.7
847	30.0	35.0	41.0	45.5	49.5	54.0	57.0	44.5
861	30.0	...	...	45.0	...	...	58.0	44.9
866	29.0	...	...	48.0	...	...	57.5	45.4
879	...	...	...	48.0	...	...	...	46.5
887	30.0	...	...	47.0	...	...	60.0	46.2
895	...	...	...	49.0	...	...	...	47.5
913	32.0	...	...	50.0	...	...	62.5	48.8
930	33.5	...	...	52.5	...	...	62.0	50.0
952	33.5	...	...	52.5	...	...	63.0	50.6
957	36.0	43.0	48.0	54.0	58.0	62.0	64.7	52.2
969	36.0	...	...	51.0	...	...	64.0	51.0
993	37.5	...	...	54.5	...	...	66.0	53.6
1010	38.5	...	...	56.5	...	...	66.5	54.7
1040	40.0	...	...	59.0	...	...	69.5	57.0

Let Y = the average percentage reductions in height occasioned by the sets of three blows (169.5, 339.0, and 508.5 ft.-lb.).

Let Z = the average percentage reductions in height occasioned by the single blows of 339 ft.-lb.

Then

$$\begin{aligned} \text{For steel 1 } & \begin{cases} X = 1.02Y \\ X = 0.94Z + 1.16 \end{cases} \\ \text{For steel 2 } & \begin{cases} X = 1.02Y - 0.2 \\ X = 1.02Z - 3.1 \end{cases} \\ \text{For steel 3 } & \begin{cases} X = 1.02Y - 0.7 \\ X = 0.91Z + 3.2 \end{cases} \\ \text{For steel 4 } & \begin{cases} X = 1.03Y - 0.8 \\ X = 0.94Z + 1.4 \end{cases} \end{aligned}$$

By applying these formulæ the estimated average values for the percentage reductions in height of the normal $\frac{1}{2}$ -inch samples quoted in the ninth columns of Tables I to IV have been obtained. These estimated average values are those which have been employed in plotting the curves of Figs. 2 to 5, which represent in graphical form the influence of forging temperature on the malleability of these steels in terms of the estimated average percentage reduction in height of normal $\frac{1}{2}$ -inch samples under seven blows of energy content varying from 169.5 ft.-lb. to 508.5 ft.-lb.

A discussion of the influence of the constitution of the four steels investigated on their malleabilities is here possible.

Steel 1—0.08 per Cent. Carbon (Fig. 2).—As the temperature of this steel is raised from 600° C. its malleability increases with increasing rate up to about 780° C. (A2), at which temperature there appears to be a reduction in the malleability of the steel, which continues uniformly until the temperature 870° C. (A3) is attained.¹ At 870° C. (A3) the steel is subject to a marked increase in its resistance to deformation, such that it does not recover the malleability it possesses at this temperature until a temperature of about 955° C. is reached.

The Ac1 and Ac3 points for this steel were found at 744° C. and 890° C. respectively, the Ar1 and Ar3 points at 698° C. and 863° C. respectively.

Reference to Fig. 2 will show that the carbon change has little, if any, effect upon the malleability of this steel. It will also show that the A3 point (870° C.), as determined by the forging tests, lies almost midway between the Ac3 and Ar3 points, as determined pyrometrically, being, as might be expected, slightly closer to the former than to the latter.

¹Whether the change in the direction of the curves at A2 should be *exactly* as shown in Figs. 2 and 10 is quite open to question. That the rate of change of malleability with temperature is subject to change at 780° C. is, however, beyond doubt.

Steel 2—0.23 per Cent. Carbon (Fig. 3).—As the temperature of this steel is raised from 600° C. its malleability increases with increasing rate up to about 750° C. (A1), at which temperature a reduction in the rate of increase in malleability ensues.

The Ac1 and Ac3 points for this steel lie in the vicinity of 735° C. and 850° C. respectively, the Ar1 and Ar3 points at 694° C. and 838° C. respectively.

Reference to Fig. 3 will show that in this case the carbon change has a marked effect upon its malleability.

The existence of A2 and of A3 is not made clear. The author, however, is not satisfied that further work would entirely confirm the shape of his curve for temperatures about 750° C., there being a marked lack of agreement in this case between the estimated average values for the percentage reductions under the single blows, under the three blows, and under the seven blows, which rendered it difficult to decide on the detail of the curve. The general form (as shown in Fig. 3) is, however, undoubtedly correct.

It is noteworthy that A1 (750° C.), as determined by the forging tests, is in this case well above Ac1 (735° C.). It may be that the proximity of A2 (768° C.) has occasioned the apparent elevation of A1 in this instance.

Steel 3—0.40 per Cent. Carbon (Fig. 4).—As the temperature of this steel is raised from 600° C. its malleability increases with increasing rate up to about 730° C. (A1), at which temperature this property of the steel is subject to a marked reduction. The malleability of the steel does not increase again with any marked rapidity till a temperature of about 760° C. is reached. The Ac1 and Ac3 points of this steel lie in the vicinity of 735° C. and 805° C. respectively, the Ar1 and Ar3 points at 695° C. and 762° C.

An interesting peculiarity of this steel is the increased resistance offered to forging at temperatures in excess of about 930° C. This increased resistance to forging appears to be well established.

It is noteworthy that a peculiarity in the behaviour of medium carbon steels in this vicinity has been noted by Boboschin,¹ who, according to Saldau,² has recorded the existence of a change in medium carbon steels at temperatures between 880° C. and 950° C. The author has, so far, been unable to obtain information as to the nature of the change observed by Boboschin. Saldau³ himself noted a drop in the rate of increase of electrical resistivity of carbon steels at about 980° C., the

¹*Journal of the Russian Society of Metallurgists*, 1911, p. 97.

²*Carnegie Scholarship Memoirs*, 1916, vol. vii, p. 206.

³*Loc. cit.* (See also Andrew, Rippon, and Miller, *Journal of the Iron and Steel Institute*, 1920, No. I, p. 572.)

observed depression of the temperature coefficient of electrical resistivity being considered by him to be due to a decrease in the rate of growth of the austenite crystals of which steels are constituted at temperatures in excess of A_{c3} . If Saldau's view be accepted as correct, it would be inferred that a decrease in malleability should ensue at about 980°C . That such a decrease in malleability does occur, though at a lower temperature, has, it is thought, been proved. It is, of course, impossible to decide whether the 930°C . and 980°C . inversions are in any wise associated. Whether the form of the curve shown in Fig. 4 for temperatures in excess of 930°C . is exactly as shown depends on the value placed upon the estimated average values for single blows made on this steel at 952°C ., 993°C ., and 1040°C . respectively. Even though these values be discarded, however, there is still no question as to a change in the malleability of steel at 930°C .

Steel 4—0.56 per Cent. Carbon (Fig. 5).—This steel behaves in a manner similar to that of the steel of lower carbon content. The malleability-temperature relationship between 730°C . (A_1) and 930°C ., however, is linear in character.

At 930°C . a change in malleability ensues, which is of the same nature as that observed in the case of the 0.40 per cent. carbon steel.

B. *The Energy-Temperature Relationship*

In Figs. 6 to 9 are shown typical energy-deformation curves for the four steels that have been investigated. These curves are based upon the values given in Tables I to IV for the complete series of tests. From these curves it was possible to obtain values for the energy required to cause certain definite amounts of deformation in the normal samples of the four steels at different temperatures. These energy values were employed in the preparation of Figs. 10 to 13, which show graphically the energy required to deform normal $\frac{1}{2}$ -inch samples of the four steels by 20 per cent., 30 per cent., 40 per cent., and 50 per cent. of their original lengths.

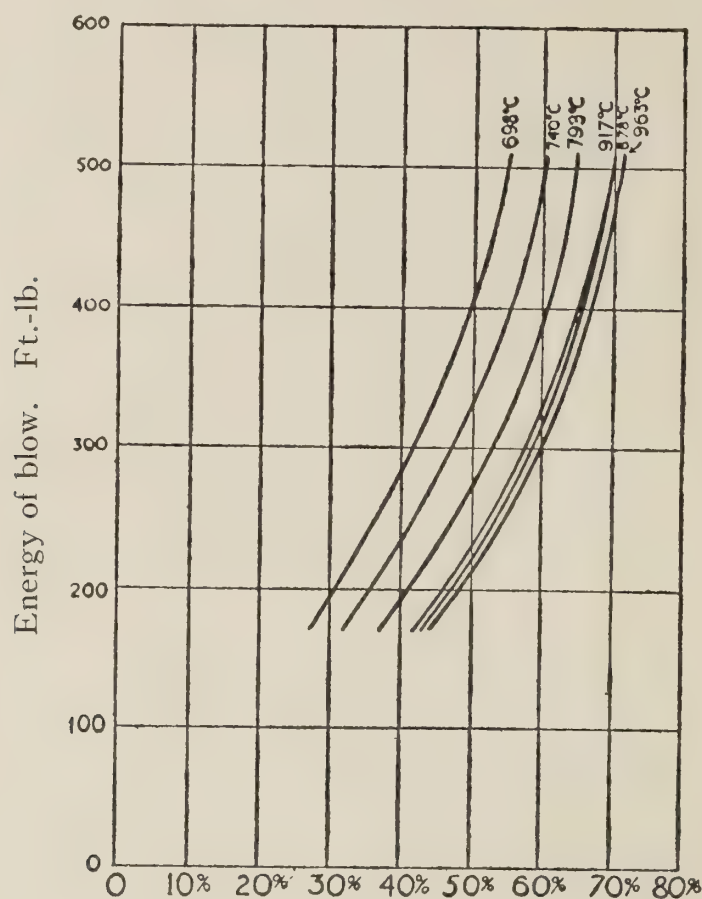
SERIES 2.—ON THE INFLUENCE OF FORGING TEMPERATURE ON THE HARDNESS OF FORGED STEELS

Experiments with Normal $\frac{1}{2}$ -inch Samples

The Rockwell hardness numbers of most of the normal $\frac{1}{2}$ -inch samples of the four steels investigated were determined subsequent to forging. Not less than six observations were made in respect of each of all the samples tested, while in a majority of cases as many as twelve readings were taken. Only the hardness numbers of those samples which had

been forged under blows of 339 ft.-lb. energy content are considered in this report, though tests which confirmed the general trend of the above figures were made on samples which had been forged under other blows.

As already stated, the $\frac{1}{16}$ -inch ball and the 100-kilogramme load were employed in these tests, so that all the hardness numbers referred to in this section of the paper are on the Rockwell B scale.



Reduction in height of normal $\frac{1}{2}$ " sample.

Fig. 6.—Steel No. 1. Carbon, 0.08 per Cent.

The results of the hardness tests are given in Table V, and are shown graphically in Figs. 14 to 17. In these diagrams (Figs. 14 to 17) not only have the average hardness numbers been linked up by straight lines, but smooth curves, which are intended to represent the general trend of hardness with increase of forging temperature, are drawn. The smooth curves will be considered in detail below.

Steel 1—0.08 per Cent Carbon (Fig. 14).—Increase in forging temperature beyond about 560°C . results in a lowering of the hardness of the forged steel. The number of tests in this vicinity (560°C .) has, however, been insufficient to allow of a really satisfactory relationship between the factors hardness and forging temperature to be drafted.

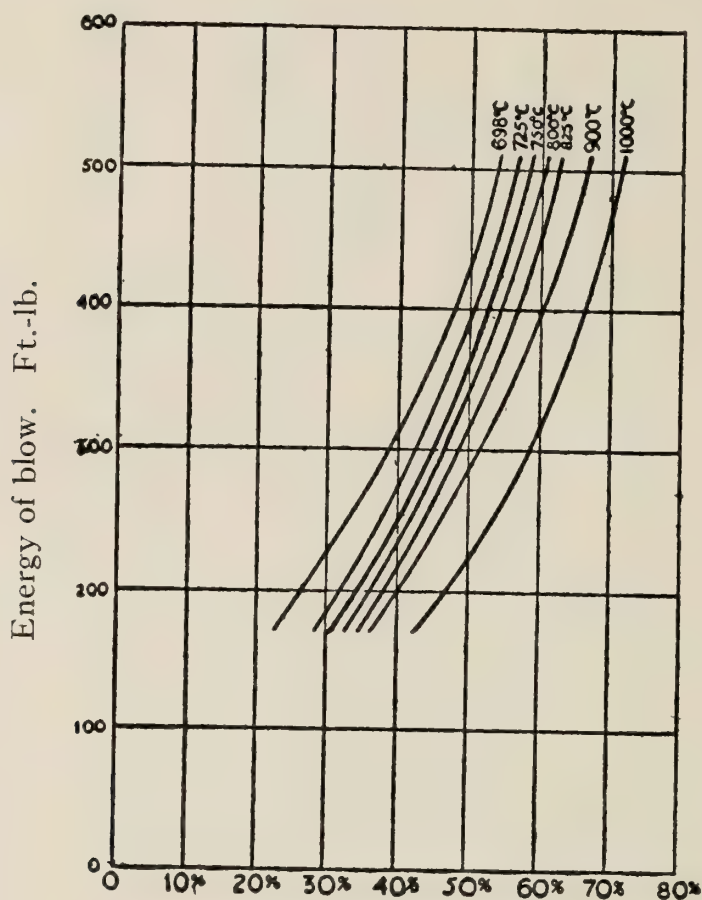
At a forging temperature of about 735°C . (A1), the hardness of this steel is subject to a marked increase. This increase in hardness is

TABLE V—HARDNESS OF FORGED STEELS. NORMAL $\frac{1}{2}$ -INCH SAMPLES
Rockwell Hardness Numbers (B Scale)

Forging Temperature Degrees C.	Steel No. 1 (C, 0.08 per Cent.)	Steel No. 2 (C, 0.23 per Cent.)	Steel No. 3 (C, 0.40 per Cent.)	Steel No. 4 (C 0.56 per Cent.)
540	...	...	95.8	100.1
548	...	...	99.3	...
566	85.0	90.2	101.3	...
584	84.5	87.5	95.0	99.0
602	...	89.3	...	...
608	...	...	...	99.4
620	...	88.5	98.5	97.7
630	...	...	96.0	...
639	...	87.2	96.1	97.5
653	...	...	...	98.5
656	...	83.9	...	...
674	79.4	85.6	95.0	102.2
693	81.2	...	94.7	96.5
698	81.2	...	...	...
708	79.9	91.9	101.8	96.9
714	79.8	...	104.6	98.3
722	...	90.6	102.8	102.7
734	80.6	...	105.2	109.1
739	84.5	90.0	99.3	106.2
754	82.7	90.8	101.7	106.3
773	83.3	89.0	99.4	106.2
792	82.8	86.9	...	108.1
800	82.0	89.5	103.5	109.5
808	...	87.9	104.4	...
822	81.0	...	105.0	110.4
827	78.9	86.7	105.2	...
838	...	...	106.2	...
847	79.6	90.0	106.5	107.1
854	77.0	...	...	...
861	...	76.5	105.1	...
866	...	...	...	109.6
878	78.6	...	...	...
887	72.9	...	108.1	109.4
895	...	76.3	...	...
913	71.0	78.5	107.5	109.5
930	76.9	...	105.5	110.4
947	...	79.7	...	...
952	77.0	...	106.3	...
961	...	75.7	...	...
969	73.8	...	105.5	109.2
978	...	74.8	...	...
993	72.0	...	107.2	109.6
1010	...	...	112.4	111.0
1040	...	...	111.2	111.9

of special note, in view of the fact that the malleability-temperature relationship for this steel is not characterized by an inversion at this temperature.

The effect of forging temperatures in excess of 735°C . is to reduce the hardness of the finished forgings. This reduction in hardness with forging temperature is completed at about 910°C . Further increase in forging temperature leads to a marked increase in hardness, while still higher forging temperatures result in a lowering of the resistance of the steel to penetration.



Reduction in height of normal $\frac{1}{2}$ " sample.

Fig. 7.—Steel No. 2. Carbon, 0.23 per Cent.

Steel 2—0.23 per Cent. Carbon (Fig. 15).—Increase in forging temperature beyond about 560°C . leads to a diminution of the hardness of the forged steel. At a forging temperature of about 700°C . (A1) the hardness of this steel is subject to change, suffering a clearly defined rise. Increase in temperature beyond A1 but slightly softens the steel until 850°C . (A3) is reached, when the steel is subject to a remarkable diminution in hardness. Beyond A3 the steel appears to increase in hardness with forging temperatures up to about 947°C . Higher forging temperatures, however, result in a lowering of hardness.

Steel 3—0.40 per Cent. Carbon (Fig. 16).—Increase in forging temperature beyond about 560°C . leads to a diminution of the hardness of

the forged steel. At a forging temperature of about 700°C . (A1), however, the hardness of this steel rapidly increases. The interpretation of the curve at this point is a little difficult, but it appears that the steel tends to soften with increase of forging temperature up to about 740°C ., beyond which temperature, however, the cooling effect of hammer and die becomes sufficient to cause hardening of the steel. Increase of forging temperature in this case results, therefore, in increase in hardness, but this increase in hardness, it is thought, is due almost entirely to the quenching effect of the hammer and die already referred to; microscopic examination of samples forged at these higher temperatures having revealed structures characteristic of this steel as quenched from temperatures in excess of Ac_{32} .

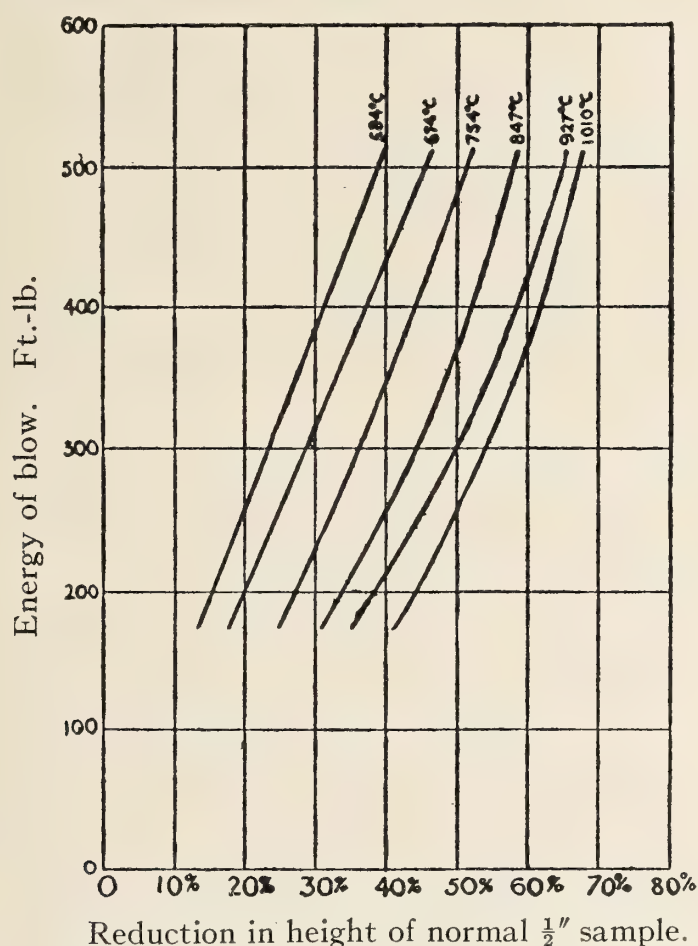
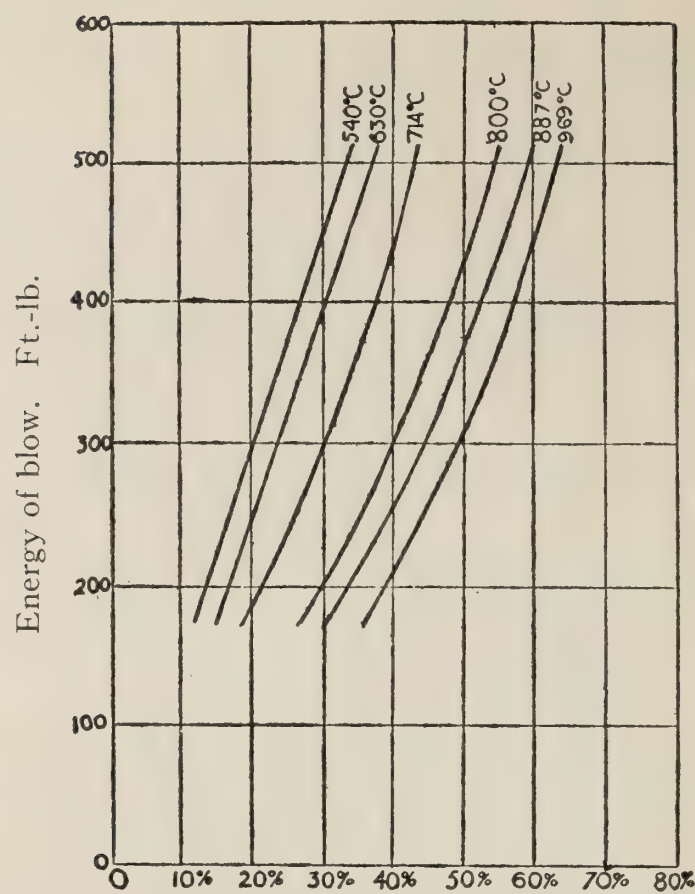


Fig. 8.—Steel No. 3. Carbon, 0.40 per Cent.

Steel 4—0.56 per Cent. Carbon (Fig. 17).—Increase in forging temperature beyond about 560°C . leads to a reduction in the hardness of the forged steel.¹

At a forging temperature of about 715°C . (A1), however, the hardness of this steel rapidly increases. Forging temperatures in excess of about 735°C . result in increasing hardness of this steel, the increasing hardness being occasioned by the quenching effect of the hammer and die, as in the case of steel No. 3.

¹The anomalous result obtained at 674°C . is inexplicable, and is here disregarded.



Reduction in height of normal $\frac{1}{2}$ " sample.
Fig. 9.—Steel No. 4. Carbon, 0.56 per Cent.

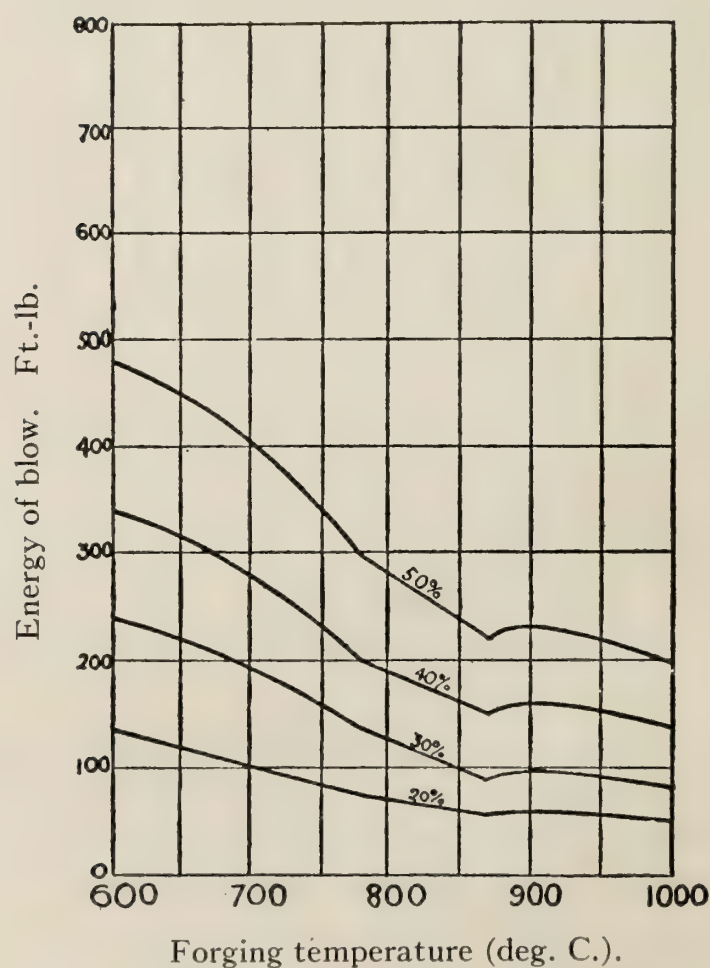


Fig. 10.—Steel No. 1. Carbon, 0.08 per Cent.

Experiments with Normal 1-inch Samples.

The results of tests made on normal 1-inch samples of 0.11 per cent. carbon steel are given in Table VI, and are shown in Figs. 18 and 19.

TABLE VI—RESULTS OF MALLEABILITY AND HARDNESS TESTS ON NORMAL 1-INCH SAMPLES OF 0.11 PER CENT. CARBON STEEL

Temperature of Forging Degrees C.	Malleability. Per- centage Reduction in Height of Sample ¹	Hardness of Forging. Brinell Number (3000 kg., 10-mm. ball)
650	10.3	127.6
700	11.7	125.2
750	13.4	124.0
775	14.6	126.4
800	15.8	125.2
825	17.8	124.8
850	19.5	121.6
875	20.1	119.8
880	21.0	117.6
890	20.2	113.2
900	20.9	116.4
925	20.5	116.0
950	20.0	119.8
975	22.2	115.2
1000	21.3	114.0

Reference to Fig. 18 will show that the deformation-temperature relationship for this steel is of a similar form to that given in Fig. 2 for the 0.08 per cent. carbon steel, which was tested in the form of normal $\frac{1}{2}$ -inch samples. The effect of the β - γ transformation is clearly marked; that of the magnetic point, however, is not so decisive.

The results of the hardness tests made on the forged samples of this steel are in the main confirmatory of those made on the forged samples of the 0.08 per cent. carbon steel (*vide* Fig. 14). The hardness tests in this case, however, were made with a 10-millimetre ball under a load of 3000 kilogrammes—the hardness values quoted in Table VI and Fig. 19 are, therefore, Brinell numbers.

It will be noted that the hardness of this steel is subject to decrease with increase of forging temperature up to about 750° C., at which temperature the hardness rises. The forged steel then becomes softer as the forging temperature is raised to about 890° C., when it suffers an increase in hardness, which reaches a maximum at about 950° C. Beyond 950° C. the forged steel softens again.

¹Representing the average of the reductions obtained under blows of 452 and 508½ ft.-lb. energy content. These values represent, therefore, the percentage reduction under a blow of approximately 480 ft.-lb. energy content.

SERIES 3.—ON WHETHER A CRITICAL RANGE OF COMPLETED DEFORMATION EXISTS IN THE ENERGY-DEFORMATION RELATION FOR METALS SUBJECTED TO SINGLE BLOWS UNDER THE DROP-HAMMER.

In Table VII and in Fig. 20 are shown the results of a series of forging tests on 99.95 per cent. copper normal $\frac{1}{2}$ -inch samples. The results of these tests appear to indicate that no critical range exists in the energy-deformation relation of copper under the conditions of these experiments.

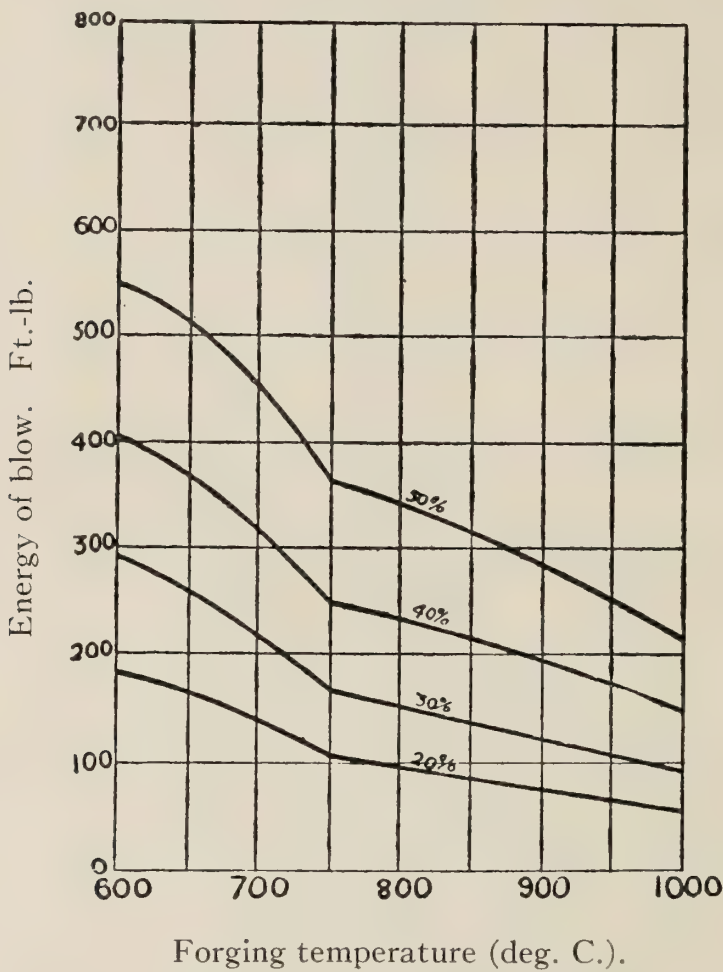


Fig. 11.—Steel No. 2. Carbon, 0.23 per Cent.

TABLE VII—RELATION BETWEEN ENERGY OF BLOW AND PERCENTAGE DEFORMATION OF COPPER AT 20° C.

Energy of Blow Ft.-lb.	Percentage Reduction in Height of Normal $\frac{1}{2}$ -inch Sample	Energy of Blow. Ft.-lb.	Percentage Reduction in Height of Normal $\frac{1}{2}$ -inch Sample
28 $\frac{1}{4}$	12.5	226	41.2
56 $\frac{1}{2}$	18.1	282 $\frac{1}{2}$	45.3
84 $\frac{3}{4}$	23.1	339	51.0
113	26.9	395 $\frac{1}{2}$	53.8
141 $\frac{1}{4}$	30.8	452	57.8
169 $\frac{1}{2}$	36.6	508 $\frac{1}{2}$	61.5

Reference to the complete series of tests recorded in Tables I to IV and to the curves in Figs. 6 to 9, which are based on certain of these complete series of tests, will show that no critical range characterizes the energy-deformation relation of steel under the conditions of these experiments.

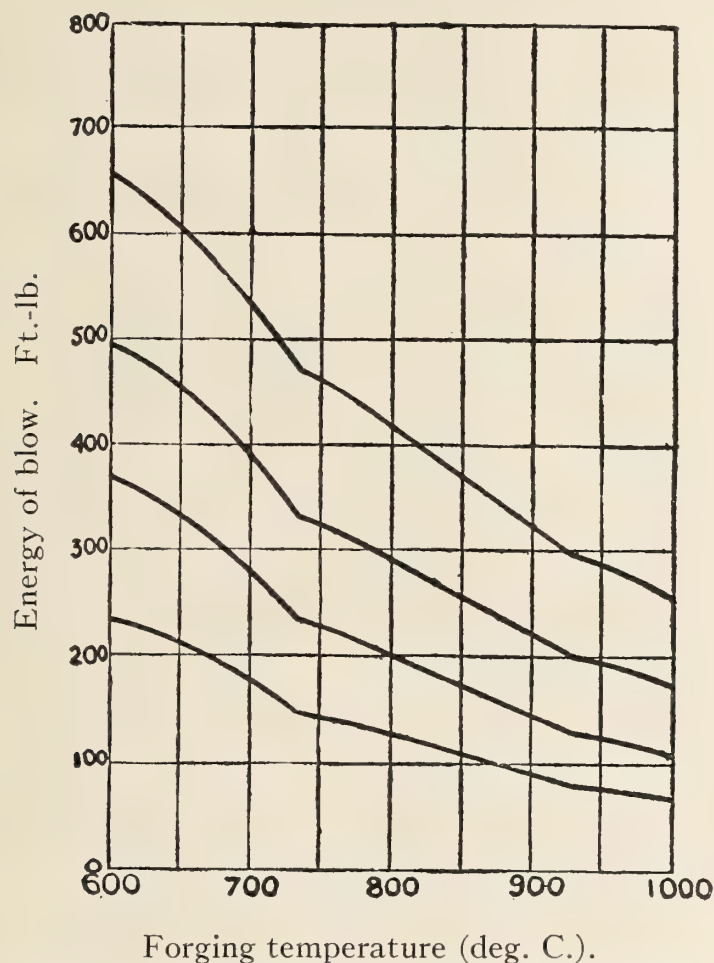


Fig. 12.—Steel No. 3. Carbon, 0.40 per Cent.

For every percentage reduction in length of normal samples of copper and of steel, and possibly of all metals and alloys, there exists then a blow of definite energy content for every temperature. It is therefore impossible to effect a given percentage reduction by a blow of less energy content than that normal to the steel for any given temperature—as might be the case did a critical range of completed deformation exist.

SUMMARY OF THE RESULTS OF THE INVESTIGATION

1. The constitution of steel has a marked effect upon its malleability. That this is the case is proved by the fact that it is possible roughly to determine the positions of certain of the critical points by examination of the malleability-temperature curve for a given steel. The critical points as so determined for the steels investigated by the author are as follows:

Carbon Content	See Steel Number	A1	A2	A3
0.08	1	Not visible	780° C.	870° C.
0.23	2	750° C.	Not visible ¹	Not visible
0.40	3	730 "	"	"
0.56	4	730 "	"	"

2. The atomic rearrangement which occurs at Ac3 appears to result in an increase in the resistance of iron to deformation (*vide* Figs, 2, 10, and 18).²

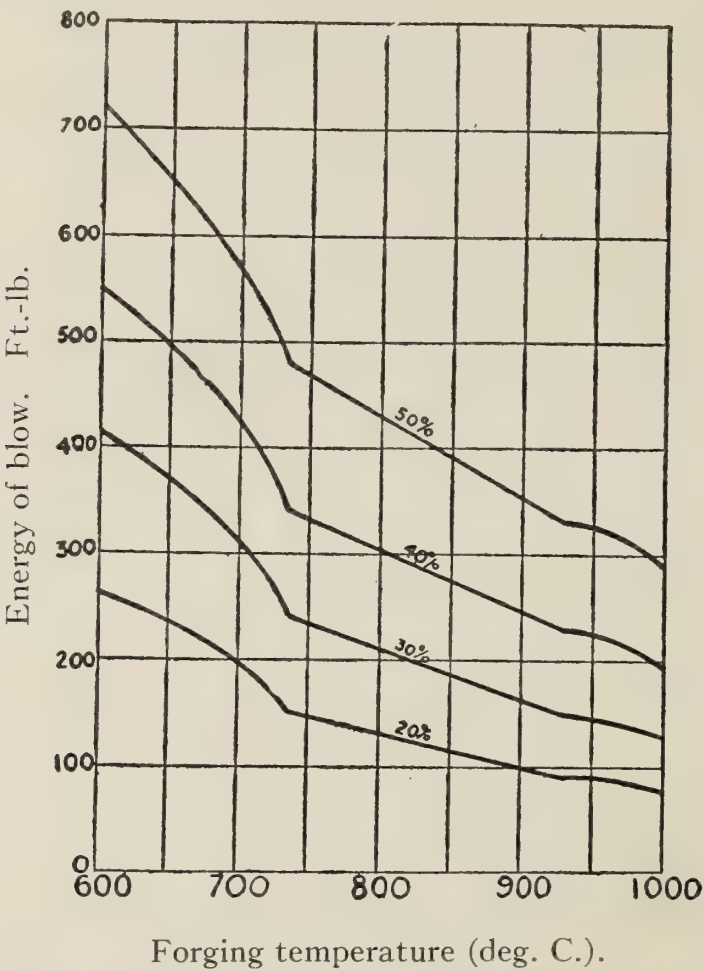


Fig. 13.—Steel No. 4. Carbon, 0.56 per Cent.

3. Evidence is forthcoming of a peculiarity in the behaviour of steels at temperatures in the vicinity of 930° C. This peculiarity is exhibited in the case of all the steels investigated. In the case of the

¹A2 may be merged with A1. See remarks above (p. 178).

²Since this discovery was made—it was reported by letter to the Secretary of the Iron and Steel Institute on September 1, 1922—Professor Sauveur has independently noted the inferior malleability of γ -iron relative to β -iron in the neighbourhood of A3 (First Dr. Henry M. Howe Memorial Lecture, American Institute of Mining and Metallurgical Engineers, 1924, see *Iron Age*, vol. cxiii, No. 8, p. 581).

0.08 per cent. carbon steel the hardness-temperature relationship (cf. also the hardness-temperature relationship for the 0.11 per cent. carbon steel, Fig. 19) shown in Fig. 14 reveals it at 930° C., at which

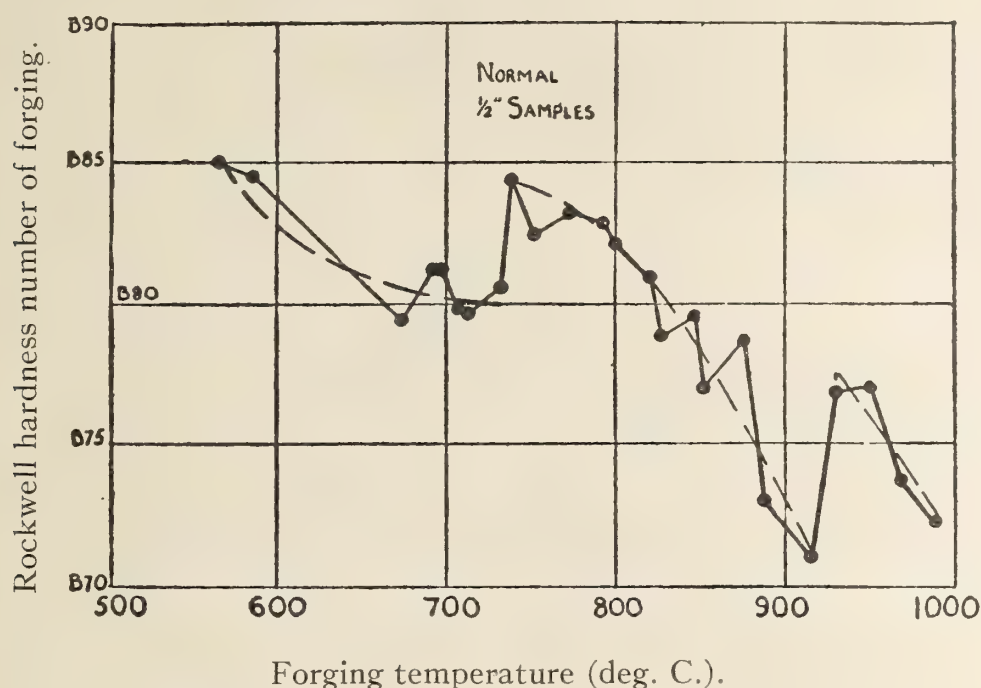


Fig. 14.—Steel No. 1. Carbon, 0.08 per Cent.

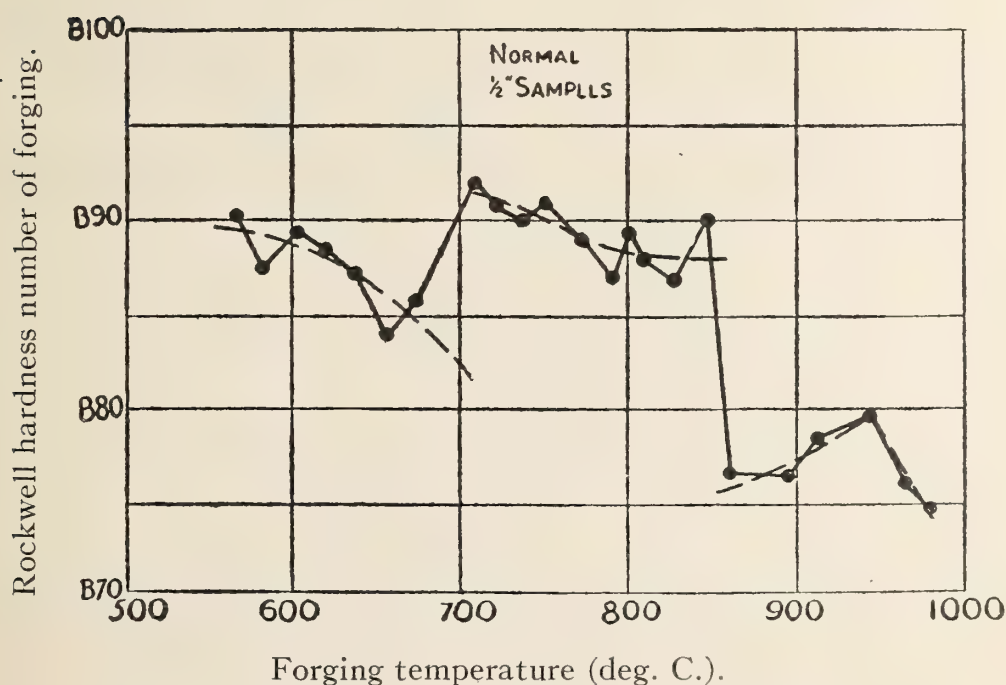


Fig. 15.—Steel No. 2. Carbon, 0.23 per Cent.

temperature the forged samples suffer a marked increase in hardness. In the case of the 0.23 per cent. carbon steel the same relationship (Fig. 15) reveals it, though not so clearly, at about the same temperature,

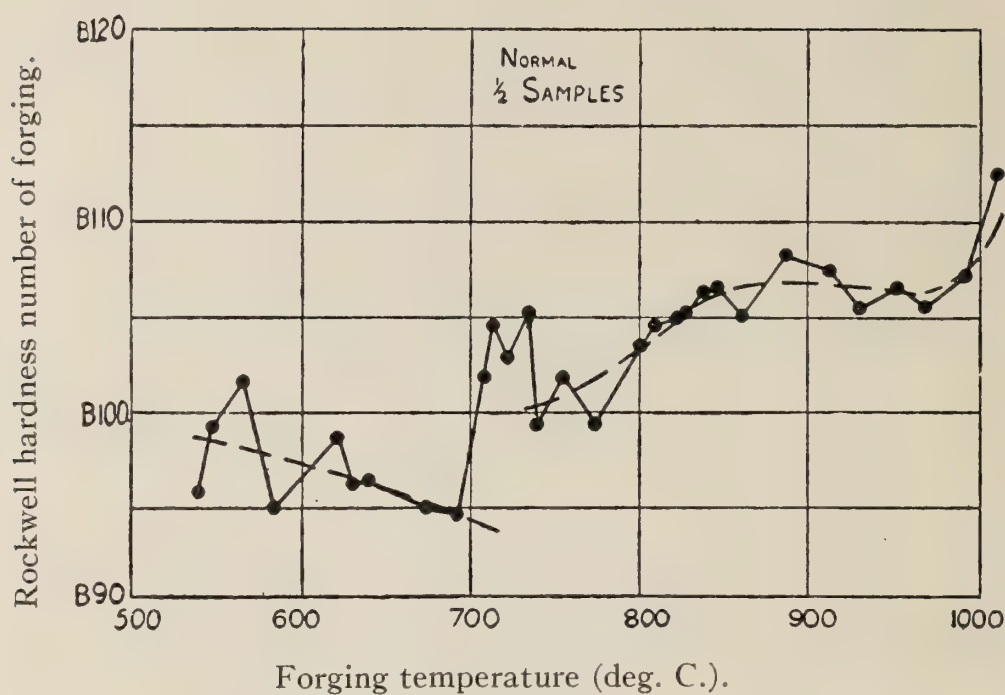


Fig. 16.—Steel No. 3. Carbon, 0.40 per cent.

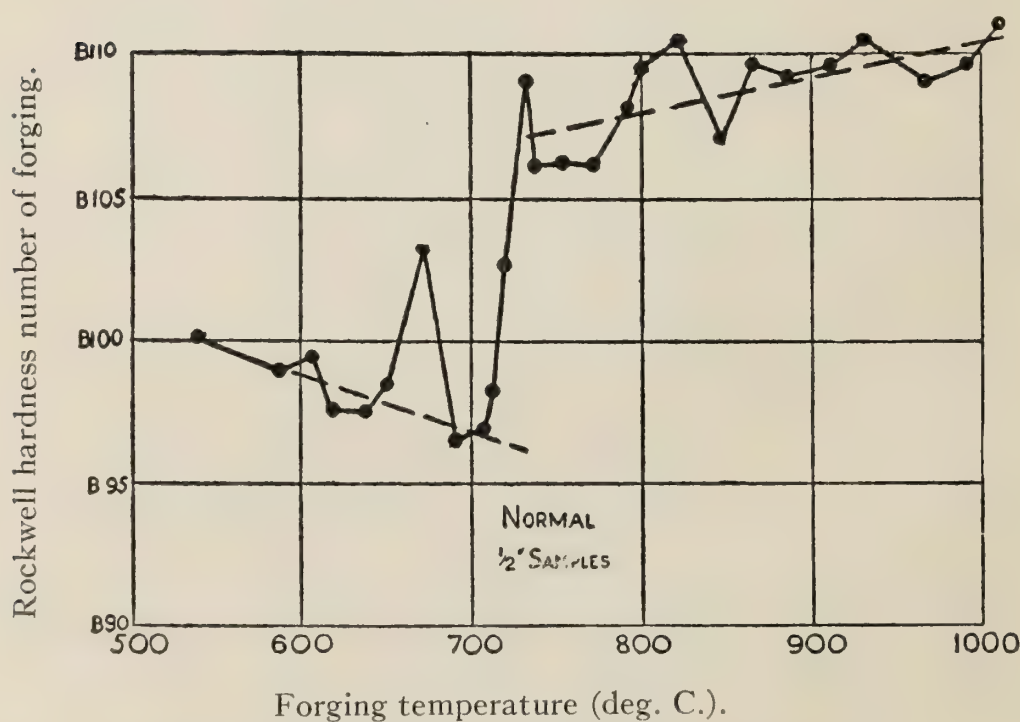


Fig. 17.—Steel No. 4. Carbon, 0.56 per Cent.

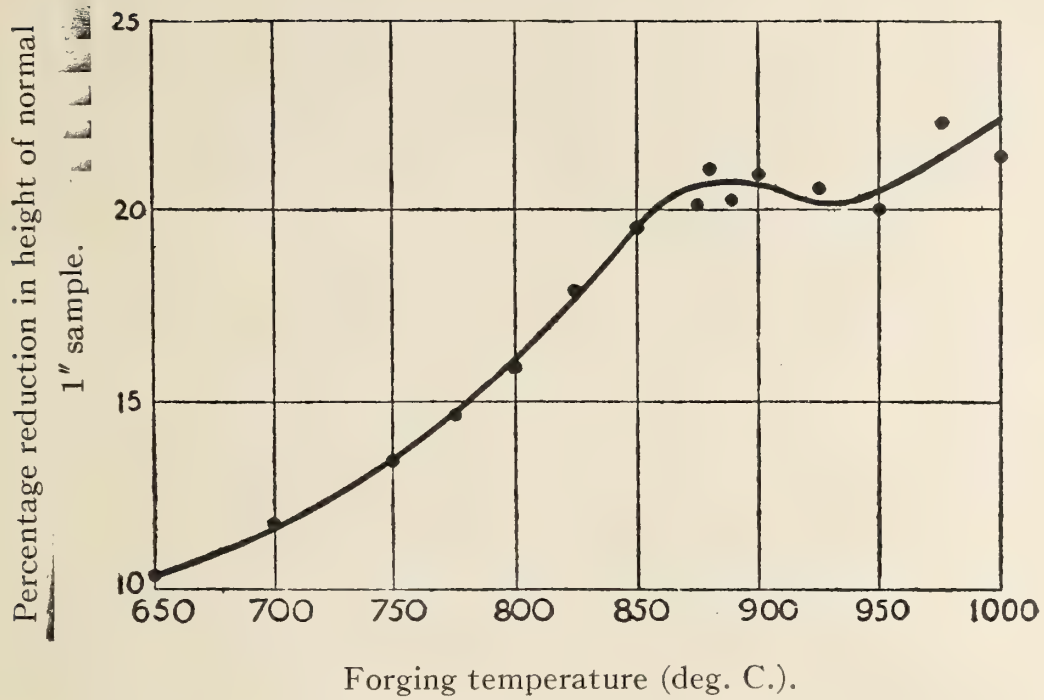


Fig. 18.—Steel No. 5. Carbon, 0.11 per Cent.

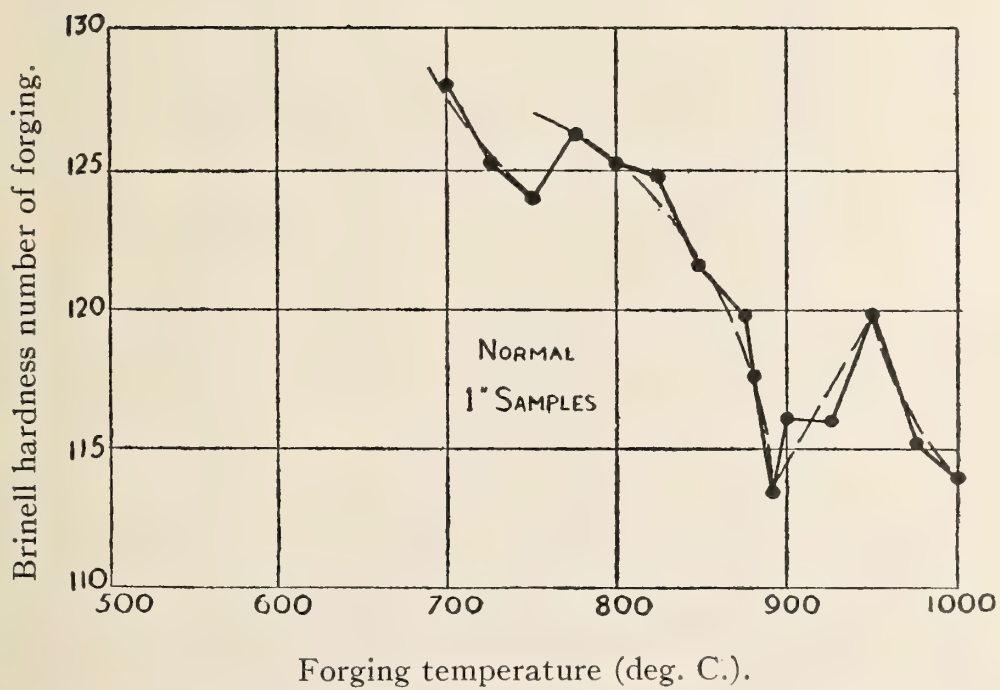


Fig. 19.—Steel No. 5. Carbon, 0.11 per Cent.

930° C. In the case of the higher carbon steels the malleability-temperature curves show an inversion at about 930° C., the malleability falling slightly at this temperature. It appears likely that the change in the rate of growth of the austenite crystals, referred to by Saldau¹ as a possible cause of the change in resistivity which occurs in this vicinity (actually at 980° C. in the case of Saldau's experiments), may account for the observed phenomena.

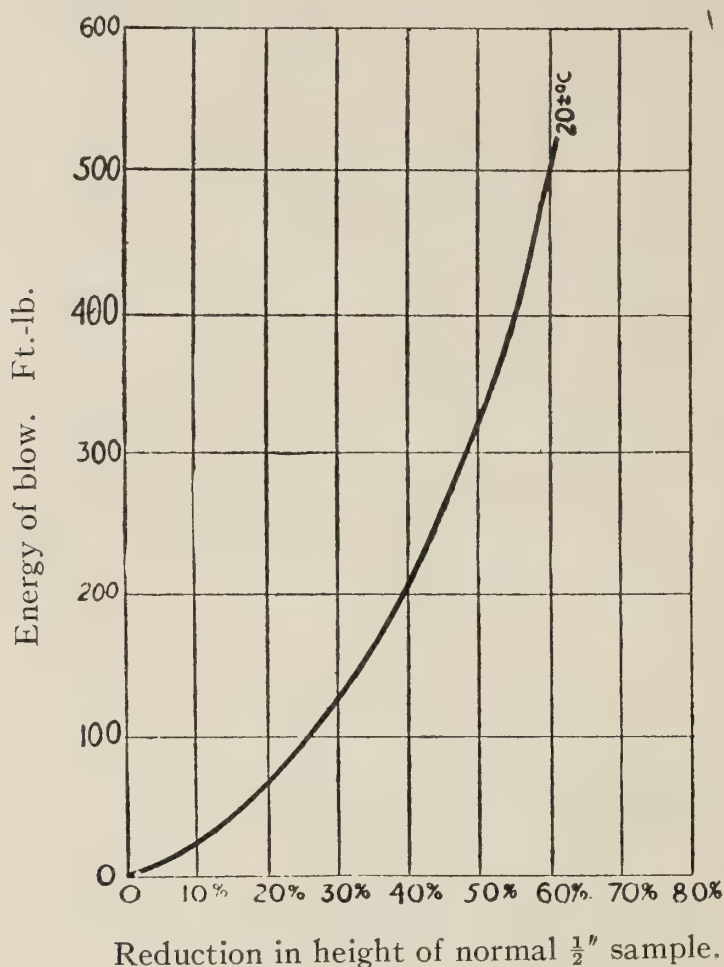


Fig. 20.—Copper.

4. There is no evidence forthcoming of the existence of a critical range of completed deformation in the case either of steel or of copper.

APPENDIX I

The Relation between the Percentage Reduction in Height and the Percentage Increase in Diameter of Forged Normal Samples.

In order that the malleability of steel may be expressed in terms of the percentage increase in the diameter² of normal samples as the result of forging reference may be made to Fig. 21, which shows graphically the relation existing between these two factors. This relationship is, of course, independent of temperature.

¹*Loc. cit.*

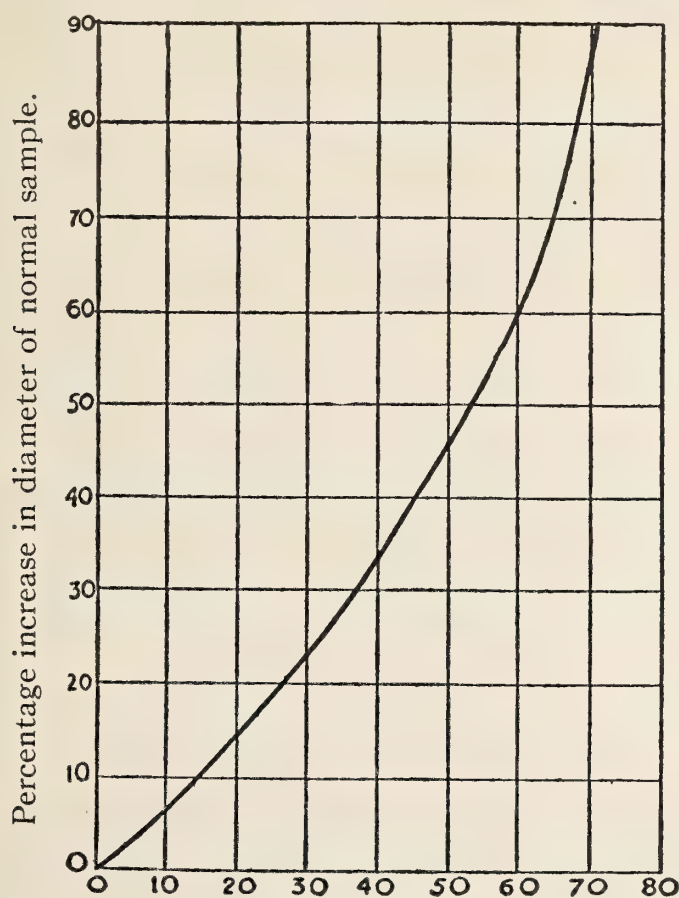
²The diameter referred to here includes diametrical bulge.

APPENDIX II

Application of the Results of the Investigation to other Practice

How far the results detailed above may be applied to other practice it is difficult to say. According to Tresca,¹ the energy required to produce analogous changes of shape of geometrically similar bodies in similar physical state varies as the volumes or weights of the bodies. Assuming the above theorem to be true, one would expect that, other things being equal, the energy required to reduce a normal 1-inch sample of metal would be 8 times, and that required to reduce a normal 2-inch sample of metal would be 64 times that required to reduce a normal $\frac{1}{2}$ -inch sample of the same metal.

That the above theorem roughly applies within the limits of the author's experiments is shown by the fact that, while at 870° C. the energy required to reduce 0.08 per cent. carbon steel normal $\frac{1}{2}$ -inch samples by 20 per cent. in height is 60 ft.-lb., that required to reduce a somewhat similar steel (the 0.11 per cent. carbon steel) in the form of normal 1-inch samples by the same amount is practically 480 ft.-lb., as would have been predicted on the Tresca hypothesis.



Percentage reduction in height of normal sample.

Fig. 21.

¹*Comptes Rendus*, 1869, vol. lxxviii, p. 1197; 1883, vol. xcvi, p. 816.

It is of interest in this connection also to compare certain of the author's results with certain of those obtained by Robin,¹ whose work on the drop-forging of steel has been of much help to the author in the prosecution of his research. To Robin (p. 98 of his Report) are due the following values² for the energy required to compress normal samples of a 0.53 per cent. carbon steel by 20 per cent. of their initial height at 1000° C.

Diameter of Sample. (Inch)	Height of Sample (Inch)	Volume of Sample (Cubic Inch)	Energy required to Compress Sample 20 per Cent. (Ft.-lb.)
0.59	0.59	0.162	117
0.39	0.39	0.048	37

For comparison with the above the following results of the author's tests on the 0.53 per cent. carbon steel at 1000° C. are worthy of note (see Fig. 13). With this steel, in the form of normal ½-inch samples, the following test results were obtained:

Diameter of Sample. (Inch)	Height of Sample (Inch)	Volume of Sample (Cubic Inch)	Energy required to Compress Sample 20 per Cent. (Ft.-lb.)
0.50	0.50	0.098	74

How far the Tresca theorem applies here may be judged from the following table.

Diameter of Normal Sample (Inch)	Volume of Normal Sample (Cubic Inch)	Energy required to Compress Normal Sample 20 per Cent. in Height		Observer
		Observed (Ft.-lb.)	Estimated (Tresca Theorem) (Ft.-lb.)	
0.59	0.162	117	.. ³	Robin
0.50	0.098	74	71	Ellis
0.39	0.048	37	35	Robin

Within what limits the Tresca theorem may be employed as a basis for calculation, using such results as those of the author (Figs. 10 to 13),

¹*Carnegie Scholarship Memoirs*, 1910, vol. ii, p. 1.
²These values have been calculated by the author from Robin's figures.
³The result observed in this case was employed as the basis in estimating the energy values quoted below.

it is hard to say, but the belief that the limits are fairly wide appears to have been accepted both by Tresca and Robin. There can be no

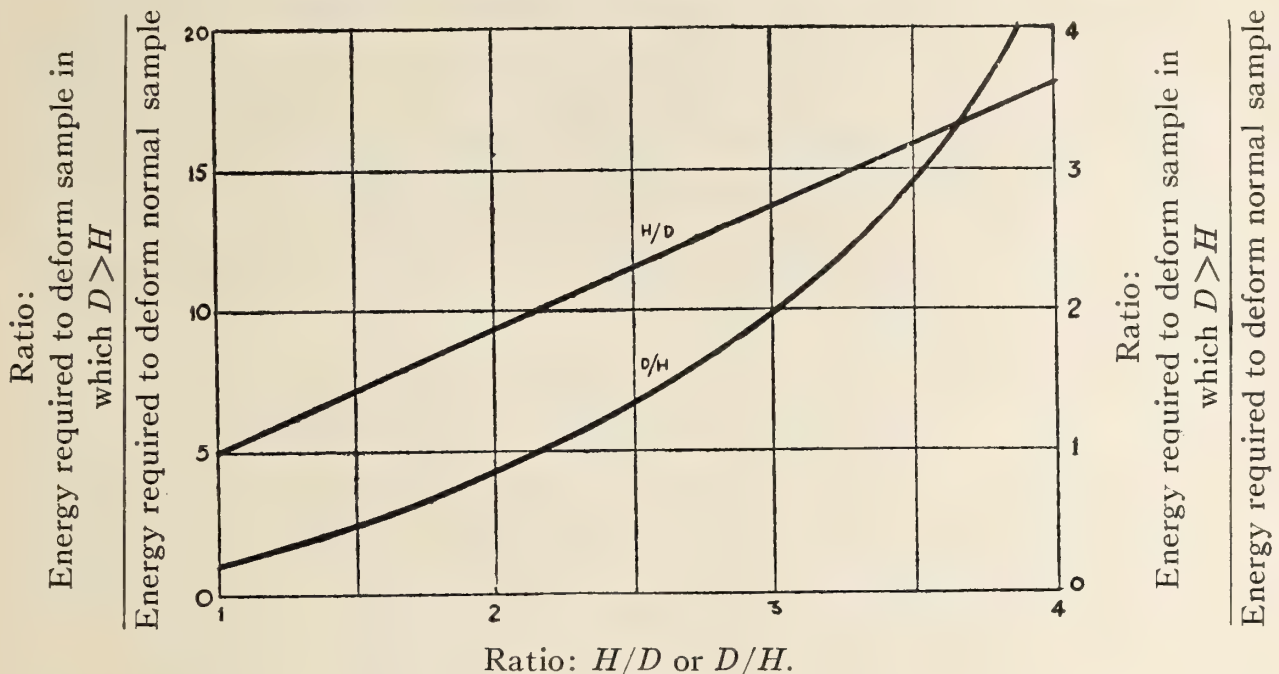


Fig. 22.

N.B.—These curves apply only to percentage reductions in height of samples of 20.

question, however, that the law holds in the case of normal samples of steel up to 2 inches in diameter (Tresca), and it is likely that it may be carried further.

APPENDIX III

The Influence of the Form of the Test Samples upon the Energy required to Deform the Same.

The above research has dealt with normal samples only.

In order to determine the effect of variations in the length or the height of the test samples, a number of tests was made on copper at room temperature and on steel at elevated temperatures. It was found that the results could be successfully related with those of similar experiments by Robin.¹ In Fig. 22 are shown, therefore, curves based upon these tests, which represent graphically the influence of variations in the ratio of the factors diameter and height upon the energy required to deform metallic samples generally. Of these curves that relating the ratio D/H (D =diameter of sample; H =height of sample) to the ratio E'/E (E' =energy required to deform sample in which D/H or H/D is greater than 1; E =energy required to deform normal sample by the same amount) may be considered as closely approximating to the truth at all temperatures. That relating the ratio H/D to the ratio E'/E is somewhat less accurate, but nevertheless is probably sufficiently correct for most practical purposes.

¹*Loc. cit.*, p. 79 ante.

CONCLUSION

The author wishes to thank Prof. G. A. Guess, M.A., for his encouragement and assistance; Mr. D. A. Schemnitz, M.A.Sc., who assisted the author in the earlier stages of this research; Mr. A. A. Bell, B.A.Sc., Research Assistant in Metallurgical Engineering, who helped the author in the later stages of the research; Professor L. J. Rogers, who made analyses of the steels; Mr. J. J. Stark, for his valued assistance; and Mr. W. K. Simpson, for the preparation of numerous test samples.

EXPERIMENTS ON THE HEAT TREATMENT OF ALPHA-BETA BRASS

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Paper presented at the Milwaukee Meeting of the American Institute of
Mining and Metallurgical Engineers, October, 1924

Certain alloys¹ that, as a result of quenching, are retained in the form of homogeneous solid solution are known to increase in hardness and strength on standing at room temperature or on heating at slightly elevated temperatures. Other alloys² that, on rapid cooling, are held in the form of homogeneous solid solution (as, for example, certain steels) behave in like manner on heating to somewhat higher temperatures. The observed increase in the hardness and strength of these alloys, referred to as "aging" or as "secondary hardness," is widely accepted as being occasioned, either in whole or in part, by the precipitation in submicroscopic form of such constituents as were held in supersaturated solid solution as a result of cooling in excess of a certain critical rate.

In earlier discussions of the cause of age hardening or secondary hardness, the question of the hardness of the precipitated particles does not appear to have been raised. At present, however, the general opinion appears to be that age hardening and secondary hardness are occasioned by the precipitation of *hard* particles within the supersaturated solid substance.³

The authors may be wrong in their belief that the last is a prevailing opinion. Nevertheless, it is thought that the following experiments,

¹Wilm, *Metallurgie* (1911), 225; Jeffries, *Jnl. Inst. of Metals* (1919), 22, 239; Merica, Waltenberg, and Scott, *Trans.* (1920), 64, 41; Fraenkel and Send, *Zeit. für Metall.* (1920), 12, 225; Rosenhain, Archbutt, and Hanson, *Inst. Mech. Eng.*, Eleventh Report to the Alloys Research Committee (1921), etc.

²Scott, *Trans. Am. Soc. Steel Treat.* (June, 1921), 511; Bain, *Trans. Am. Soc. Steel Treat.* (Jan., 1924), 89.

³See, for example, the remarks of Jeffries and Archer, *Trans. Am. Soc. Steel Treat.* (1923), 4, 283: "It is postulated that the increase in hardness of freshly quenched martensite on standing at room temperature or after mild tempering is due to the precipitation of cementite, the *hard* cementite particles 'keying' the slip planes of ferrite grains." This statement, of course, cannot be taken as representing the sentiment of these authors in respect of all types of "aging" and of "secondary hardness." [*Italics are the authors'.*]

which were carried out mainly with the view of determining whether the precipitation of *soft* particles within a supersaturated solid solution would not enhance its hardness, should be of interest.

EFFECT OF REHEATING QUENCHED ALPHA-BETA BRASS ON ITS MICROSTRUCTURE

The question whether the reheating of true alpha-beta brasses⁴ quenched from such temperatures and at such rates as to obtain homogeneous beta solid solution would result in mechanical improvement was taken up by one of the authors in 1918-19. The results of preliminary experiments did not encourage him to proceed with the investigation, it being noted that a structural arrangement likely to cause mechanical weakness was always obtained. This structural arrangement is shown in Figs. 1 and 2, which are photomicrographs of sections of two brasses which, prior to reheating at 400° C. for 40 hr., were in the form of homogeneous beta crystals. In both sections, it will be noted, excess alpha has been deposited, not only within the crystals, but at the crystal boundaries. Stead and Stedman⁵ have clearly shown that this structural arrangement conduces to mechanical weakness. This structural arrangement, it should be noted, was found in these alloys subsequent to annealing at 400° C. for periods of less than one hour (see Table 8).

The deleterious effect on tensile strength that might have been expected to result from the production of such a structural arrangement as is shown in Figs. 1 and 2 has been fully confirmed by the tensile tests made in connection with the experiments herein described. The results of these tensile tests, which are quoted in Tables 2, 4 and 6, prove that, while the mechanical condition of the alpha-beta alloys may, under certain circumstances, be improved by heat treatment and even by quenching, the effects of such treatments as comprise quenching and reheating to temperatures below the $\alpha + \beta \rightarrow \beta$ transformation temperature are little short of disastrous.

But the tests herein recorded were not made to confirm the opinion that the reheating of quenched alpha-beta brass would be of no commercial value. They were made primarily because it was recognized that alpha-beta brass was an alloy that could be obtained in the form of a supersaturated solid solution wherein could be deposited, as a result of reheating, particles (alpha) actually softer than the solid solution (beta) in which they were generated.

⁴Brasses that contain both the alpha and beta constituents after reheating to 400° C. subsequent to quenching with the view of obtaining pure β .

⁵Jnl. Inst. of Metals. (1914, 11, 119).

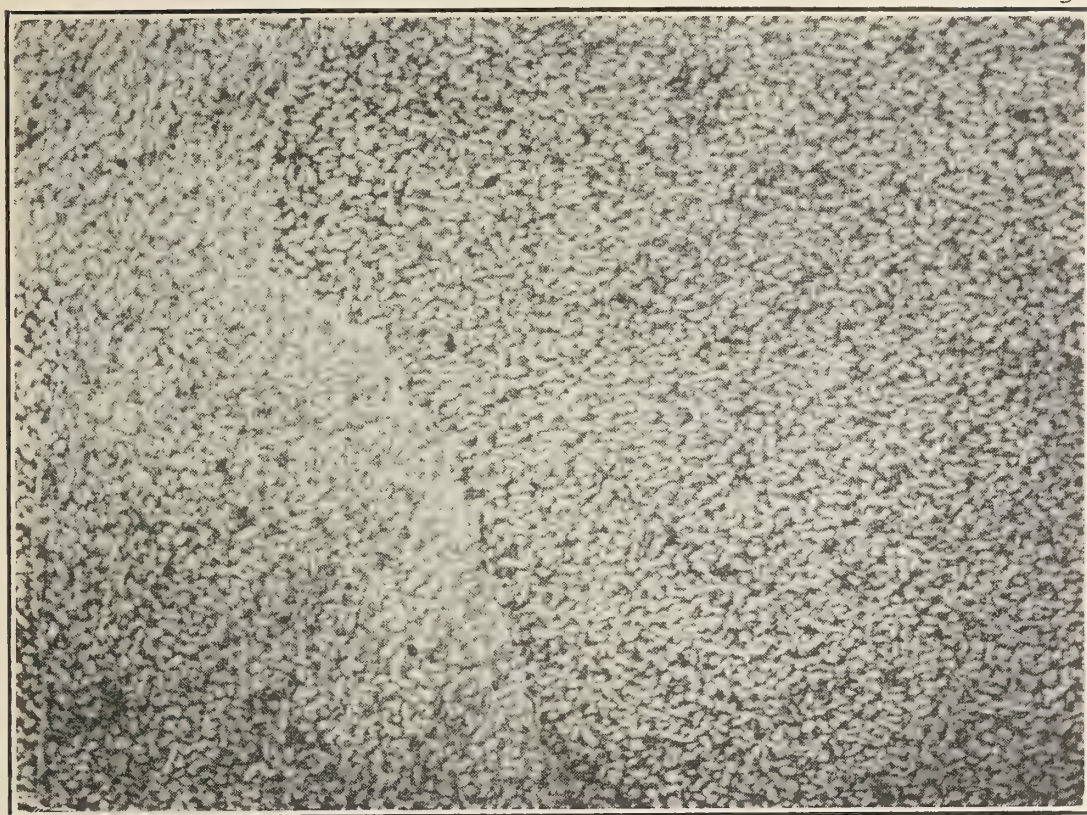


FIG. 1.—Brass containing 57.40 per cent. copper; reheated subsequent to quenching (700° C.) at 400° C. for 40 hr.; alpha precipitated at beta boundaries. $\times 75$.

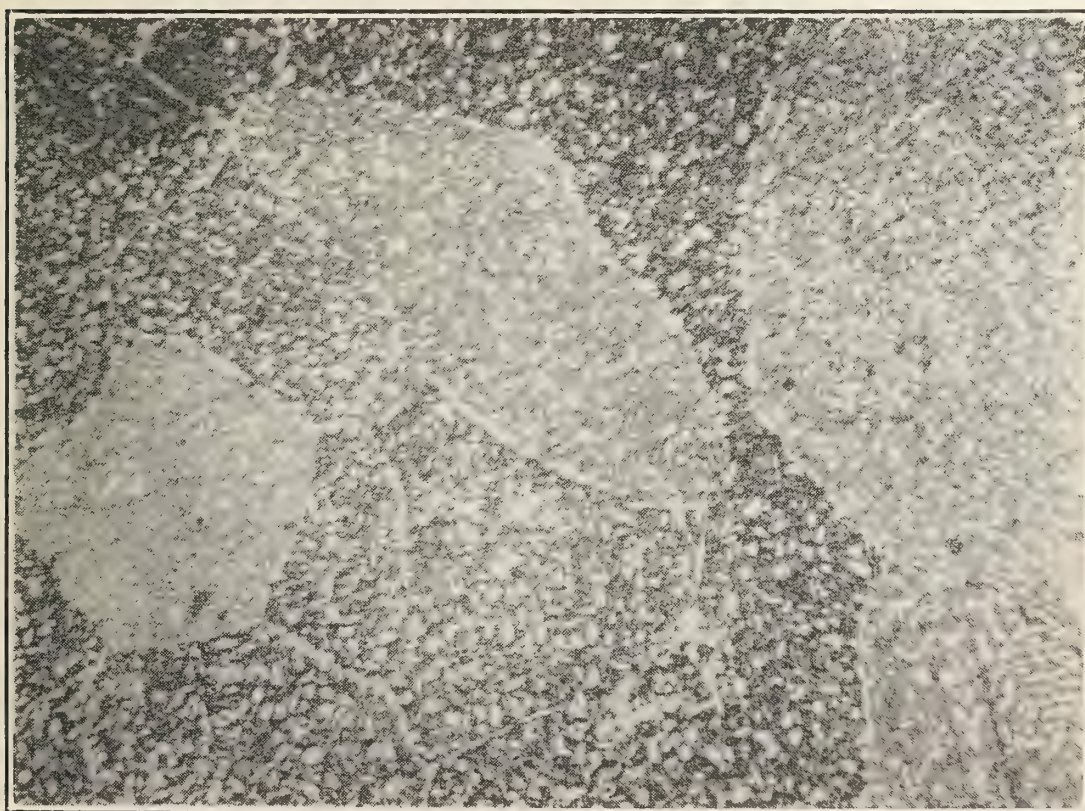


FIG. 2.—Brass containing 56.51 per cent. copper; reheated subsequent to quenching (700° C.) at 400° C. for 40 hr., alpha precipitated at beta boundaries. $\times 75$.

ALLOYS USED IN THE EXPERIMENTS

There were used in the experiments two alloys of the following compositions:

Alloy	A Per Cent.	B Per Cent.
Copper.....	56.66	52.02
Nickel.....	nil.	3.86
Zinc.....	difference	difference

which will be referred to throughout this paper as alloys A and B, respectively.

The alloys were in the form of chill castings, from which prismatic samples $\frac{1}{2}$ by $\frac{1}{2}$ by 3 in. were cut for purposes of test. These samples were annealed in bulk at a temperature of 725° to 750° C. for 1 hr. prior to other treatment. The mechanical properties of the alloys, subsequent to the preliminary anneal, were as follows:

Alloy	A	B
Maximum stress, lb. per sq. in.....	54,100	51,200
Percentage elongation.....	39.5	32.2
Brinell hardness number.....	102	116

The test specimens used in making the mechanical tests were small, having gauge lengths of only 1.155 in. and diameters of 0.326 in. The gauge lengths of the specimens were four times the square roots of their cross-sectional areas. The Brinell tests were made with a 10-mm. ball and a load of 500 kg. It will be noted that in so far as their mechanical properties were concerned the annealed castings were of fair average quality.

By the expedient of quenching a number of small samples of the two alloys subsequent to their complete attainment to various temperatures, the $\alpha + \beta \rightarrow \beta$ transformation temperatures were approximately determined. For the pure brass, the transformation point under these conditions was found to be in the vicinity of 670° C. (under conditions of equilibrium the transformation point is lower); for the nickel brass it was found to be in the neighbourhood of 720° C. The test samples for these experiments were, therefore, quenched in ice water from 720° C. and 770° C. (50° C. above the $\alpha + \beta \rightarrow \beta$ transformation temperature) respectively. Subsequent to quenching the samples were examined microscopically to confirm the absence of alpha.

NATURE OF EXPERIMENTS

After quenching, the samples were reheated; some to 100° C., some to 200° C., some to 300° C., and some to 400° C. Prior to reheating, a determination of the Brinell hardness number of each sample was made.

A similar determination was made after the thermal treatment and the percentage increase in hardness, if any, was then calculated. This method of estimating the observed increase in the hardness of the alloy seemed to be justified by the fact that the maximum variation from the average hardness number of the test samples prior to reheating was, in both alloys, less than 3.0 per cent.; while the percentage increase in hardness as a result of reheating reached, in some instances, as high as 12.8 per cent.

Reheating of the test samples was conducted for time periods varying from $\frac{1}{4}$ to 40 hr. The samples were in all cases introduced into a furnace kept at the reheating temperature. They were then allowed to reach the temperature of the furnace, the time of reheating being measured from the time at which the samples first attained the temperature of the furnace.

Subsequent to reheating, the samples were thoroughly examined microscopically for traces of alpha. It was found in most cases extremely difficult to determine whether alpha had been precipitated at the boundaries of the beta grains, but it is certain that, in some instances, such precipitation had occurred when microscopic examination of both etched and unetched samples at high powers failed to reveal its presence. A variety of etching reagents was employed in this connection.

The samples were, in all cases, tested by the Brinell test and, in some instances, in tension by the usual methods of tensile test.

RESULTS OF REHEATING QUENCHED ALPHA-BETA BRASS AT 100°, 200° AND 300° C.

Reheating at 100° ± 2° C.

The reheating at 100° ± 2° C. was carried out in an electric drying oven thermostatically controlled; the results of the hardness tests and microscopic examinations made on the two series of samples are given in Table 1.

It should be noted that the authors think that in many cases alpha had been precipitated at the beta grain boundaries, though they were unable to detect its presence directly. The grain boundaries subsequent to etching always appeared as dark lines even when the samples were examined at high magnification. It was thought that this might be due to the fact that such light as arrived at the surface of the etched samples in the neighbourhood of the boundaries was reflected away from the optical axis of the microscope. Many samples were, therefore, examined in the unetched state, but alpha could not be detected even under these conditions. The colour of the fractures of the tensile test samples

(Table 2), however, afforded indirect proof of this precipitation. It can be stated with confidence that in no case had visible precipitation of alpha occurred within the beta crystals. The results of the tensile tests made on the two series of samples are recorded in Table 2.

Certain of the broken test specimens were characterized by what has been referred to in the table as a duplex fracture. The normal fracture of quenched alpha-beta brass is a dull brown-red. Certain of the above test specimens, however, possessed a coarsely crystalline

TABLE 1.—EFFECT ON HARDNESS OF REHEATING QUENCHED SAMPLES OF ALLOYS A AND B AT 100° C.

Alloy	Brinell Hardness Numbers		Time of Reheating Hours	Percentage Increase in Hardness	Structure Revealed by Microscope
	Prior to Reheating	Subsequent to Reheating			
A	136	136	¼	nil	Beta
	130	130	½	nil	
	133	133	1	nil	
	130	130	2	nil	
	136	138	4	1.5	
	136	143	6	5.1	
	130	143	8	10.0	
	133	150	16	12.8	
	130	146	32	12.3	
B	146	146	¼	nil	Beta
	150	150	½	nil	
	143	143	1	nil	
	143	143	2	nil	
	146	146	4	nil	
	146	146	6	nil	
	150	158	8	5.3	
	143	158	16	10.5	
	143	158	32	10.5	

bright yellow fracture—the yellow approaching in tint that of the fracture of alpha brass of maximum zinc content. Others (those having a duplex fracture) had fractures wherein were exposed to view constituents having both the dull brown-red and the bright yellow colors. It has been assumed by the authors that the bright yellow of these fractures has been occasioned by precipitation of the alpha constituent at the grain boundaries, such bright yellow fractures having always been found in those samples reheated for long periods at 400° C., which showed clear microscopic evidence of the precipitation of alpha at the grain boundaries.

TABLE 2.—EFFECT ON TENSILE STRENGTH OF REHEATING QUENCHED SAMPLES OF ALLOYS A AND B AT 100° C.

Alloy	Condition of Sample	Time of Reheating Hours	Tensile Strength Pounds per Square Inch	Elongation Per Cent.	Fracture
A	Quenched	nil	46,320	17.4	Dull brown red
	Reheated	6	46,700	13.0	Dull brown red
	Reheated	8	42,700	12.2	Dull brown red
	Reheated	16	49,360	20.0	Duplex
	Reheated	32	7,860	3.0	Duplex
B	Quenched	nil	45,150	Not noted	Dull brown red
	Reheated	6	6,600	2.6	Duplex
	Reheated	8	12,780	5.2	Duplex
	Reheated	16*			Duplex
	Reheated	32	14,040	2.0	Bright yellow

*Broken while being machined.

Whereas the presence of nickel retards the precipitation of the alpha constituent, it does not oppose the deleterious ultimate effect of that precipitation.

TABLE 3.—EFFECT ON HARDNESS OF REHEATING QUENCHED SAMPLES OF ALLOYS A AND B AT 200° C.

Alloy	Brinell Hardness Numbers		Time of Reheating Hours	Percentage Increase in Hardness	Structure Revealed by Microscope
	Prior to Reheating	Subsequent to Reheating			
A	130	130	¼	nil	Beta
	133	133	½	nil	
	133	133	1	nil	
	130	130	2	nil	
	133	133	4	nil	
	136	138	8	1.5	
	130	140	12	7.7	
	133	150	16	12.8	
	130	143	40	10.0	
B	146	146	¼	nil	Beta
	150	150	½	nil	
	150	150	1	nil	
	150	150	2	nil	
	150	150	4	nil	
	146	146	8	nil	
	146	152	12	4.1	
	146	158	16	8.2	
	150	158	40	5.3	

TABLE 4.—EFFECT ON TENSILE STRENGTH OF REHEATING QUENCHED SAMPLES OF ALLOYS A AND B AT 200° C.

Alloy	Condition of Sample	Time of Reheating, Hours	Tensile Strength Pounds per Square Inch	Elongation Per Cent.	Fracture
A	Quenched	nil	46,320	17.4	Dull brown red
	Reheated	8	41,880	12.2	Duplex
	Reheated	12	49,300	21.7	Dull brown red
	Reheated	16	38,370	6.1	Duplex
	Reheated	40	31,800	8.7	Duplex
B	Quenched	nil	45,500	Not noted	Dull brown red
	Reheated	8*			Bright yellow
	Reheated	12	18,120	2.0	Bright yellow
	Reheated	16	5,580	4.3	Bright yellow
	Reheated	40*			Bright yellow

*Broken while being machined.

It is of interest, also, to note that reheating has reduced the ductility of the A alloy in certain cases, even though no evidence of the precipitation of alpha was observed either in the fracture of the test specimen or in etched sections of the same.

Reheating at 200° ± 10° C.

The reheating at 200° ± 10° C. was carried out in an electric-resistance crucible furnace, controlled by a variable external resistance. The results of the hardness tests and microscopic examinations made on the two series of samples are given in Table 3. The results of the tensile tests made on the two series of samples are recorded in Table 4.

The same remarks may be applied in respect of the results given in these tables as were applied to the results obtained with the samples reheated at 100° C.

Reheating at 300° ± 10° C.

The reheating at 300° ± 10° C. was carried out in an electric-resistance crucible furnace, controlled by a variable external resistance. The results of the hardness tests and microscopic examination made on the two series of samples are given in Table 5. The results of the tensile tests made on the two series of alloys are recorded in Table 6.

In these experiments no doubt as to the precipitation of alpha existed, except in those samples that had been reheated for the shorter periods of time. In the case of these samples, however, the fractures of the tensile test specimens gave indirect evidence of the precipitation of

TABLE 5.—EFFECT ON HARDNESS OF REHEATING QUENCHED SAMPLES OF ALLOYS A AND B AT 300° C.

Alloy	Brinell Hardness Numbers		Time of Reheating, Hours	Percentage Increase in Hardness	Structure Revealed by Microscope
	Prior to Reheating	Subsequent to Reheating			
A	133	136	$\frac{1}{4}$	2.3	Beta
	133	138	$\frac{1}{2}$	3.8	Beta
	133	140	1	5.3 <i>B</i>	Beta + Alpha
	133	146	4	9.8 <i>B</i>	
	133	150	8	12.8 <i>B</i>	
	133	148	16	11.3	
	133	143	32	7.5 <i>B</i>	
	136	143	48	5.3 <i>B</i>	
B	143	150	$\frac{1}{4}$	4.9	Beta
	143	152	$\frac{1}{2}$	6.3	Beta
	146	152	1	4.1	Beta
	150*	162	16	8.0 <i>B</i>	Beta + Alpha
	143	156	32	9.1 <i>B</i>	

*This anomalous result, the authors are unable to explain.

TABLE 6.—EFFECT ON TENSILE STRENGTH OF REHEATING QUENCHED SAMPLES OF ALLOYS A AND B AT 300° C.

Alloy	Condition of Sample	Time of Reheating, Hours	Tensile Strength, Pounds per Square Inch	Elongation, Per Cent.	Fracture
A	Quenched	nil	46,320	17.4	Dull brown red
	Reheated	$\frac{1}{2}$	27,480	4.3	Duplex
	Reheated	4	38,700	2.0	Duplex
	Reheated	8	33,600	4.3	Duplex
	Reheated	32	32,820	8.7	Duplex
B	Quenched	nil	45,500	Not noted	Dull brown red
	Reheated	1*			Bright yellow
	Reheated	16*			Bright yellow

*Broken while being machined.

alpha at the boundaries of the beta grains. This precipitation was such as to render the nickel alloy so fragile that it was impossible to turn samples for tensile test from the treated prisms.

AGING AT ROOM TEMPERATURE

To determine the effect of reheating the quenched alloys at 400° C., a series of samples of alloy A that had been quenched two weeks pre-

viously were taken for test. Before reheating at 400° C. the hardness numbers of the samples were checked, when it was found that the hardness had increased during the fortnight. The quenched simple brass is, therefore, subject to “aging” in the most precise sense of this word. In Table 7 are given the results of the two series of tests made, the first immediately after quenching, the second two weeks later.

TABLE 7.—AGING OF QUENCHED BRASS AT ROOM TEMPERATURE

Original Hardness Number	Hardness Number after Standing Two Weeks	Percentage Increase in Hardness
133	145	9.0
133	149	12.0
136	149	9.6
136	148	8.8
133	143	7.5
Average increase.....		9.4

Reheating at 400±10° C.

The samples were retreated subsequent to the second series of hardness tests; that is, they were requenched and then reheated to 400° C. for various periods of time. The results of the hardness tests and microscopic examination made on the brass samples reheated at 400° C. are given in Table 8.

TABLE 8.—EFFECT OF REHEATING QUENCHED SAMPLES OF ALLOY A AT 400° C.

Alloy	Brinell Hardness Numbers		Time of Reheating, Hours	Percentage Increase in Hardness	Structure Revealed by Microscope
	Prior to Reheating	Subsequent to Reheating			
A	136	148	nil*	8.8	Beta + Alpha
	133	143	¼	7.5	
	133	141	½	6.0	
	133	140		5.3	
	133	138	4	3.8	
	133	137	8	3.0	
	133	137	16	3.0	

*Sample heated until it just reached 400° C.

A definite loss in the percentage increased in hardness resulting from reheating was recorded for all periods of time in excess of that required to raise the alloy to 400° C., this simple treatment (mere heating to 400° C.) being, in itself, sufficient to cause precipitation of alpha. No tensile tests were made on these samples, it being considered unnecessary to carry the matter further in view of the poor results obtained as a result of reheating at the lower temperatures.

SUMMARY

By reheating alpha-beta brass, which as a result of quenching is retained at room temperature in the condition of homogeneous beta solid solution, it is possible to cause precipitation of alpha in sub-microscopic form.

This precipitation of alpha has a deleterious influence on the tensile properties of the quenched brass, but has the effect of increasing the Brinell hardness of the material. This latter phenomenon is felt to be worthy of remark, as the precipitated alpha is appreciably softer than the quenched beta whence it is generated. It is well known that age hardening can be occasioned by the precipitation of hard particles (as in the case of duralumin); reheated quenched brass, however, affords an example of a substance subject to age hardening caused by the precipitation of soft particles.

It is difficult to imagine how soft particles can act as "keys" to prevent the relative slip of the atom-bearing planes of a solid solution. If, however, it is postulated that the submicroscopic particles are of the nature of colloidal particles whose boundaries are controlled by surface tension and that at these boundaries there is distortion of the space lattices, both of the particles and of the surrounding solid solution, the increased resistance to the relative slip of the atom-bearing planes of the solid solution occasioned by the presence of the precipitated particles, whether they be hard or soft, can, it is thought, be explained.⁶ The distortion of the space lattices at the boundaries of the particles referred to above opposes the propagation of slip *via* the slip planes of the precipitated substance. The hardness of the material wherein the precipitated substance is generated is therefore increased. The greater the number of such particles (particles whose boundaries are controlled by surface tension) the greater will be the strength of the alloy wherein they occur.

In the process of time, the forces of crystallization will exceed those surface-tension forces that control the boundaries of the particles and growth of the particles will proceed in accordance with the normal laws of crystallization. The spherical particles will change into minute allotriomorphic crystals at whose boundaries no space-lattice distortion will exist and which, therefore, will offer considerably less obstruction to the relative movement of adjacent atom-bearing planes of the matrix than is offered by the spherical particles, this diminution in the resistance offered to slip by the precipitated solute being due to the fact that slip in the matrix can be propagated with relative ease *via* the slip planes of

⁶O. W. Ellis: The Structure of Metals. See report of Winter Sectional Meeting of Am. Soc. for Steel Treating, Rochester, Iron Trade Rev. (1924), 74, 430.

the solute. A reduction in hardness will be the outcome of this change in the form of the precipitate.

To put it briefly: while spherical particles, whether soft or hard, are in process of formation the hardness of the alloy will increase, because of the increasing resistance offered to slip (owing to space lattice distortion at the boundaries of the particles) as a result of the precipitation of particles of this form. When the spherical particles begin to change into minute allotriomorphic crystals the hardness of the alloy will tend to decrease and will actually be lowered (1) when the rate of formation of allotriomorphic crystals exceeds the rate of formation of spherical particles and (2) as the allotriomorphic crystals themselves increase in size.

ON THE MECHANISM OF THE INHIBITION OF THE CATALYTIC ACTION OF PLATINUM BLACK AND PARTIALLY REDUCED NICKEL OXIDE BY CHLORINE

(Reprinted from the Journal of Physical Chemistry)

By M. C. BOSWELL, *Associate Professor of Organic Chemistry*,
and C. H. BAYLEY, *Research Assistant*

Recently the senior author and R. R. McLaughlin have published¹ experimental results which seem to justify the conclusions that a normal platinum catalyst consists of particles of platinum with an interior content of oxygen, the particles being surrounded by a surface layer of dissociated water in the form of charged hydrogens and hydroxyls, and that this external layer is the seat of the catalysis of oxidation, reduction and hydrolysis, commonly observed with platinum black. Analogous results and conclusions had been reached with regard to the nature of partially reduced nickel oxide.² If these conclusions are accepted as a working hypothesis, it may reasonably be deduced, that the observed interference by chlorine, even in very small concentrations and very small absolute amounts, with the normal behaviour of these complexes as catalysts, is due to the disturbance or destruction of these surface films. It was also shown in these two papers the remarkable protection afforded by these surface films for the underlying interior content of oxygen, and also the necessity for the presence of this interior oxygen in order to maintain the surface films in their active condition, especially in the catalysis of hydrogenation reactions where the surface oxygen and interior oxygen are, to some extent, removed. If, then, the so-called poisoning action of chlorine on these catalysts is due to the destruction of the protective surface films, this should show itself by a very greatly increased accessibility of the interior oxygen, and increased ease of its removal by free hydrogen. The experiments described in this paper have given results which confirm these deductions from the theory. As a consequence not only can a fairly satisfactory picture be given of the mechanism of this poisoning action but also confidence in the original working hypothesis of the mechanism of catalysis by these platinum and nickel complexes is considerably increased.

¹Proc. Royal Soc. of Can., 17, 1 (1923).

²Proc. Royal Soc. of Can., 16, 1 (1922).

EXPERIMENTAL

Materials

Platinum catalyst: This was prepared by a modification of Loew's method as described in the above-mentioned platinum paper. The moist platinum black so obtained was mixed with an equal weight of fine asbestos previously purified by acid extraction and washing. The whole was dried at 110° C. for three hours and transferred to the U-tube employed in the experiments. Care was taken to prevent contamination of this platinum by laboratory gases.

Nickel oxide: This was prepared by igniting crystallized nickel nitrate in a porcelain casserole, dissolving the resulting oxide in nitric acid, evaporating to dryness on the water bath, dissolving 5 grams of the nitrate in distilled water, adding 5 grams purified asbestos, the whole ignited and cooled in a vacuum desiccator and one half placed in a U-tube used in the experiments, care being taken to prevent contamination from laboratory gases.

Hydrogen: Electrolytic hydrogen was used. This was freed from traces of oxygen by passing over hot copper gauze.

Oxygen: Compressed oxygen from liquid air and containing only traces of inert gases was used.

Nitrogen: Compressed nitrogen was used containing less than $\frac{1}{2}$ per cent. oxygen which was freed from oxygen by passing over hot copper gauze.

Apparatus

The apparatus used, except where otherwise described, was that used in the platinum and nickel experiments referred to above, and described and illustrated in the paper on nickel catalysis.³

NICKEL EXPERIMENTS

The U-tube containing nickel oxide on asbestos was made the central tube of a train of five U-tubes, the other four containing sulphuric acid (sp. gr. 1.84) on pumice. A calibrated gas burette of almost 400 c.c. capacity was connected at each end of the train. The air was expelled by nitrogen while the nickel oxide was heated to 275° C. by a sodium nitrite bath. A measured volume of hydrogen was now passed between the two burettes across the heated oxide.

The above mentioned paper on catalysis by nickel shows that at this point a large proportion of the oxygen of the original oxide still remains. The rate of action of the hydrogen has become very slow. The surface film has formed, furnishing marked protection for the underlying interior content of oxygen.

³Proc. Royal Soc. of Can., 16, 1 (1922).

Volume of hydrogen at outset.....	762 c.c.
“ “ “ after 1st passage.....	363
“ “ “ 2nd “	308
“ “ “ “ 3rd “	274
“ “ “ “ 4th “	235
“ “ “ “ 5th “	231
“ “ “ “ 6th “	183
“ “ “ “ 7th “	178
“ “ “ “ 8th “	176
“ “ “ “ 9th “	168
“ “ “ “ 10th “	160
“ “ “ “ 11th “	158
“ “ “ “ 12th “	156
“ “ “ “ 13th “	154
“ “ “ “ 14th “	152
“ “ “ “ 15th “	150
“ “ “ “ 16th “	148
“ “ “ “ 17th “	146
“ “ “ “ 18th “	145
“ “ “ “ 19th “	145
“ “ “ “ 20th “	144
“ “ “ “ 21st “	144
“ “ “ “ 22nd “	144
“ “ “ “ 23rd “	144

The total water formed was .4301 g., equivalent to 534 c.c. hydrogen.

Total hydrogen used up = 618 c.c.

Hydrogen remaining in the nickel = 84 cc.

The tube containing the partially reduced nickel oxide was allowed to cool to 23° C., the free hydrogen being completely expelled by nitrogen. It has been shown that no hydrogen is given off in this operation at temperatures up to 300° C. 5 c.c. chlorine mixed with nitrogen was now introduced into the tube containing the catalyst and the whole allowed to stand for 3 hours at room temperature. The catalyst tube was now connected up with the original train of U-tubes and burettes, and heated to 275° C. in an atmosphere of nitrogen in order to sweep out any free chlorine as well as any water formed by the action of the chlorine on the nickel-hydrogen-oxygen complex. This passage of nitrogen was continued for 2 hours.

Water formed = .0022 g.

Hydrogen equivalent of water = 2.7 c.c.

The water evolved was thus very small in comparison with the hydrogen remaining on the catalyst.

The catalyst at 275°C. was now treated with hydrogen just as at the outset, the water formed being determined after each passage.

	1st passage	2nd passage	3rd passage
Hydrogen disappeared.....	32.0 c.c.	36.0 c.c.	28.0 c.c.
Water formed.....	.0141 g.	.0080 g.	.0101 g.
Hydrogen equivalent of water.	17.5 c.c.	9.9 c.c.	12.5 c.c.
Hydrogen remaining on nickel.	95.8 c.c.	121.9 c.c.	137.4 c.c.

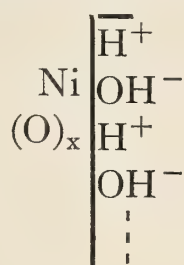
The apparatus was allowed to stand over night at room temperature, filled with nitrogen. The following day the catalyst was heated to 275° C. and hydrogen passed as before.

	1st passage	2nd passage	3rd passage
Hydrogen disappeared.....	79.0 c.c.	17.0 c.c.	29.0 c.c.
Water formed.....	.0407 g.	.0001g.	.0265 g.
Hydrogen equivalent of water.	50.6 c.c.	12.4 c.c.	33.0 c.c.
Hydrogen remaining on nickel.	165.8 c.c.	170.4 c.c.	173.9 c.c.

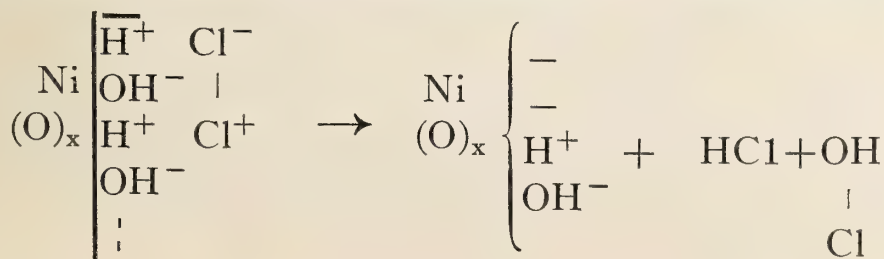
The action of chlorine did not result in the evolution of any considerable amount of water, but an amount approximately equivalent to the chlorine used (.0022 g. found, .004 g. calculated). Were all the hydrogen held on the catalyst evolved as water .0675 g. would have been obtained. That a deep-seated action occurred, however, is shown by the gradually increasing and relatively large volumes of hydrogen used upon passing this gas over the poisoned catalyst, as well as by the very large increase of hydrogen remaining on the catalyst, after each action of hydrogen. These two behaviours are characteristic of the action of hydrogen on the interior content of oxygen during the reduction of the original nickel oxide and during the process of establishing the surface film which is the seat of the normal catalytic action (see papers on the mechanism of nickel and platinum catalysis). The action of the chlorine appears to consist eventually in the destruction of the surface catalytic film, thus allowing the free hydrogen greater access to the interior content of oxygen. The presence of this oxygen being necessary for the maintenance of the surface film, the catalytic activity of the nickel complex is quickly destroyed.

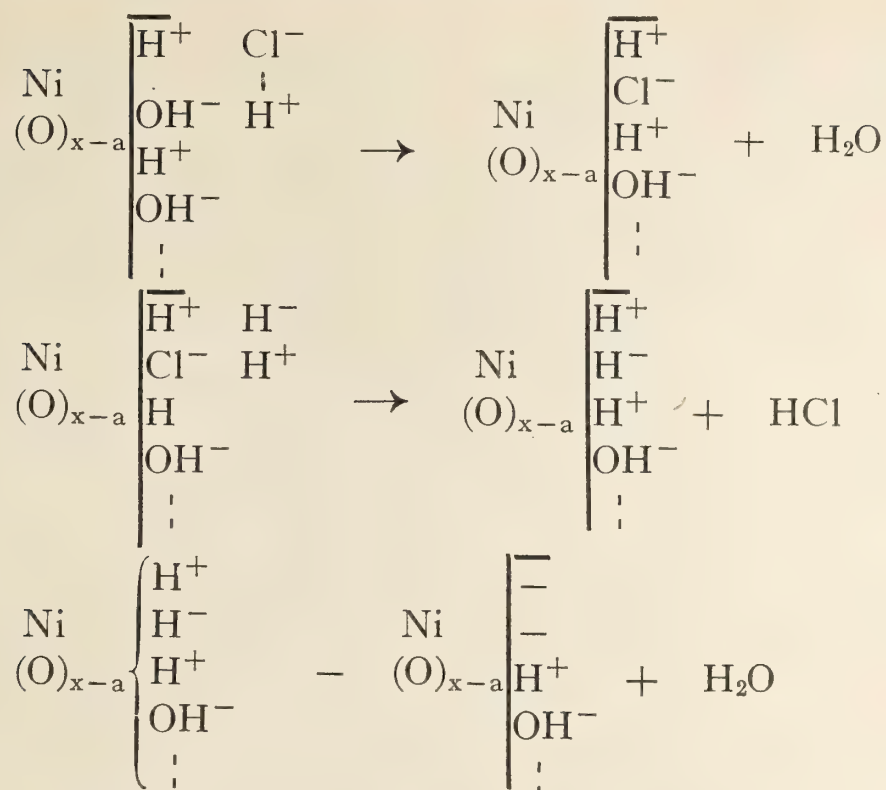
Upon now passing hydrogen once more 15 c.c. only disappeared. However, a relatively very large evolution of water occurred all at once (.1124 g. with a hydrogen equivalent of 139.7 c.c.). Thus the amount of hydrogen held by the catalyst increased rapidly with each passage of hydrogen until the hydrogen remaining on the catalyst had increased from 84.0 c.c. before the action of chlorine to 173.9 c.c. after treatment with chlorine and subsequent treatments with hydrogen. Then suddenly 139.7 c.c. was evolved as water and the hydrogen remain-

ing on the catalyst fell to 49.2 c.c. It seems, then, that the deep-seated change brought about by only 5 c.c. of chlorine gas, by which the normal catalytic properties of a normal nickel catalyst are destroyed, is due to the above mentioned changes in the surface film of the catalyst. It is a common explanation of the poisoning of a catalyst to say that the catalyst adsorbs the poison and thus prevents the adsorption of the bodies whose reaction is normally accelerated by the unpoisoned catalyst. This is very probably the whole explanation in certain cases where the poisoning material is present in so large amount as to enable the entire surface of the catalyst or a large part of it to adsorb the poisoning compound. However, in those cases usually designated as catalytic poisoning such conditions do not exist, the catalytic poison exerting its full effect even at very small concentrations and in minute absolute amounts. This latter is true in the case under consideration. It seems more reasonable to say that the poisoning is probably started by adsorption of the poisonous gas resulting in the destruction of something vital to the catalysis by the normal catalyst, and that the poisonous gas is regenerated or some active alteration product of it formed, capable of being again adsorbed by a further amount of the normal catalyst, followed by the above mentioned destructive action and the accompanying regeneration of the poison. Thus the poison is really a catalyst for poisoning. From this point of view the poisoning of nickel catalyst by the "poison catalyst" chlorine may, in terms of our original working hypothesis of the constitution of the normal nickel catalyst, be represented as follows: The normal nickel catalyst is represented thus



where Ni represents, not a nickel atom but a nickel particle made up of a number of nickel atoms, the number depending on the conditions of preparation, fineness of division, etc. This nickel particle has an interior content of oxygen (O) and the whole is surrounded by a surface film of charged hydrogens and hydroxyls. When a large amount of this catalyst is acted on by a very small amount of chlorine the following occurs:





followed by the adsorption of the evolved hydrochloric acid, and so the cycle continues.

PLATINUM EXPERIMENTS

In the paper on platinum catalysis already referred to, it was shown that on passing alternately hydrogen and oxygen over platinum black an equilibrium condition is reached where the hydrogen used up, the oxygen used up and the water formed in each of these two actions all become approximately constant. In this equilibrium condition the surface film has been fully formed and the normal platinum catalyst has been prepared. It was with this catalyst, using 2.5 g. platinum black, so prepared, that our experiments were performed. The following readings were obtained, the catalyst being at 150° C.:

Hydrogen c.c. disappeared	Oxygen c.c. disappeared	Water formed g.	H equiv. c.c. of water	H thus added to Pt
75		.0510	63.4	11.6
	5	.0043	5.3	6.3
37		.0225	28.0	15.3
	7	.0053	6.6	8.7
25		.0174	21.6	12.1
	8	.0054	6.7	5.4
19		.0124	15.4	9.0
	6	.0022	2.7	6.3
20		.0121	15.1	11.2
	6	.0013	1.2	10.0
20		.0140	17.3	12.7
	6	.0006	0.7	12.0
20		.0142	17.6	14.4

Hydrogen disappeared c.c.	Oxygen disappeared c.c.	Water formed g.	H equiv. of water c.c.	H on Pt
		.0020	2.4	12.0
6		.0027	3.3	14.7
	5	.0010	1.2	13.5
5		.0030	3.6	14.9
	5	.0046	5.7	9.2
10		.0041	5.1	14.1
	7	.0012	1.5	12.6
17		.0013	16.2	13.4
	6	.0012	1.5	11.9
24		.0160	19.9	16.0
	8	.0057	7.1	8.9
29		.0185	23.0	14.9
	9	.0023	2.8	12.1
33		.0260	32.3	12.8
	15	.0001	0.1	12.7
50		.0275	34.2	28.5
	10	.0001	0.1	28.4
57		.0244	30.3	55.1
	11	.0000		55.1
53		.0293	36.4	71.7
	11	.0001	0.1	71.6
50		.0494	61.4	60.2
	25	.0117	14.5	45.7
38		.0282	35.1	48.6
	12	.0018	2.2	46.4
35		.0213	26.5	54.9
	2	.0000		54.9
42		.0278	34.6	42.0
	11	.0016	1.9	40.1
38		.0209	26.0	52.1
	26	.0330	41.0	11.1
12		.0079	9.8	13.3
	10	.0021	2.6	10.7
40		.0250	31.1	19.6
	11	.0033	4.1	15.5
40		.0238	29.6	25.9
	10	.0052	6.4	19.5
38		.0225	27.9	29.6
	16	.0054	6.7	22.9
38		.0224	27.8	33.1
	20	.0157	19.5	13.6
83		.0359	44.6	52.0
	2	.0010	0.1	51.9
44		.0225	27.9	68.0
	11	.0103	12.8	55.2
43		.0149	18.5	79.7
	25	.0520	64.6	15.1
25		.0157	19.5	20.6

At this point the hydrogen used up was constant, the oxygen used up was constant and the amount of water formed approximately constant. The hydrogen in the apparatus was now displaced by nitrogen, (it has been shown that no hydrogen is given off in this operation; see platinum paper, p. 191) allowed to cool to room temperature and 5 c.c. chlorine introduced as in the nickel experiment just described. This was allowed to stand over night at room temperature. The following morning the apparatus was swept out by a large volume of nitrogen. Hydrogen and oxygen were now separately passed over the catalyst at 150°C ., the volume of gas which disappeared and the amount of water formed by each passage of gas being measured.

The first reading under the heading "water formed" is the water formed after treatment with chlorine. This was .0020 g. compared with .0022 g. in the case of the nickel experiment. The value calculated from the theory, as already explained, is .004 g.

From the above results it is seen that the behaviour with platinum is quite similar to that with nickel. The hydrogen which remained on the platinum after each gas treatment becomes gradually greater until it is suddenly evolved in large amount as water. The process is then repeated with again a sudden large removal of hydrogen as water. In the case of every hydrogen action there was an increase of hydrogen remaining on the catalyst. This behaviour is characteristic of the action of hydrogen on the interior content of oxygen, either when a portion of the surface film has been removed, or before the surface film has been fully formed (see paper on platinum catalysis). It seems that here, just as in the poisoning of nickel catalyst, the action of the chlorine has been to partially remove the surface film and render the interior oxygen content of the catalytic complex accessible to free hydrogen. And the same interpretation of the facts already given in the case of nickel may be used here.

It seems advisable, in connection with the general question of the mechanism of catalysis, particularly of hydrogenation by means of nickel and platinum catalysts, to point out a difficulty which has lately been stressed by several authors, and to offer a suggestion which may help to remove it. These authors question the necessity for the presence of oxygen in these catalysts. This conflict of opinion appears to be due to the failure to recognize that the catalysis, say of hydrogenation, can be accomplished chiefly in two distinct ways. One of these involves the use of finely divided metals which by their method of preparation (as, for instance, nickel from nickel cyanide) could not contain any oxygen, while the other by means of oxides of metals reduced by hydrogen at relatively low temperatures, when oxygen is certainly present in the catalysts. In the former case the catalysis is probably due to the

adsorption of hydrogen, or of the compound hydrogenated, or of both by the catalyst, while in the latter case the catalysis has, we believe, quite a different mechanism, as outlined in the two papers referred to above. Whether all of the details of the mechanism suggested by the senior author are finally shown to be correct or not the following statements appear to be true, and must along with other facts receive representation in any adequate theory of catalytic action by means of partially reduced oxides, viz., (1) The catalyst contains oxygen in two distinct conditions, (2) in one of these conditions the oxygen is associated with hydrogen and is easily reactive, (3) in the other condition the oxygen is protected in some way and is much less active, (4) this protection is removed by the action of small quantities of so-called poisons, so as to render the less active oxygen more reactive. The senior author believes that these facts receive an adequate representation by the theory advanced in the two papers above referred to, and in the present paper.

SUMMARY

Experiments on the poisoning of nickel and platinum catalysts by chlorine seem to indicate that the poisoning is accomplished by the destruction of the surface film on the catalytic particles, which film is the seat of the normal catalytic action, thus rendering the interior oxygen content accessible to free hydrogen. This interior oxygen so vital to the maintenance of this surface film, and hence of catalytic action in the normal catalyst, is thus quickly removed. The experiments appear also to lend additional support to the theory of mechanism of catalytic action already advanced by the senior author.

HISTORY OF THE RENAISSANCE OF ARCHITECTURE IN FLORENCE

By K. F. NOXON, *Research Assistant, Department of
Architecture*

Causes of the Renaissance.

The Renaissance of Architecture in Florence was a definite section of, and bore an important relation to the Renaissance of architecture in the rest of Italy, which in its turn was only one of the manifestations of the general movement of the Renaissance.

The nature of the whole movement of the Renaissance, which is merely a name for a period of history, it is often claimed, was the revival of antique learning. A truer and more exhaustive definition is that the nature of the movement was an intellectual upheaval, an awakening of reason. The true nature may be most readily grasped by an examination of the effect of the movement, shortly stated as the gradual merging of Mediaevalism into Modernism. From the fourth to the fourteenth century the world was in the grasp of mediaevalism. Reason lay dormant and the mental condition of the time was one of ignorant prostration before the Church. "Beauty was a snare, pleasure a sin, the world a fleeting show, man fallen and lost, judgment was inevitable, Hell everlasting and abstinence and mortification were the only safe rules of life." In the closing years of the fifteenth century and the opening years of the sixteenth a great change had taken place. Modernism was born and man had found himself and the world in which he lived. Just as the sleeping intellect of the middle ages was shown by the stagnation of the times the awakened reason was manifested in a thousand different ways. The revolution in literature started by the seeds planted by Dante, Petrarch and Boccaccio had almost reached maturity. The great princes were the tutelary saints of the philosophical doctrines of Aristotle and Plato. The discovery of the solar system by Copernicius and Galileo was accomplished and the core was forever removed from the mythology of the Church which was the most powerful agent in the enforcement of its dogma. Vasallius gave the world new light on anatomy and Harvey on the circulation of the blood. Politically, popular freedom was a definite goal; monarchies were established and a limitation was set on papal authority. In law the true text of the Roman code was used as a basis to introduce rational method into the theory of modern jurisprudence. That men were no longer content to live in ignorance of the world about them led to the discovery of America and the East. Inventions such as

printing, engraving, the compass, telescope, paper and gunpowder all were manifestations of the change of intellect which permitted and encouraged their adoption.

It is almost an impossible task to ascertain the cause of this emancipation of reason in Italy. The enigma of organic life has always defied analysis. Perhaps the pendulum which governs the cycles of the world's progress had reached its furthestmost limit and a return was the natural course or perhaps a single intellect achieved light through some "sport" of nature and had the power to spread the doctrine of freedom. This cause we are searching for was only the cause for the first heart beat of the new movement. Subsequently cause and effect were inseparably intermingled. The fall of Constantinople and the resulting spread of antique documents certainly contributed to the advancement of reason but it is interesting to note that the same documents were always available in mediaeval times without use being made of them. Many of the inventions which are ascribed as causes of the Renaissance were rejected by the mediaeval mind but, when they were adopted by the people of developed intellect, were important means of spreading and directing the movement.

The Renaissance in architecture was only one of the manifestations of the new movement but on account of its ethnographical significance as the most utilitarian of arts it may be considered the most important. There were several reasons for the Renaissance in architecture to take the nature of a revival of antique form. In the first place it was a true reversion to type for the squatter styles of the mediaeval ages were always exotic and never expressive of national conditions or ideals. The people of Italy could never exchange the horizontal lines of the Roman Basilica with its flat roof for the vertical Gothic with its converging roof lines nor could they be expected to adopt the unnecessary light of many great windows in place of the welcome shadow of blank wall. The least foreign of mediaeval styles was that developed in Lombardy but the political downfall of that state caused the doom of the nationalization of the Lombardy style of architecture. The different styles existing in Italy during the middle ages can be considered as intervals in the natural progression of classical architecture and the revival of the Roman forms was as natural a step as the Roman revival of Greek forms over a thousand years before. The second reason was largely accidental and may be traced to the fall of Constantinople and the ready access to the culture of Roman days. Dante, Petrarch and Boccaccio seized upon this fund of information for inspiration and stimulated a real enthusiasm throughout Italy for all things antique. Another great influence on the tendency of the new style was exerted by the immediate existence of

many antique monuments which often served as direct inspiration for expression of the new intellect.

The next group of conditions affecting the movement in architecture are usually indiscriminately classed as causes or effects of the movement as a whole but are in reality reasons for the continuance of the movement along already established channels. Commercial prosperity of the nation made it possible for great patrons to support many artists working on elaborate undertakings. The political system of decentralization divided Italy into many small states and created many wealthy masters who vied with each other in the richness of their buildings and the luxury of their manner of living. The newly acquired forces of the nation were unexhausted and the people themselves had an enormous capacity for enjoyment. Their fresh and unperverted senses rendered them keenly alive to the beauties of art and their appreciation was repaid by an increase of effort on the part of the artist. Exaltation of the personality of the artists themselves was a great factor in the production of the masterpieces of the period. Dante in his *Inferno* refers to this trait which was evidently pronounced as early as the thirteenth century:

"This miserable fate
Suffer the wretched souls of those, who lived
Without or praise or blame; with that ill band
Of angels mix'd, who nor rebellious prov'd
Nor yet were true to God, but for themselves were only."

Strong individuality and thorough developement in certain lines commanded great admiration. The giants of the Renaissance were produced in this manner and it is to be deplored that the modern standard of perfection demands a total lack of defects rather than the full development of all qualities, good or bad, which might produce the unlawfulness of the period of the Renaissance but would tend to create greater excellence of human endeavour.

The awakening of the intellect and the resulting Renaissance of architecture did not take place suddenly. Mediaevalism merged into modernism as Winter merges into Spring. It is true that there was a sudden departure from the old movement in the work of Brunelleschi which is unparalleled in the history of architecture but it is possible to find out-croppings of evidence of the new manner as far back as the middle of the thirteenth century. Owing to its dependent nature, good architecture must always follow in the wake of commercial prosperity and it is at the time of the greatness of Pisa that we first discover a tendency for the rejection of mediaeval ideas. The republic of Pisa was the first to acquire riches and power by the aid of commerce and liberty. Her conquests by sea brought her both wealth and antique examples, the

necessary adjuncts for the development of her architecture. Niccolo Pisano observed early in the thirteenth century that the human figures, on an ancient sarcophagus which had been brought from the East as a spoil of war, expressed a hitherto unknown freedom of treatment. He set himself to copy the work and before long his fame as a sculptor and architect had spread throughout Italy.

At the present day the famous group containing the Duomo, Baptistry, Campo Santa and Bell Tower remains as a monument of the grandeur of Pisa. The Duomo was started as early as the eleventh century when Pisa astonished the shores of the Mediterranean by the strength of her fleet, her part in the crusade and her conquest of Sardinia and the Balearic Isles. At the height of her fame, in the thirteenth century, Niccola Pisano worked on the bell tower of the Augustines (The Leaning Tower of Pisa) and the influence on his followers was responsible for the present architectural remains at Pisa. In 1282 Genoa and Pisa entered into a war which proved disastrous and from that date until 1404 Pisa was at the mercy of the growing power of Florence. In 1406 the Florentines, fearing a second blockade by Gian Galeazzo, Duke of Milan, determined to acquire a safe route to the sea and to this end completely subdued Pisa. The Pisans, who regarded themselves as the most illustrious of the Tuscan republics, refused to submit to the yoke of the Florentines and the most ancient and wealthy families moved from the city. Accompanying the loss of population was the loss of wealth, commerce and every remnant of prosperity resulting in the failure to produce further works of architectural interest. The work of Niccola Pisano exerted a wide influence on architecture in Italy. His principle works were at Pisa but he also erected the Chiesa del Santo at Padua, S. Trinita at Florence, The Dominican Convent at Arezzo, S. Giovanni at Siena and is credited with further work at Venice, Naples and Viterbo. His pulpits at Pisa and the Cathedral in Siena provide sufficient justification for his reputation as the first sculptor to express the freedom of the Renaissance. His most lasting influence was through the work of his followers. After his son Giovanni had constructed the Campo Santo at Pisa he was called to Naples by Charles I of Anjou where he built the Castel Nuovo and S. M. Nuovo. On his return from Naples he erected the facade of the cathedral at Siena. His work may also be found at Arezzo, Orvieto, Perugia and Pistoja. The next in line of the Pisan architects was Andrea Pisano, whose principal work was the Church of S. Giovanni at Pistoja (1337). He also fortified and increased the Ducale Palace in Florence. In Venice he designed the Arsenal. His son Tommaso Pisano completed the famous group in Pisa by finishing the chapel in the Campo Santo and the Bell Tower.

Before continuing this line of Tuscan architects, which finally led to the work of Brunelleschi and the grandeur of the complete Renaissance, it would be well to mention a rather isolated attempt at the revival of the antique that occurred in Naples. Contemporaneous with the rise of Naples and the seizure of the states of the Church by that city at the time of the Church schism at the end of the fourteenth century we find a single example of classical work. This is the Bell Tower of S. Chiara which was erected by Giacomo di Sanctis, a disciple of Massuccio II, who was a well-known builder in Naples. The Bell Tower was intended as an illustration of the five orders of architecture. The work was unfinished, the first three storeys containing the Tuscan, Doric and Ionic orders were alone completed and the work remains a sterile attempt at the revival of the antique. Future development of the movement in Naples cannot be traced to this building but entered through Florentine and Roman architects.

The next in order of the Tuscan architects was Arnolfo di Cambio whose work is the connecting link between mediaeval and modern. To-day we may stand on the Piazza Michael Angelo in Florence and the outstanding physiognomy of all we seen can be attributed to this energetic builder. The great bulk of the Duomo and the long, low mass of S. Croce as well as the walls which mark the boundary of the city we owe to Arnolfo. Giotto, by his most delicate Campanile and Orcagna, by his Church of Or S. Michele, also give evidence of the wealth of the city at that time and hint of a coming change in the methods of design and building. If more direct prophecy is required we need only to name as an example the Loggia dei Lanzi by Orcagna where the familiar pointed arch of the Gothic has been altered to the round arch of Rome. The Duomo itself is the building in which old and new merge into one. The work was commenced by Arnolfo in 1294 with a simple plan, remarkable for its excellence of proportion. At the time of Arnold's death it was completed as far as the drum of the dome. In style it was neither Renaissance nor Gothic for it lacks Renaissance detail as well as Gothic features, such as side chapels, flying buttresses or subordinate supports. It may almost be considered classical in contruction with Gothic detail. Its dome was the first work of Brunelleschi and was the definite commencement of the new manner and was, strange to say, Gothic construction with classical detail. Thus we find the genius of the Mediaeval crowned by the genius of the Renaissance.

Early work in Florence

A study of the political history of Florence reveals the reason for her early participation in the arts of the Renaissance. In 1381 the democratic

party directed by the Alberti, Ricci and Medici was deprived of power in consequence of the abuse which their associates, the Ciompi, had made of their victory. From that date until 1434 the Albizzi directed the republic and no victory of an aristocratic party ever merited a more brilliant place in history. It was during their rule that Florence became the wealthiest republic in Italy, developed art, science and literature and formed the great men from whom the Medici and their contemporaries reaped the benefit. Florence also followed the noblest policy during that time by considering herself the guardian of liberty in Italy. In turn she set limits to the ambitions of Gian Galeazzo Visconti, of Ladislaus, King of Naples and of Filippo Maria, Duke of Milan. From the year 1421, the Medici, through their acquisition of riches, became more powerful until finally in 1434 Cosimo Medici became ruler. He reigned for the next thirty years and during his time ruled the city well although the government was not democratic. By virtue of their respective positions of wealthiest man and ablest statesman, he and Capponi directed the affairs of the republic in a creditable manner. They kept the population in good humor by continual gifts and festivals. For the arts he was a great patron. Most generous with his immense fortune, he built palaces, churches and hospitals. He provided for the outstanding artists of the time and showed a true appreciation of their work. It was unfortunate for Florence that the successors of the Medici family did not possess the greatness of character of Cosimo. From his death the trend of affairs in Florence became steadily worse and the strenuous objection to the Medici rule which eventually led to their expulsion was only equal in ferocity to the methods they took to regain their power. Pietro de Medici, the successor to Cosimo, ruled poorly on account of ill-health and a tendency to quarrel with the Pitti family. In 1469 he died leaving Lorenzo and Guiliano to carry on the government, the elder of which had not reached the age of twenty-one. Both were given up to the pleasures of their age and had yet no ambition. Later they rendered themselves popular by keeping up a continual state of festivity, the expense of which was really paid by the stupid public. In 1470 the Nardi conspiracy against the Medici failed through the disinterest of the same public. Eight years later the Pazzi conspiracy resulted in the death of Guiliano and was indirectly the cause of a war between the Pope and Florence. Lorenzo deserves credit as a constant patron of the arts. He died in 1492 and the taste for pleasure that was acquired by the people under his reign and the reigns of his predecessors was caused a reaction as illustrated by the preachings of Savonarola. Suddenly Italy was surrounded by powerful enemies with whom formerly she had disdainfully considered barbarian nations. Charles VIII of France entered Italy in 1494

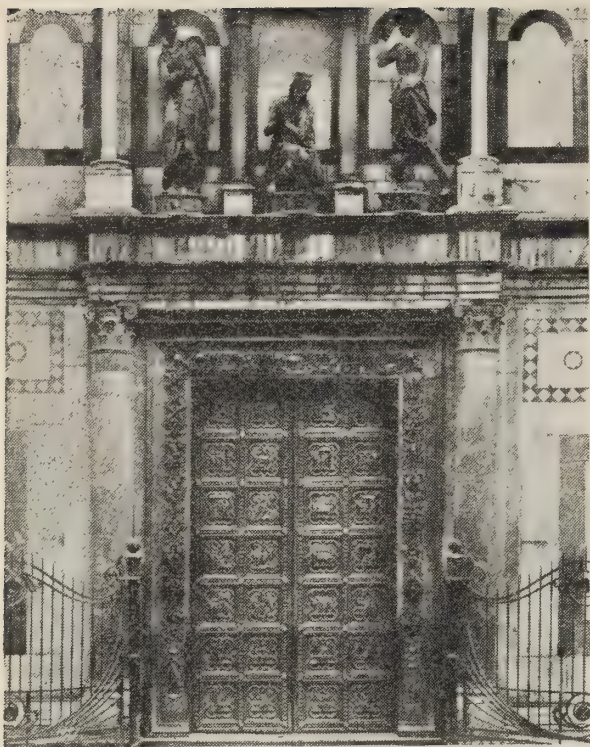
and conquered the entire country without a battle of note. The Medici's failure to stop him at Florence led to their expulsion and the nefarious means they took during the sixteenth century to regain their lost position, together with the presence of foreigners throughout Italy, caused the downfall of art in Florence.

To return to the beginning of the fifteenth century we find Florence the most intellectual and influential city in Italy. This together with the highly developed state of the art of sculpture led to the formation of a competition for the reconstruction of the north doors of the Baptistry of S. M. del Fiore. The south door was finished by Andrea Pisano in 1330, after twenty-two years of labour and the competition called for a design for the north door somewhat after the same manner.

For the purpose of choosing a sculptor for the work Jacopo della Quercia, Niccola d'Arezzo, Francesco Val d'Ombrino, Simoni da Colle, Niccolo Lamberti, Filippo Brunelleschi and Lorenzo Ghiberti were given a year in which to submit the designs in bronze of "The Slaying of Isaac". The competition was won by Lorenzo Ghiberti, whose design was superior to Brunelleschi's both in composition and treatment. Although Vasari states that Brunelleschi and Donatello were united in enjoining the committee to accept the design of Ghiberti, it is quite evident that the exclusive nature of Brunelleschi did not rest while there was another better than himself in the profession. The logical result was his determination to excel in the profession of architecture and to solve the greatest architectural problem of the day; the building of the dome of the Florentine cathedral. With this in view, he and his boyhood friend, Donatello, set out for Rome to make a complete study of the antiquities of that city. This competition of the bronze doors was not important because it resulted in the acquisition of a new set of doors which surpassed Pisano's first effort, or on account of the resulting encouragement to sculpture, but because it was the means of turning the greatest artist of the Renaissance towards the profession of architecture.

Brunelleschi was born in 1337, the son of Lippo Lapi. As he did not show an inclination to follow his father's profession of notary he was finally placed with a goldsmith. The practical subjects of sculpture and perspective soon followed. A great reason for the development of the imagination of Brunelleschi was the interest he took in the works of Dante. We are told that he never failed to be present at the disputations and preachings of learned persons. This development of the intellect, so unusual in those times, had a great influence on his departure from the Mediaeval form of building.

After arriving at Rome with Donatello he studied there for a period of eighteen years when, after many trials and tribulations he finally



NICOLA PISANO'S SOUTH DOOR TO BAPTISTERY.



DUOMO.
Dome by Brunelleschi, 1420.



PAZZI CHAPEL.
Brunelleschi, 1420.

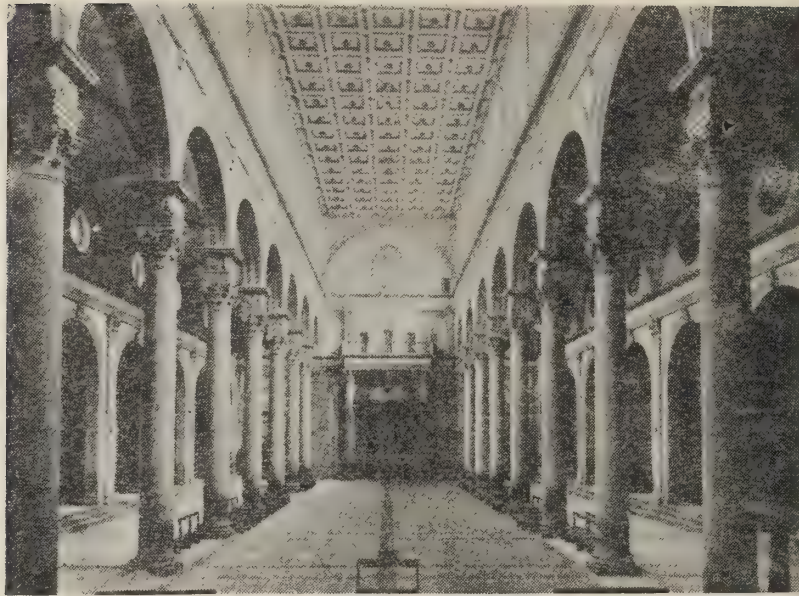


FLORENCE BAPTISTERY, EAST DOOR.
Ghiberti.

commanded sufficient confidence amongst the people of Florence to obtain the commission of building the dome. The diplomacy that he was forced to use to get rid of his associate Ghiberti, who had no just cause to share the credit of his invention; the talent with which he pursued the work and the perseverance and patience he exhibited when dealing with his employers, all entered into the struggle. The result, which was as we know, in favour of Brunelleschi, was one of his greatest achievements. The dome was raised on an octagonal drum. This was an added difficulty in construction and a step forward in the process of dome-design. In construction the dome is Gothic principle, having eight main ribs and sixteen intermediate ribs between which are stretched straight vaulting ribs. It consists of two concentric shells with complete arrangements for the drainage of water, access to the cupola and facilities for its interior decoration.

This tedious process of building the dome occupied fourteen years (1420-1434). During this period Brunelleschi also found time to erect several smaller buildings which firmly established his reputation as a great designer in the new manner. His first work was the portico of the Ospedale degli Innocenti, which consists of a series of arches directly placed on widely spaced slender columns. This work was later decorated with charming medallions by Luca della Robbia. The first ecclesiastical work of the Renaissance was the Pazzi Chapel, a mortuary chapel for the Pazzi family in the cloisters of S. Croce. It is a complete departure from the old manner of building. Its plan, although related to the plans of several of the Roman temples, is well adapted to its use and the arrangement of the site. It is in the form of a Greek cross and, while it may not express the end in view, it at least provides a proto-type for the later structure of S. M. della Carceri at Prato. The details of the Corinthian capitals, which support the blank upper storey, have a slight feeling of the Romanesque and also possess the common fault of slenderness at the supports which Brunelleschi exhibits throughout all his work. The balustrades set between the columns show one of the few applications Brunelleschi makes of the Ionic order. In this case the balusters are small Ionic columns which are so effective that it is surprising that they have not been more often made the proto-type for work on modern buildings. The decoration of the wavy fluting on the upper frieze and the sculpture work of Donatello and Settignano are admirably suited to the purpose.

Brunelleschi, by convincing Giovanni de Medici that the Prior superintendent of the church of San Lorenzo was incapable of building a church worthy of containing the chapels of the men of the great families which were to be placed there, obtained the commission of building the



INTERIOR L. LORENZO, FLORENCE.
Brunelleschi, 1425.



FLORENCE, RICCARDI PALACE, CORNICE.
Michelozzi, 1430.



FLORENCE, PALAZZO RUCELLAI.
Alberti, 1451.



RICCARDI PALACE, FLORENCE, 1430
Michelozzi, 1430

church as well as the old sacristy. The greatest point illustrated in S. Lorenzo is that Brunelleschi retained certain Gothic features of construction and clothed them with Renaissance detail. These are not, however, obvious and the general effect is strictly classical. The difficulty of slightness of supports which was noticed in the Pazzi Chapel is again evident in the internal treatment of S. Lorenzo.

The real genius of Filippo in truly using the classical details for the expression of the modern spirit is ably illustrated in his work at the Abbey Fiesolane, situated just outside Florence. Here we have a church on the conventional Latin cross plan. The straight walls are broken by simple arches which are defined by archivolt mouldings of simple but effective design. From the narrow pilasters on the corners, which are crowned by a delicate cornice, spring the arches which govern the shape and height of the ceilings of the nave and transepts. The crossing is covered by a simple Byzantine dome. The whole church is a masterpiece of simplicity. It was completed after the death of Brunelleschi (Died 1446 and completed 1462) and therefore it is unlikely that the admirable detail work of the doors and window reveals in the cloister were by his hand. The last of his ecclesiastical work that he superintended himself was the church of S. Spirito (1433). It is in the form of a simple Latin cross and exhibits a distinct Byzantine feeling in both the smaller domes on the side aisles and the large dome over the crossing. The latter, a simple dome on pendentives without a drum and obtaining the lighting from windows set in the actual dome itself, shows no advance in construction. The interior is impressive and its chief claim to fame is that it noticeably lacks defects. The exterior shows no attempt at the solution of the problem. The octagonal sacristy was added by Il Cronaca and the graceful campanile by Baccio d'Agnolo in 1487. In addition to this church Brunelleschi designed the second cloisters of S. Croce.

The first attempt of the application of the new style to domestic work is seen in the work of Michelozzi who built the Riccardi palace in 1430. Michelozzi was a pupil of Donatello for several years and was intimate with Cosimo de Medici. When Brunelleschi fell into a rage at the rejection of his design for the palace, on the grounds of over-sumptuousness and smashed his model into a thousand pieces, Michelozzi was called in to submit the new design. He did not fail to make use of his opportunity and the Riccardi palace is to-day a standard for the Florentine type of domestic work. The facade is divided into three storeys which diminish in height towards the top and the whole is crowned by a great cornice which has been cleverly designed from its Roman proto-type. The arrangement of the windows is most pleasing to the eye and the

heavy rusticated lower base tells its story of troublesome times and the patron's need of protection.

The next great residence to be built was the Pitti palace for Luca Pitti. Brunelleschi started the structure and the rugged simplicity of the work is a monument to his fame. The entire design has not been carried out, nor are Brunelleschi's intentions for it known, but the facade facing on the piazza gives a good idea of the grandeur of the scheme. The windows on the lower floor and the garden facade were constructed by Ammanati in 1568. The last of Brunelleschi's work in Florence was the Palazzo Quaratesi which lacks the crowning cornice and is scarcely superior to the work of Hichelozzi. Brunelleschi died in 1446.

It is unfortunate that we have no further domestic work of Michelozzi. In 1433 he followed his master into exile to Venice, where he worked on the library of S. Georgio Maggiore and upon his return in 1434 he spent many years repairing th Palazzo Vecchia. In 1462 he erected the Chapel Portinari in S. Eustorgio, Milan and ten years later he died.

The same year as the death of Brunelleschi, another architect in the person of Leon Battista Alberti sprang into prominence by his work in the remodelling of San Francesco at Rimini. Here he was constrained to follow the mediaeval lines of the fourteenth century church and his remodelling of the exterior was never completed. The treatment of the arches of the facade may have been inspired by the Augustan archway in the same town. He was born in 1404 and was the best educated of the architects of the time. Architecture was only one branch of his studies and the manner in which he proceeded to make journeys to measure the antique buildings is characteristic of his thorough methods. He takes his place in the architecture of Florence by the erection of the Palazzo Rucelai. This is the first house which has pilasters on the facade. The awkwardness of the design produced by the even heights of the storeys and only slightly relieved by an alternation in the width of the bays containing the windows, can be forgiven upon the realization of the fine taste of the treatment of the flat pilasters and cornices. The intermediate cornices are to be regretted as is also the reduction of the great cornice. His facade of S. M. Novella, Florence (1456) was at least an attempt at the solution of the church facade. The scroll forms on the side, hiding the side aisles, were a new feature adopted from the late Roman works and by their abuse at the hands of later Venetian architects were factors in the deterioration of Renaissance architecture. In 1460 he is credited with the design of the facade of S. Sebastiano in Mantua. Later in the same city he constructed (1472-1512) the church of S. Andrea which is an innovation in church planning, doing away with

the slender columns of Brunelleschi's churches by the substitution of built in chapels and massive piers. Alberti died in 1472.

By the work of these three architects, Florence achieved fame in a new manner but from now on we find that the other cities of the north and even Rome developing the new style in an excellent manner. In Rome the works of Filarete, a Florentine who afterwards went to Milan, and Rossellini, also of Florence who did a great deal of building for Pope Pious II in Pienza, deserve special attention and will be dealt with later. It has already been shown that Alberti and Michelozzi carried a new manner to Milan, Mantua, Prato and Venice.

The next great architect to have an effect upon the spreading of the movement outside Florence was Giulio da San Gallo (1443-1517). He first appeared in the records when working upon the cloisters of S. M. de Pazzi, Florence (1479). A year later the Palazzo Antinori was attributed to him. (Also attributed to Baccio d'Agnolo). This is of simple design and would scarcely be noticed were it not for a subtle nicety of proportion and elaborate stone joining which proclaim it a Renaissance building. The crowning cornice which had already been introduced is neglected and a wide overhanging roof is used. Another work outside of Florence is the Villa Poggio a Cajano. In Loreto he worked on the Santa Casa and in Rome built the cupola of the church of Marie de Loreto, restored the soffit of S. M. Maggiore, adorned the national church of S. M. dell'Anima and built the palace adjoining S. Pietro in Vinculis, which is not worthy of attention. He withdrew from Rome when Bramante was entrusted with the re-building of St. Peter's. His most important work in connection with the tracing of the history of architecture is the church of S. M. dei Carceri at Prato (1485). This church is an improvement on Brunelleschi's Pazzi Chapel. Its main point of difference is an increased symmetry when the four arms of the cross form the interior of the church. The dome is raised on a drum. This plan appeared again in a design by Antonio San Gallo at Montepulciano some years later. In Florence he erected the Palazzo Gondi and the Pianciaticchi-Ximenes. Later in life he was employed at St. Peter's but he was forced to leave owing to ill-health and he returned to his native country where died in 1517.

The last architect of prominence of the early period in Florence was Simeone del Pollajuolo called Il Cronaca. The records show that he fled to Rome after a brawl in Florence. He immediately went to the house of Antonio Pollajuolo who was a worker in tarsia. Here he took a delight in studying the antique monuments and upon his return to Florence in 14489 he was proficient in the knowledge of antique forms. By an accident Benedetto da Majano, who had already made a model of

and started a palace for Filippo Strozzi, was forced to leave Florence at the same time as Il Cronaca arrived. Strozzi, impressed with the young man's knowledge of Rome, and with a model he made of the courtyard and main cornice of the palace, commissioned him to proceed with the work. The palace bears a close resemblance to the Riccardi palace which was built fifty-nine years before. The chief points of difference are increased weight of rustication on the upper storeys, increased height of the upper storeys and the replacement of the large arched windows on the lower floor by small, high rectangular openings. The iron work, which includes flag-standards, is worthy of notice and is by the hand of Niccolo Grosso, called Caparra.

Il Cronaca also built the sacristy at S. Spirito which is in the form of an octagonal temple with a shallow cornice, supported by double pilasters at the angles. He also built the church of San Francesco dell' Osservanza on the hill of S. Miniato and the council chamber of the Palazzo Vecchia. After building this hall he was seized with the frenzy of religion from his contact with Savonarola and talked of nothing else. He died at the age of fifty-five in 1509.

One of the main features of the movement up to the present time has been a close connection between architecture and the craftsmen who actually perform the detail work of the building. Included in this class are the many sculptors that the period produced. The importance of their work in the final effect of the building warrants a short survey of the principal men during the fifteenth century in Florence.

The first man of note was Ghiberti (1378-1455). He has already been mentioned in connection with the competition of the Baptistry doors. His success with this work marked him as an artist of good taste with the power to tell a story plainly without dramatic vehemence and the ability to eliminate painful details of the subject. His experience as a painter, from which profession he was called to take part in the competition, doubtless accounts for his smooth style in sculpture work. In fact his pictorial, rather than sculptural, style almost enables us to call him a painter in bronze. The first of his gates was a supreme achievement of Tuscan bronze casting while the second has been criticized for its multiplicity of planes.

After Brunelleschi, Ghiberti's greatest rival in the competition was Giacomo della Quercia whose designs have been lost. A comparison may be made though, of della Quercia's work by the examination of his bas-reliefs upon the facade of S. Petronio at Bologna and round the font of S. John's Chapel in the cathedral of Siena. He possessed a style more in the nature of Michael Angelo's and in fact it can almost be assumed

that Michael Angelo derived a certain amount of inspiration from della Quercia's work.

Donatello, who shares with Ghiberti the title of the greatest sculptor of the period had a style remarkable for straightforward and truthful treatment of the subject. In his *St. George and David in the Bargello* at Florence, he has cleverly shown the eternal struggle of the soul with evil. He did a great deal of work including bas-reliefs of the dancing-boys for the cathedral at Prato and the *Cantoria* in the *S. M. del Fiore*, now in the museum behind the church, and *Judas*, which is placed in the *Loggia del Lanzi*. He made some intricate bronze bas-reliefs for *S. Lorenzo* in Florence and more interesting ones for *S. Antony* in Padua. In 1451 he was called to Padua to model the statue of *Gattamelata* which is a masterpiece in scientific bronze casting and was the first equestrian statue since the days of Rome.

The next step in the equestrian work was performed by his pupil Verrochio, who started the statue *Colleoni* which is placed in front of the *Scuolo S. Marcuo* in Venice. To all the arts he practised, Verrochio applied limited powers, a meagre manner and a prosaic mind. That he possessed the ability as a teacher is shown by the quality and quantity of his pupils who included Leonardo da Vinci, Lorenzo de Credi and Pietro Perugino. Florentines are still proud to show his "Incredulity of Thomas" on the eastern wall of *Or San Michele*, "The Boy and the Dolphin" in the court of the *Palazzo Vecchio* and the "David" in the *Bargello*.

Verrochio died in 1448 before the *Colleoni* statute was finished. The work of casting was completed and the pedestal erected by Alessandro Leopardi and to him must be given great credit for the work. Verrochio should receive great praise for the conception of the statue but he was undoubtedly influenced and aided by the work of Donatello. Leopardi, however, has, by the erection of one of the most beautiful pedestals in existence, presented the casting in its best light. Had Leopardi left no other work than the moulding on the base of this pedestal he would still deserve credit as one of the greatest artists of the Renaissance. The flag-standards in the *Piazza, S. Marco*, also by his hand, show a wealth of beautiful detail and, what is more characteristic of Leopardi, a highly studied and graceful profile.

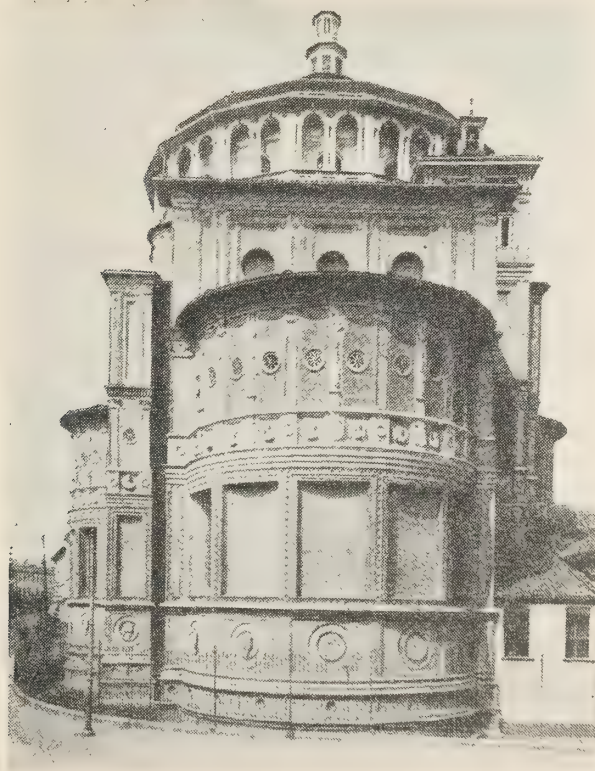
A contrast to Ghiberti and Donatello is offered in the work of Luca della Robbia, whose masterpieces in glazed earthenware are perhaps better known than any other type of sculpture of the Renaissance. His work is most life-like without the rugged realism of Donatello or the somewhat effeminate graces of Ghiberti. He has used remarkable judgment in keeping the technique of his work exceptionally simple and



VENICE, COLLEONI MONUMENT.
Verrochio and Leopardi.



FLORENCE, FOUNTAIN BY VERROCHIO.



S. M. DELLE GRAZIE. 1492.

although he uses colour his subjects are invariably white on a pale blue ground and are wholly free from affectation. His followers, Andrea, who was his nephew and his four sons, Giovanni, Luca, Ambrosio and Girolama, continued the manufacture of the glazed terra cotta but they lacked the fine taste of their teacher and gave way to the temptation of using many varied colours which robbed the work of its simplicity and therefore much of its charm.

Rossellini, Civitale and Mina da Fiesole may be grouped together, inasmuch as all their work is distinguished by sweetness, grace, gentility and self-restraint. The fine taste of Mina da Fiesole was also endowed upon his friend Desiderio da Settignano, who was one of Donatello's few scholars. The chief point of difference was his bolder inventive faculty. The bust he made of Marietta di Palla degli Strozzi is a work of most captivating dignity. Settignano's work in connection with architecture has already been pointed out in the description of the Pazzi Chapel. An artist who claimed a third place beside Mina and his friend was Benedetto da Majano. In his bas-reliefs at S. Gemignano we have a pictorial treatment of legendary subjects which show study of Gherlandajo's frescoes. He also shows Ghiberti's influence by a liberal use of landscape and architectural backgrounds. His chief claim as an artist is through his work as architect of the splendid Strozzi Palace.

It will be noticed that all the sculptors mentioned so far have been Tuscan. This has not arisen through the wish of the writer to group the artists according to their province but is a natural fact and shows the true position of Tuscany and the development of the arts. However, these sculptors, when they carried on their work in other cities, combined the individuality of the place in which they worked with their native technique. A striking example of this is the work on the Certosa di Pavia which was the centre of a school which had little in common with Florentine work. Amadeo and Fusina working in conjunction with Borgognone, a painter, gave the facade of the Certosa that rich and complex effect of decorative beauty which many generations of artists were destined to continue and complete.

The influence exerted by Florence on the arts was felt throughout the length and breadth of Italy. Even the papal city was no exception. During the period of medievalism Rome had fallen upon evil days. Her population had been depleted by plague and famine and her very existence had been threatened by the removal of the seat of the supreme pontificate to Avignon. In 1377 Pope Gregory IX sacrificed the gay life and agreeable climate of the Rhone valley to return to Rome and this transition made the Renaissance in Rome possible. From that time the arts in Rome were dependent upon the vicissitudes of the various Popes and

while they might be stimulated by the generosity of one holy father they were liable to be starved by the niggardliness of the next.

Three architects, all Florentine, governed the trend of the architectural movement in Rome during the fifteenth century. Pope Nicholas V (1447-1455) employed Bernardino Rossellino to further his scheme of improvement of the city and did much to lessen the dangers of the plague and famine by his hygienic reform. His successor, Eugenius IV, considered such innovations pagan and there was a return to the evil days. Fortunately for Rome the next man to ascend the papal throne was Sixtus IV (1471-1484) who, although he came from the head of one of the mendicant brotherhoods which were so bitterly denounced by the humanists as hot-beds of ignorance, made himself the champion of intellectual progress. Sixtus IV earned for himself the title of "The Great Builder" but the excellence of the work was due to Baccio Pontelli. The first work of this architect in 1466 was the Pal. Ducale at Urbino which had a lasting influence by the impression it created on the mind of Bramante. In Rome his works consisted of S. Pietro in Montorio (1472-1505), Ospedale S. Spirito (1473), the porch of S. Pietro in Vinculis, the commencement of the Church of S. Sisto, the rebuilding of S. M. del Popolo and, in association with Pietro da Cortonna, the erection of the Church of S. M. della Pace.

The third Florentine architect to leave his stamp upon Rome was Antonio Filarete, who with Simone, the brother of Donatello, constructed the bronze gates for St. Peter's as early as 1431. Filarete was also responsible for the inauguration of the movement in Milan by the erection of the Ospedale Maggiore, a building which became an example for a great number of Northern works. The early work in Milan cannot be dismissed without mentioning the work of Bramante's early life. Influenced by the work of Pontelli and Lauranna at Urbino Bramante derived from Alberti's work at Rimini a sense of grandeur and from his teacher, Leonardo da Vinci, a charming touch for decorative form. His early work at Milan consisted of several churches, all of which give promise of the genius that was destined to startle Rome at the beginning of the sixteenth century.

In this period other work of note was accomplished at Verona where Fra Gioconda built the Pal. del Consiglio and at Brescia where La Loggia by Formentone (1489) compares favourably with building in any other city.

Venice by her tyrannical political rule and geographical isolation was slow to absorb the Florentine influence. Her tardy commencement was compensated by thorough later use of the style and her immunity from sack by her enemies enables her to boast to-day of a most complete collec-

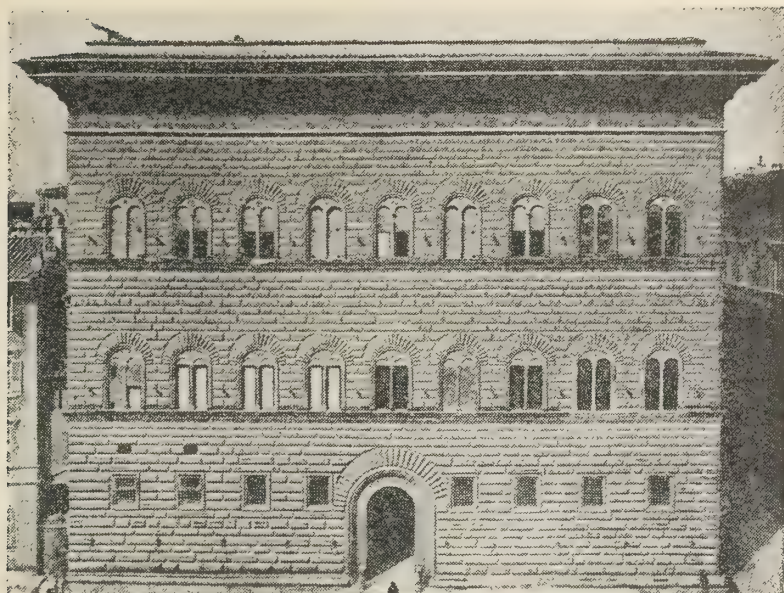
tion of buildings of every period. Her first architect was B. Buon, a native of Bergamo and his inspiration for classical work would be hard to find as it is certain that he never saw the work of Brunelleschi in Florence. Both Alberti and Michelozzi are attributed work in Venice but it remained for the members of the Lombardi family to truly express the ideals of Venice by the adaption of the new manner.

From the years 1500 1550 has been arbitrarily defined the period in which the culmination of the Renaissance style took place. This applies more to Rome than to the rest of Italy for we must certainly consider the work of Vignola, Palladio and Alessi as belonging to the period of ascendancy. The main factions to notice were the growing power of Rome and the slackening prosperity of Florence. In reality the brilliant period of architecture in Florence closed with the fifteenth century.

In Rome Bramante ushered in the new era in a most auspicious manner. He was followed by Peruzzi, Raphael, Antonio San Gallo, the younger and the great Michel Angelo and no more brilliant succession of architects have ever adorned a single city. Not even the sack of Rome could arrest the progress in the art of building.

In Florence war and politics drove out her native artists. Owing to her exposed position Florence was always visited and usually violated by the many foreign armies which invaded Italy during the sixteenth century. The deposed Medici finally succeeded, by allying themselves with these enemies, in regaining their leadership in Florence but the genius which had caused the rise of the famous family had died out and their rule became one of the great factors in the downfall of Florentine art. Of the few buildings of note of the period were works by Il Cronaca and Baccio d'Agnolo, both architects of the first period. The most brilliant work was by the hand of Michael Angelo. He built the new sacristy on Brunelleschi's church of San Lorenzo and filled it with the Medici tomb. His other work in Florence was the Laurentian Library (1523).

Venice continued to be the scene of excellent work during the period of idleness at Florence. Her architects, Scarnagnino, Bergamasco and Sansovino, reared the palaces and public buildings which to-day proclaim the glory of the city. The Venetian states also produced great men. Verona had San Michele to erect her palaces and gates; Siena, Peruzzi; and Mantua, G. Romano. Venice was the only city to carry on good work after the decline had become fairly general at the end of the sixteenth century. The high tide of art at Rome had three important issues; the work of Vignola at Caprarola, Palladio at Vicenza and Alessi at Genoa. Each was responsible for a style peculiar to themselves and which expressed the conditions and ideals of its location in almost a perfect manner.



FLORENCE, PALAZZO STROZZI.
B da Majano and il Cronaca. 1489-1533.



MAIN FACADE. 1517.



PAL. CONSIGLIO. 1599-1614.

Florence never regained her lost prestige. The seventeenth century produced few men of note and the work though plentiful was of mediocre quality. Michelozzi, whose ancestor had such an important part in the moulding of the movement, built the Pal. Tornabuoni, the Villa Carreggi and the outside court of the Pal. Vecchia (1565). The greatest man of the period was B. Ammanati who was responsible for the two bridges, the Ponta alla Carraja (1559), the Ponta S. Trinita (1567) and the Fountain of Neptune and the Tritons, all of which added materially to the fine appearance of the city. His other works were the Pal. Guigni (1564), Pal. Pazzi and the re-construction of S. Giovanni degli Scolopi. Notable for the excellence of his work was Buontalenti who, in 1565, constructed the Pal. Ricardi-Mannelli, in 1570 the Casino Medici and the Church of S. Trinita, and in 1592, the Pal. Nonfinito. Vasari, besides completing his valuable lives of the architects, left in Florence as monument to his fame as an architect, the Pal. Uffizzi and the stairway of the Laurentian Library (1571). The latter work, considered by many to contain the first germ of the decline of the movement, was built under the supervision of Vasari from the designs by Michael Angelo and its meaningless curves might certainly have been the inspiration for later men such as Bernini and Borromini who seized upon the shape and made of it a structural form.

Thus we see that Florence occupied an important place in the History of Renaissance art. Her early interest in the arts, coupled with the stabilizing influence of the Medici rule and her commercial prosperity led her to be the starting point of the upheaval. The subsequent development of architectural design which took place in the fifteenth and sixteenth centuries occupies a place equal to the art of Athens or Rome. But it is not so much for this development that we owe our debt to Florence as to the part she played as sponsor of the movement, which once started has had the impetus to continue and which continues to have a direct influence on the building of modern times.



THE DOME OF ST. PETER'S, ROME

BUILDINGS OF THE RENAISSANCE IN FLORENCE

NAME	YEAR	ARCHITECT	THIS NUMBER REFERS TO MAP
BRIDGE			
Carraja.....	1559.....	Ammanati.....	1(C.2)
CHURCH			
S. Trinità.....	1567.....	Ammanati.....	2(C.2)
CASINO			
Mediceo.....	1570-76.....	Buontalenti.....	1(A.4)
di Livia or San Marco.....		Buontalenti.....	9(B.4)
CHURCH			
Abbey of the Cannons Regular.....	1412.....	Brunelleschi.....	
S.S. Annunziata.....	1250.....		
	1470-6.....		
	1470-6.....	Alberti.....	1(B.5)
	1601.....	Cacini.....	1(B.5)
La Badia.....	1625.....	Segolani.....	1(C.4)
Badia Fiesolana.....	1462.....	Brunelleschi.....	
Cathedral (S.M. del Fiore).....	1294.....	Arnolfo di Cambio, etc.	2(C.4)
S. Croce.....	1294.....	Arnolfo di Cambio.....	1(D.4)
	1435.....	cloisters by Bramante.....	1(D.4)
	1420.....	Pazzi Chapel by Brunelleschi....	1(D.4)
S. Felicita.....		dome by Brunelleschi.....	1(D.3)
S. Giovanni degli scolopi.....	1580.....	remodelled by B. Ammanati....	10(B.4)
	1616.....	Peruzzi.....	10(B.4)
S. Lorenzo.....	1425.....	Brunelleschi.....	1(B.3)
S. Marco.....	1550 (ap.) ..	church restored.....	1(B.4)
	1437.....	convent by Michelozzi.....	1(B.4)
S.M. Carmine.....	1771-82.....	Mannaioni.....	1(C.1)
S.M. Madalena di Pazzi.....	1479.....	cloisters by A. San Gallo.....	1(C.5)
S.M. Novella.....	1278.....	restored by Sisto.....	2(B.3)
	1456.....	Alberti, etc.....	2(B.3)
S. Francesco dell'Osservanza.....		Il Cronaca.....	
S. Pierino.....			
S. Romolo.....			
S. Salvatore del Monte.....	1504.....	Il Cronaca.....	
S. Salvatore d'Ognissanti.....	1554.....		1(B.2)
	1627.....		1(B.2)
S. Spirito.....	1433.....	Brunelleschi.....	1(D.2)
S. Trinità.....	1250.....	Nicola Pisano.....	2(C.3)
	1570.....	Buontalenti.....	2(C.3)
FORTRESS			
S. Giovanni Battista.....	1534.....		
FOUNTAIN			
Neptune and the Tritons.....	1564-75.....	Ammanati.....	3(C.3)

NAME	YEAR	ARCHITECT	THIS NUMBER REFERS TO MAP
GARDENS			
Boboli.....	1550.....	Il Tribolo and Buontalenti.....	1(E.2)
HOSPITAL			
degli Innounti.....	1419-45.....	Brunelleschi.....	2(B.5)
LIBRARY			
Laurentian.....	1524-71.....	Michelangelo.....	3(B.3)
LOGGIA			
dei Lanzi.....	1376.....	Tolenti.....	4(C.3)
S. Poalo.....	1451.....	Brunelleschi.....	4(B.3)
del Pesce.....		Vasari.....	5(C.3)
MARKET			
Mercati Nuova.....	1547-51.....	Tasso.....	6(C.3)
MONASTERY			
S. Marco.....	1442-43.....	Michelozzi.....	2(B.4)
MUSEUM			
S. M. dei Fiore.....			3(C.4)
Nazionale or Bargello.....			
PALACE			
Albizzi.....	16—		8(C.4)
Antinori.....	1480.....	G. da SanGallo.....	
Arcivescovile.....	1573.....	Dosio.....	7(C.3)
Bartolini-Salimbeni.....	1520.....	Baccio d'Agnolo.....	(D.5)
Buturlin.....		Domenico d'Agnolo.....	8(B.4)
Castellani.....			
Corsini.....	1556.....	Silvani and Ferri.....	3(C.2)
Corsini.....			
Ginori.....	1700.....	Baccio d'Agnolo.....	3(B.4)
Gondi.....	1490.....	G. da SanGallo.....	4(C.4)
Guadagni.....	1490.....	Il Cronaca.....	2(D.2)
Guigni.....	1560.....	Ammanati.....	5(B.5)
Guintini.....		Brunelleschi.....	
Lardarel.....	1558.....	Dosio.....	8(C.3)
Nonfinito.....	1592.....	Buontalenti and Cigoli.....	5(C.4)
Panciatichi.....	1700.....	C. Fontana.....	5(B.4)
Panciatichi-Ximenes.....	1490.....	G. da SanGallo.....	2(C.5)
Pandolfini.....	1520.....	Raphael etc.....	1(A.4)
Pazzi.....	1568.....	Ammanati.....	
Pitti.....	1440.....	Brunelleschi.....	3(D.2)
Capponi.....	1660.....	Silvani.....	6(B.4)
Quaratesi.....	1442-1446....	Brunelleschi.....	6(C.4)
Riccardi or Guadagni.....			7(C.4)
Riccardi.....	1430-33.....	Michelozzi.....	4(B.4)

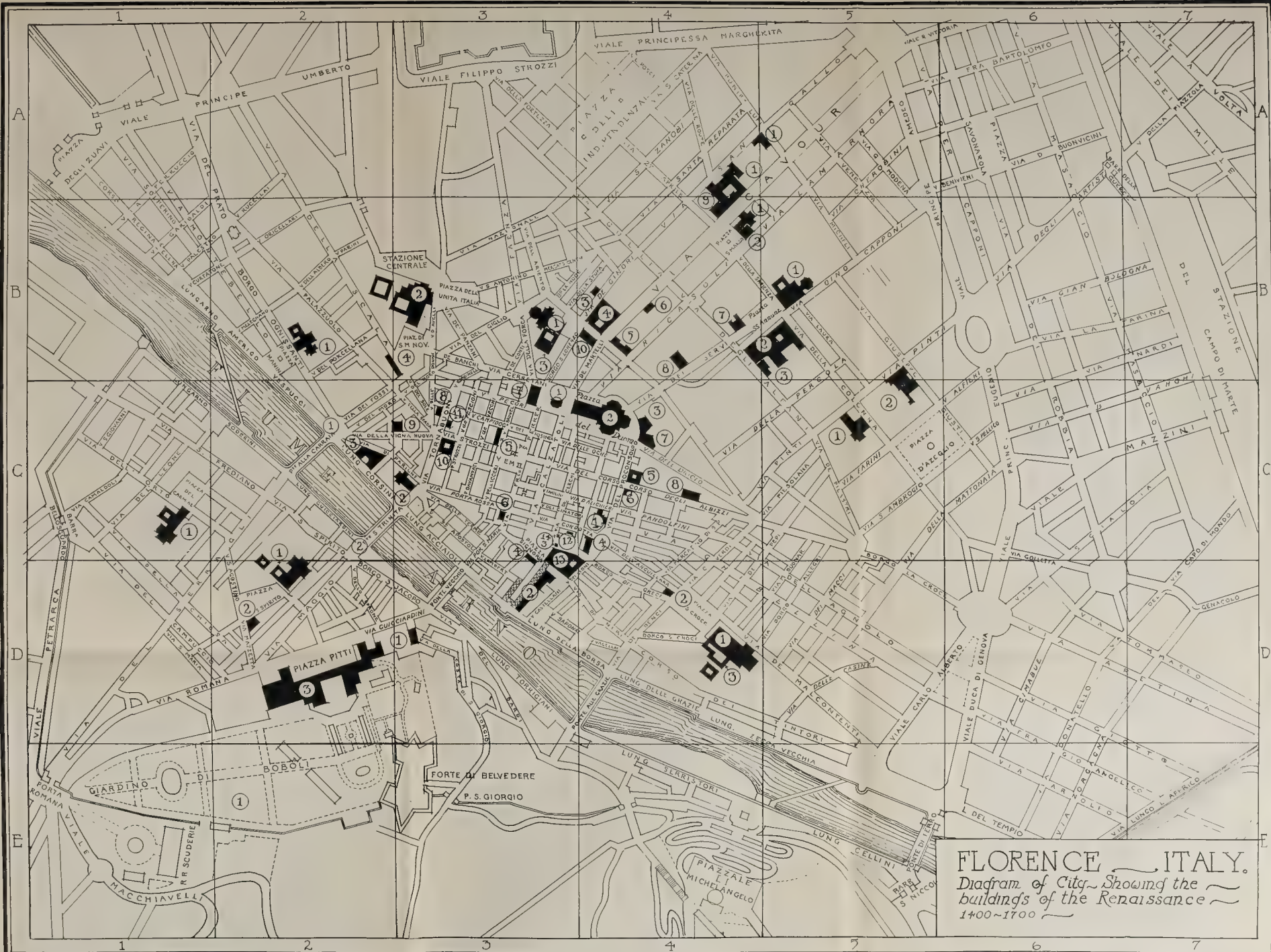
NAME	YEAR	ARCHITECT	THIS NUMBER
	PALACE		REFERS TO MAP
Riccardi-Mannelli.....	1565.....	Buontalenti.....	7(B.4)
Rucellai.....	1451.....	Alberti.....	4(C.3)
Serristori.....	1510.....	Baccio d'Agnolo.....	2(D.4)
Strozzi.....	1489-1553....	B. da Majano and Il Cronaca...	10(C.3)
Tornabuoni.....	1555.....	Michelozzi.....	11(C.3)
del Uffizzi.....	1560-74.....	Vasari.....	2(D.3)
Uguccioni.....	1550.....	Folfai.....	12(C.3)
Vecchio.....	1298.....	Arnolfo di Cambio.....	13(C.3)
	1540.....	Vasari, etc.....	13(C.3)
Zuccari.....	1590.....	Zuccari.....	

STATUE

Cosimo I.....	1594.....	Giov. da Bologna.....	14(C.3)
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VILLA

Bondi or Dante.....			
Campi.....			
Carreggi.....	1555.....	Michelozzi.....	
Corsini.....			
Gamberaja.....			
Petraja.....			
Palmieri or Schifanoja.....			
Poggio a Cajano.....	1480-85.....	G. da SanGallo.....	
Poggio Imperiale.....	1622.....		
Reale.....			



THE COLORIMETRIC DETERMINATION OF PLATINUM BY POTASSIUM IODIDE

By E. G. R. ARDAGH, *Associate Professor of Chemical Engineering*,
F. S. SEABORNE and N. S. GRANT

HISTORY OF PLATINUM

As early as the first half of the sixteenth century, it appears to have been noticed that the gold ore in the Spanish Mines of Darien included grains of white metal, resembling the noble metals, yet distinctly different from silver. This fact remained unknown throughout Europe generally, because the Spanish government, having found that this new metal lent itself admirably to the adulteration of gold, prohibited its exportation.

About the middle of the eighteenth century, platinum began to find its way into Europe, from South America, under the name of "Platina del Pinto" (the little silver of the River Pinto).

La Torre, a Spaniard, published in 1748 the first article referring to platinum. Sir William Watson (1750) was the first to describe it as a new metal.

Its chemical individuality and qualities were established by the successive labours of Scheffer (1752), Margraft (1757) and Bergmann (1777).

Count von Sickingen (1772) was, so far as is known, the first to succeed in working the metal.

The first platinum crucible was produced it is thought by Achard in 1784.

The auriferous sands of the Urals were shown in 1819 to contain a white metal which in 1823 was recognized to be platinum. In 1829 a special expedition of scientific men, of whom G. Rose and Humboldt were two of the most prominent, was organized to investigate these deposits.¹ In consequence of the attention that was focussed at this time on the metals of the platinum group we find such famous men as Berzelius and H. Rose very busily engaged in 1829 investigating the chemical and metallurgical properties of these metals. It was in 1829 that Lassaigne² discovered that a precipitate of platinic iodide dissolved in an excess of potassium iodide solution to give a carmine-red liquid. In his *Manual of Analytical Chemistry* (Griffin's translation), published in 1831, H. Rose³ also refers to this carmine coloured solution, and the writings of other experimenters of the time make reference to the same phenomenon.^{4 5 6 7}

It was not, however, until 1881 that any attempt was made to use the reaction between platinic solutions and an excess of KI solution as a test for minute amounts of platinum.

The remarkable rise in the price of platinum† has naturally brought in its train the need for an accurate and reliable method of determining quantitatively very small amounts of the metal not only in ores, sands and concentrates, but in filings, sweepings, plating-solutions, and other by-products that accumulate on the premises of the manufacturing jeweller.

The earliest work done on this problem was that of Frederick Field.⁸ Much work of a preliminary nature was carried out by this investigator, but all of it was qualitative. Field's researches were based upon the characteristic property of very dilute platinic solutions giving with a solution of potassium iodide a beautiful rose-pink colour, proportional in intensity to the quantity of platinum present in the solution. He found the delicacy of the test to be quite remarkable, one part of platinum in 2,000,000 being quite readily detected, and he pointed out that a drop or two of hydrochloric acid hastens the development of the colour, whereas all sulphites, hyposulphites, as well as ammonia and mercuric chloride, completely destroy it. Heating the solution after the colour had been developed also caused it to fade.

Field made a number of qualitative experiments to determine the effect of the presence of other metals upon the speed of development and the intensity of the colour. The presence of zinc, iron (ferrous), cadmium, manganese and silver, had no effect upon the intensity of the colour; nickel, cobalt, copper and gold, however, interfered. The interference of gold presented considerable difficulty. Field recommended the use of oxalic acid for its precipitation.

Following Field's work, and in all probability as a direct consequence of it, other experimenters and authors have included in their writings the KI colour reaction for the qualitative detection of minute quantities of platinum.^{9 10 11 12 13}

Although Field had broken the ground as early as 1881, no one appears to have made a quantitative study of the KI colour reaction until 1908, when J. C. H. Mingaye attacked the problem.¹⁴ Mingaye uses the method to determine platinum in sands. His process involves the following steps:

†Encyclopedia Britannica quotes platinum:—

1874—£1 5 2 (\$6.10) per ounce Troy.

1908—£5 2 6 (\$24.90) per ounce Troy.

1924—The price is in the neighbourhood of \$125 per ounce Troy.

1. Concentration of the ore.
2. Obtaining the platinum as a platinic chloride solution.
3. Adding either stannous chloride or potassium iodide to develop the colour.
4. Comparing the colour produced with a set of standards made from a solution of pure platinic chloride.

The standard solution used for this purpose contained one grain of pure platinum (as chloride) per 100 c. c. of solution. Mingaye found that dilute solutions of platinic chloride "deposited" after long standing, and, therefore, it was necessary to make them up just prior to use.

This colorimetric method has been used in Australia, in conjunction with fire-essays, and it is said to give excellent results. Mingaye states: "The method is simple, easily performed, accurate, and enables a large number of estimations or checks to be put through in a comparatively short time."

Our own experience, however, has led us to the conclusion that there are a number of factors to be taken into account, if dependable results are to be secured. In other words, the method is not by any means so simple and so free from pitfalls as Mingaye declares it to be.

THE COLORIMETRIC DETERMINATION OF PLATINUM

In the following work it has been our endeavour to study the reactions between potassium iodide and platinic solutions from a quantitative, rather than from a qualitative point of view.

THE COLORIMETRIC METHOD.—The first step in the use of any colorimetric method is to prepare a standard, or a set of standards to which the solutions of unknown value may be compared. The standard stock solutions used for this work were prepared in the following manner:

STANDARD STOCK SOLUTIONS.—Some soft platinum in sheet form was carefully cleaned in nitric acid, followed by hydrochloric and then heated to bright redness.

Exactly 0.1000 gram of this clean platinum was dissolved in aqua regia, the solution evaporated nearly to dryness in a porcelain casserole, and then 5-10 c. c. more hydrochloric acid added.

The exact weight (0.06 g.) of NaCl necessary to form the relatively stable Na_2PtCl_6 was added, and the solution evaporated just to dryness to remove the excess of acid, so that when distilled water was added, a clear, yellow neutral solution of sodium chloroplatinate was obtained.

Care must be exercised while evaporating to dryness, that the sodium chloroplatinate is not decomposed by excessive heating. By carrying out the latter stages of the evaporation on a water bath, all danger of decomposition was eliminated.

If the salt does not dissolve in distilled water to form a clear, dark yellow solution, but remains cloudy or murky, it is quite probable that some decomposition has taken place. Should this occur, evaporate to dryness, add concentrated HCl (or aqua regia, if necessary), and proceed as before.

Solution I, containing 1 mg. of platinum per c.c.

The above aqueous solution, containing 0.1000 gram of platinum, was made up to 100 c.c. with distilled water, giving a solution each c.c. of which contained 1 mg. of platinum as Na_2PtCl_6 .

Solution II, containing 0.1 mg. of platinum per c.c.

A second standard solution was prepared from I by making exactly 50 c.c. of No. I up to 500 c.c., and thus a solution containing 0.1 mg. per c.c. was obtained.

It was observed that this latter solution tended to "deposit" after standing several weeks. This may be prevented by making the solution slightly acid with hydrochloric.

A set of standard comparison tubes were then prepared by:—

1. Pipetting a known volume of one of the standard solutions into a 50 c.c. Nessler tube, wide form.
2. Adding 25-30 c.c. distilled water.
3. Adding potassium iodide.
4. Making the solution up to the mark with distilled water.

The characteristic rose-pink colour made its appearance at once in the stronger solutions, but the more dilute ones (those containing less than two parts of platinum per million) did not show a trace of colour for

TABLE NO. 1

SOLUTION No. I		SOLUTION No. II		SOLUTION No. III	
0.1 mg. Pt. in 50 c.c. 2 parts per million Excess KI. No acid.		0.2 mg. Pt. in 50 c.c. 4 parts per million. Excess KI. No acid.		0.4 mg. Pt. in 50 c.c. 8 parts per million. Excess KI. No acid.	
Apparent content of platinum in mg.	Elapsed Time	Apparent content of platinum in mg.	Elapsed Time	Apparent content of platinum in mg.	Elapsed Time
0	5 min.				
Visible	10 min.	Visible	10 sec.	Visible	At once
.075	3 hours	.075	5 min.	.2	5 min.
		.10	30 min.	.25	30 min.
.075	3 hours	.15	3 hours	.35	3 hours
.10	16-18 hrs.	.20	16-18 hrs.	.40	16-18 hrs.

some time, and a much longer period elapsed before the colour attained its maximum value.

All our colour comparisons were made by daylight, at a north window. When we used Nessler tubes, we placed them on a sheet of white paper, directly between the observer and a large window, and made our comparison by looking down the tubes. There does not appear to be any appreciable gain in accuracy from using a Nessler stand with adjustable mirrors. This is a matter of the personal equation, however.

It will be shown later that there are a number of factors which affect the rate of colour development. Table No. I was compiled from observations taken, using freshly made stock solution. The colour in solutions that have stood for some time (about ten days to two weeks), develop more rapidly. So far, no way, through which the solution may be aged more rapidly, has been discovered, nor is it known what chemical action takes place during this "ripening".

DEVELOPMENT OF COLOUR

The amount of KI added, in excess of the minimum quantity necessary, has very little effect upon the time required for the colour to reach its maximum value. This was ascertained in the following manner, using the shelf reagent, containing 20 g. potassium iodide per litre.

TABLE NO. 2

0.2 mg. Pt. in 50 c.c. 0.2 c.c. KI		0.2 mg. Pt. in 50 c.c. 0.5 c.c. KI		0.2 mg. Pt. in 50 c.c. 1.0 c.c. KI		0.2 mg. Pt. in 50 c.c. 5.0 c.c. KI	
Apparent content of platinum in mg.	Time	Apparent content of platinum in mg.	Time	Apparent content of platinum in mg.	Time	Apparent content of platinum in mg.	Time
Visible	At once	Visible	At once	Visible	At once	Visible	At once
.05	30 sec.	.05	30 sec.	.05	30 sec.	.05	30 sec.
.10	2 min.	.1	1½ min.	.1	1½ min.	.1	1½ min.
.15	5 min.	.15	4 min.	.15	4 min.	.15	4 min.
.175	1½ hrs.	.175	1½ hrs.	.175	1½ hrs.	.175	1½ hrs.
.2	10-12 hrs.	.2	10-12 hrs.	.2	10-12 hrs.	.2	10-12 hrs.

Thus, for a 50 c.c. tube of solution, 1 c.c. of KI will be ample to bring up the colour quite satisfactorily.

So far, the work had been done entirely at room temperature. The effect of heat upon solutions and their reactions was next investigated.

EFFECT OF HEAT

Heating the solution to 40°C. before adding the KI caused the colour to develop more rapidly, but at temperatures near boiling, no colour will develop.

Colour once developed is not so easily destroyed. We found that platinum solutions, in which the rose-pink had been already developed, could be heated to 60°C. and kept at that temperature for some time, without perceptibly fading. In time, however, at this temperature, the colour gradually changes from pink to yellow, and finally the solution becomes almost colourless.

At 50°C. the colour develops quite rapidly, but never reaches the maximum value obtainable by working at room temperature. The acceleration of colour development on heating is especially noticeable in the case of very dilute solutions, which would ordinarily require 10-12 hours to develop fully.

Continued heating, however, at this temperature (50°) for three hours, caused complete disappearance of the colour of solutions up to 0.1 mg. of platinum in 50 c.c., while those above this strength were greatly faded.

The colour was observed to fade quite rapidly from platinum solutions, even when deeply coloured, when they were heated to

TABLE NO. 3

Effect of heating solutions containing Na₂PtCl₆ (10 parts of platinum per million) plus KI (slight excess).

0.5 mg. Pt. in 50 c.c. plus KI at 30°C.		0.5 mg. Pt. in 50 c.c. plus KI at 40° C.		0.5 mg. Pt. in 50 c.c. plus KI at 50° C.	
Apparent content of platinum in mg.	Time	Apparent content of platinum in mg.	Time	Apparent content of platinum in mg.	Time
Visible	At once	Visible	At once	Visible	At once
.05	1 min.	.05	1 min.	.05	1 min.
.10	2 min.	.10	2 min.	.10	1½ min.
.15	5 min.	.15	5 min.	.15	4 min.
.20	14 min.	.20	12 min.	.2	8 min.
.2	15 min.	.225	15 min.	.25	15 min.
.225	40 min.	.25	20 min.	.30 max.	20 min.
.25	60 min.	.30	30 min.	.275	25 min.
Approximately same from here on as cold solutions.		.35	60 min.	.250	30 min.
		Development very slow from here on.		.225	35 min.

temperatures much in excess of 60° . C. W. Davis¹⁵ infers that the rose-pink colour is not destroyed by boiling, and that in consequence the test enables one to differentiate between platinum and tellurium by boiling the rose-pink solution. This is not the case when the quantity of platinum is small enough to make a quantitative reading possible.

Colours that have been destroyed in neutral solutions may often be restored to a considerable extent by adding a few drops of HCl and KI to the colourless solution. This restored colour never attains the original intensity, and hence, once the KI has been added, the solution should never be warmed above 40°C .

EFFECT OF ACIDS ON COLOUR DEVELOPMENT

SULPHURIC.—The addition of a small quantity of sulphuric acid hastens the development of the colour, but, on allowing the solution to stand, the colour fades.

NITRIC ACID.—The addition of nitric acid imparts a yellowish-green tinge to the solution, completely obscuring any pink colour.

ACETIC ACID.—Has the effect of retarding the development of the colour, and, when much is present, entirely prevents its appearance.

HYDROCHLORIC ACID.—As Field⁸ pointed out, the addition of HCl gave the most satisfactory results. One drop of concentrated HCl (sp. gr. 1.19) was sufficient to make an appreciable difference in the speed of development in 50 c.c. of solution. A large excess of HCl, however, must be avoided, as it tends to change the colour from rose-pink to reddish-yellow, or even to brown. To obtain the best results, the amount which may be added to 50 c.c. of solution (containing from 0.05 to 0.2 mg. Pt.), lies between 0.5 c.c. and 1 c.c. of normal HCl.

EFFECT OF AIR AND LIGHT

Now, since time was found to affect so greatly the depth of the colour, even under the most favourable conditions, careful investigations were made to determine the effect of long standing, with and without acid, in open and sealed vessels, exposed to and protected from the sunlight.

It was found that, not only did the colour require a definite length of time, depending upon the conditions, to reach a maximum value, but also that, as soon as it reached this value, it began slowly to fade.

Tubes of platinum solution were found to fade in the dark, as well as in the sunlight. Hence the fading is not due to light. Stoppering the tubes carefully also failed to prevent fading.

PERMANENT STANDARDS

It is evident that a standard solution of platinum falls short of the ideal as a standard of comparison in colorimetric work, since sets of colours prepared from it do not remain constant in tint and intensity; and, hence, a set of standard colours must be prepared every day or so, from the standard platinum solution.

It is, therefore, desirable to prepare or obtain some solution of the same shade of colour that will not fade, or to use a scale of standard tints made from glass. It has been pointed out by Field that solutions of manganese and cobalt salts exhibit colours very similar to the platinic chloride-potassium iodide rose-pink.

Aqueous solutions of cobalt salts, when carefully prepared and suitably toned down, proved to be the most satisfactory of those suggested for use as colour standards for this work.

If one takes the sulphate of cobalt alone, however, its aqueous solution is too pink. It lacks the necessary yellow shades required to match the rose-pink produced in the platinum solutions. This is also true of cobalt nitrate, acetate and chloride. The addition of a small amount of $K_2Cr_2O_7$ gives the desired tone to the colour.

In making up the cobalt solutions, the following proportions give a very close match.

(1) Dissolve 10 grams of cobalt sulphate hexahydrate† in water, add 1 c.c. concentrated H_2SO_4 and make up to 100 c.c.

(2) Dissolve 0.1 gram of $K_2Cr_2O_7$ in water, add 1 c.c. concentrated H_2SO_4 and make up to 100 c.c.

Solution No. 2 is then run into solution No. 1 drop by drop, from a burette, until the desired tint is obtained.

Small amounts of this solution, when sufficiently dilute, match the platinum solutions, especially those containing 1-4 parts of platinum per million. Four parts of platinum per million (0.2 mg. in 50 c.c.) is the ideal standard of comparison as stronger or weaker solutions are not so readily or accurately matched with each other.

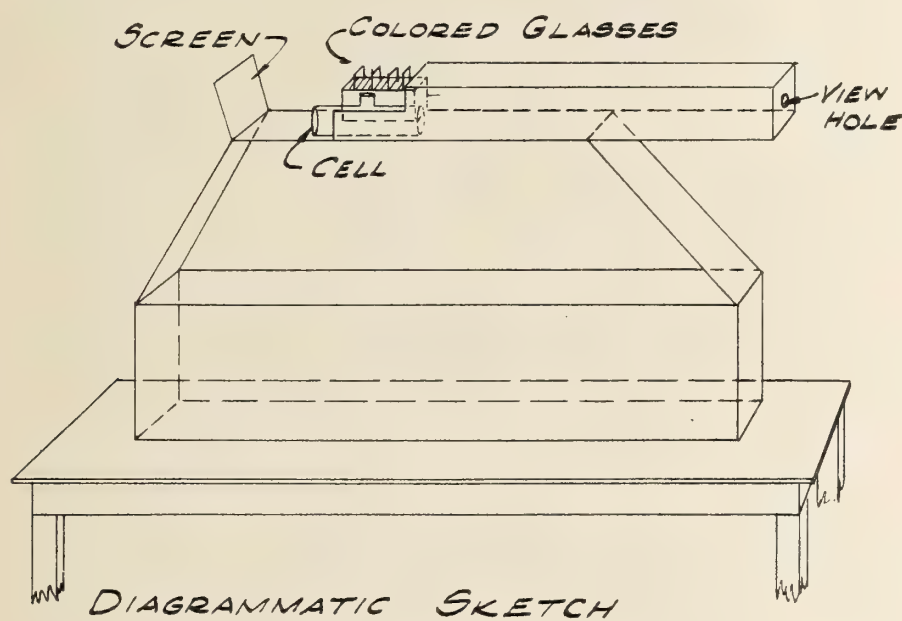
TABLE NO. 4

To Match Solutions Containing in 50 c.c.	Use of Solution No. 1	Use of Solution No. 2
0.1 mg. Pt.	3.5 c.c.	1.5 c.c.
0.2 mg. Pt.	9 c.c.	3.0 c.c.
0.3 mg. Pt.	14.5 c.c.	4.5 c.c.
	Make up to 50 c.c.	

†The hydrate may be readily determined by total dehydration. Thus 10 grms. heptahydrate lose 4.5 gms. of water, while 10 gms. of hexahydrate lose 4.1 gms. of water on dehydration.

LOVIBOND TINTOMETER.—We found, however, that we were able to increase the delicacy and accuracy of our readings very appreciably by using a Lovibond Tintometer. In this instrument, the solution undergoing examination is placed in a cell and matched in colour with tinted glasses. The cell being plane parallel at the ends, gives a column of solution of definite length.

The Lovibond Tintometer, though originally designed for testing beers and ales for colour, has since been adapted to cotton seed oil and many other liquid controls and tests. By using a special cell, designed by ourselves, we found the instrument to be admirably suited to our purpose. (See Fig. I).



DIAGRAMMATIC SKETCH
OF THE
TINTOMETER

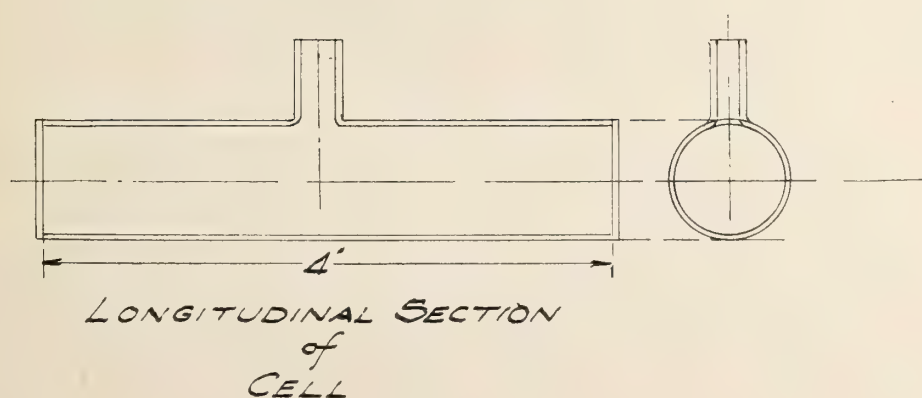


Fig. I

The cell was made from a piece of heavy walled, soft glass tubing (Carius tubing), to the centre of which a side tube for filling and emptying was sealed. The ends were ground until the tube was exactly four inches long, and then closed by cementing on end pieces with an acetone solution of pyroxylin.

We found that the red and yellow glasses (neutral tint) that are used by cottonseed oil refiners, would give us perfect colour matching.

A graduated scale of readings of solutions, containing definite amounts of platinum, must be prepared before the instrument can be used for platinum determinations. Graphs may be plotted from these readings. One thus obtains curves of colour values to which the readings of solutions of unknown strengths, between their maximum and minimum limits, may be compared.

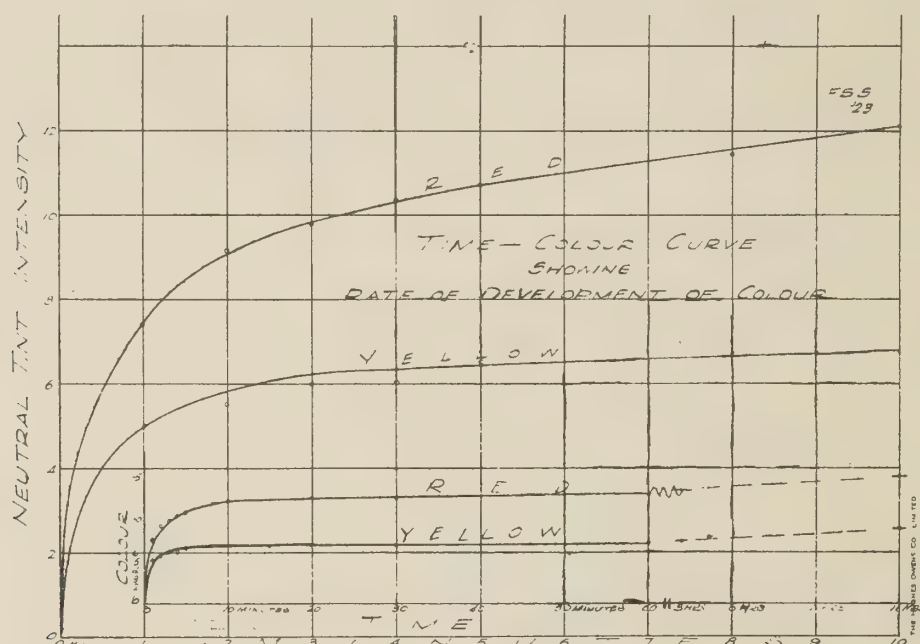


Fig. II

Curves are recommended, since they are better adapted to the rapid determination of the intensities of the tints, and hence the platinum content of the solution, than are the tabulated figures from which the curves are plotted. Curves also do away with the necessity of interpolating between the standard readings taken. Such a set of readings and curves were prepared as follows:—

- (a) Quantities of platinum solution, varying from 0.05 mg. platinum to 0.40 mg. platinum by 0.05 mg. increments were placed in a number of 50 c.c. Nessler tubes. Water was added, and then hydrochloric acid and potassium iodide, the whole made up to the mark with distilled water, and carefully mixed.

- (b) Readings, expressed in values of red and yellow, neutral tints, were taken of a four inch column of each of these solutions. The maximum readings (i.e., values at maturity) for each concentration were recorded (see table No. 5), and a graph, containing curves for red, yellow and total colour plotted from them.

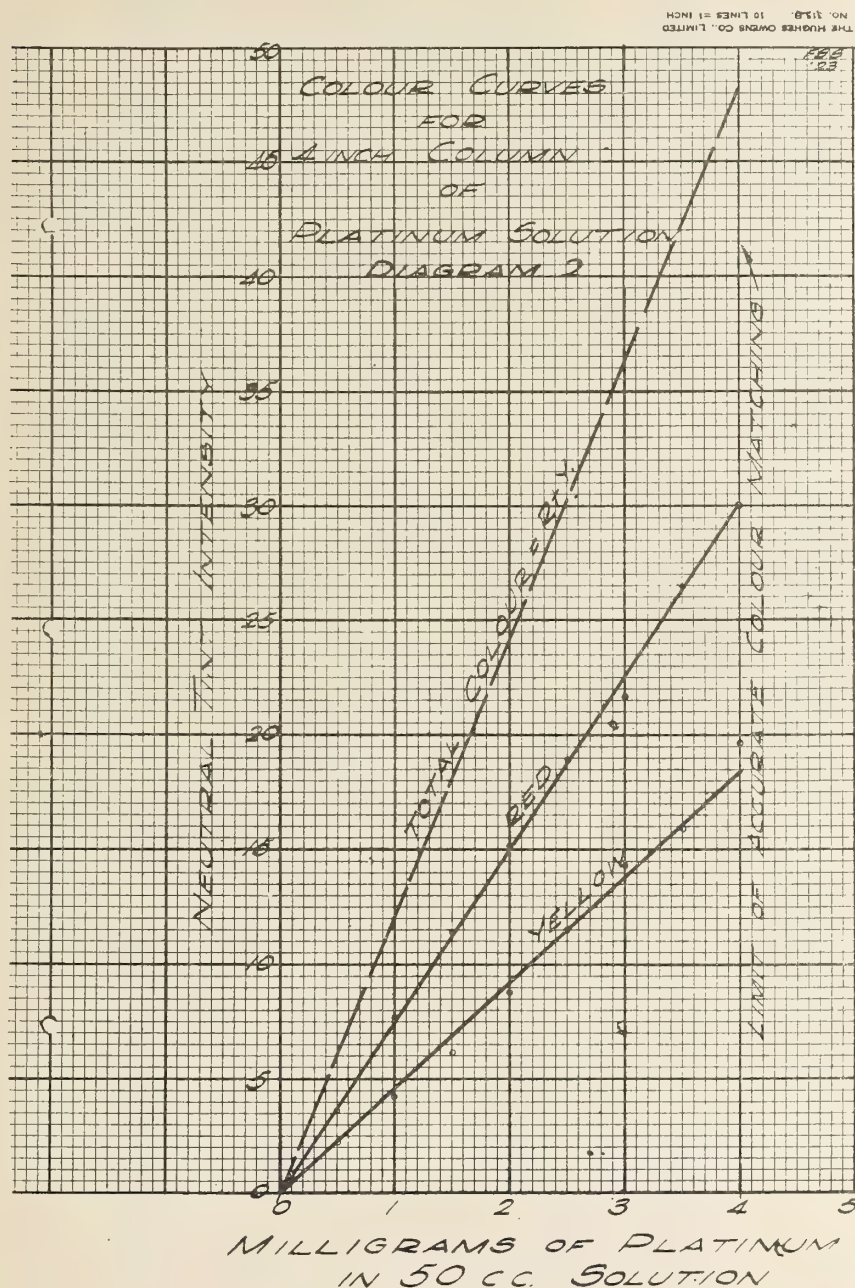


Fig. III

RATE OF COLOUR DEVELOPMENT.—It has been previously noted that the depth of colour is dependent upon the time. Choosing a 0.2 mg. platinum solution (4 parts of platinum per million), as being the most convenient and satisfactory strength with which to work, a series of readings were taken of the colour during development. From these readings, a colour curve of colour intensity against time was plotted. (See Table No. 6 and Fig. III).

TABLE NO. 5
TINTOMETER READINGS

Mg. of Platinum in 50 c.c. Solution	Length of Column	NT READINGS		NT READINGS		AVERAGE READINGS	
		Red	Yellow	Red	Yellow	Red	Yellow
0.05	4 in.	3.5	2.2	3.75	2.2	3.65	2.2
0.10	4 in.	7.8	3.7	7.5	4.5	7.65	4.1
0.15	4 in.	10.95	5.5	11.85	6.5	11.40	6.0
0.20	4 in.	14.65	8.7	15.8	8.8	15.2	8.75
0.25	4 in.	19.15	11.5	18.8	11.5	18.97	11.5
0.30	4 in.	20.45	15.0	22.8	13.8	21.65	14.4
0.35	4 in.	26.5	15.7	26.5	15.8	26.5	15.75
0.40	4 in.	30.0	21.0	30.0	18.5	30.0	19.75

TABLE NO. 6

TINTOMETER READINGS									Average Readings	
Platinum in 50 c.c.	Length of Column	Time (Mins.)	Readings		Readings		Readings			
			R.	Y.	R.	Y.	R.	Y.	R.	Y.
0.2 mg.	4"	0	0	0	0	0	0	0	0	0
+0.5 c.c.	4"	1	7.6	5.0	7.25	5.0	7.4	5.0	7.4	5.0
N HCl	4"	2	9.0	5.0	9.25	5.5	9.15	6.0	9.15	5.5
+1 c.c. KI	4"	3	9.6	5.5	9.9	6.0	9.8	6.5	9.8	6.0
	4"	4	10.25	5.5	10.45	6.0	10.35	6.5	10.35	6.0
	4"	5	10.65	6.0	10.8	6.5	10.75	7.0	10.75	6.5
	4"	8	11.55	6.5	11.35	6.5	11.45	7.0	11.45	6.7
	4"	10	12.1	6.5	12.05	6.5	12.1	7.2	12.1	6.75
	4"	15	12.3	7.0	12.35	6.5	12.35	7.2	12.3	6.8
	4"	20	12.5	7.0	12.4	7.0	12.6	7.2	12.5	7.0
	4"	30	12.6	7.2	12.6	7.0	12.6	7.2	12.6	7.1
	4"	60	13.15	7.2	13.15	7.0	13.2	7.7	13.15	7.3
	4"	3 hrs.	13.45	7.7	13.55	7.7	13.7	7.7	13.55	7.7
	4"	14-16 hrs.	15.0	8.8	15.0	8.7	15.2	8.9	15.1	8.8

N.B.—Due to lack of yellow slides, colours between 3.7 and 5.0 could not be measured.

THE DETERMINATION OF PLATINUM IN SOLUTIONS CONTAINING
OTHER METALS

1. GOLD.—Auric chloride has very strong colouring powers, especially in the presence of potassium iodide. As this yellow colour interferes so greatly with the pink of the platinum solution, the gold must be either removed or rendered inactive. The former is obviously the more practical, and may be accomplished in several ways.

(a) OXALIC ACID.—To establish the conditions surrounding this reaction, the following series of tests was made:—

1. To a solution containing 0.2 mg. of platinum in 50 c.c. was added 1 c.c. of gold solution (containing 1 mg. per c.c.).

A concentrated solution of oxalic acid, at room temperature, was made up by placing an excess of purified oxalic acid crystals in distilled water. A few c.c. of this solution were added to the gold and platinum solution, and its effect observed. In a few minutes a purplish tint developed which later deepened, reaching what appeared to be a maximum in about 30 minutes.

This solution was then filtered, and the filtrate tested for platinum. It was found that the correct depth of colour for 0.2 mg. of platinum finally developed, but the development seemed to be retarded by the excess of oxalic acid. There was no trace of gold apparent.

2. The retardation of the colour by oxalic acid was proved by a "blank" test, i.e., by using pure platinum solution and no gold. This was carried out by varying the amounts of oxalic acid added. It was found that any quantity of oxalic acid solution greater than 10 c.c. of the saturated solution retarded the colour development very noticeably.

3. It was also determined that 5 c.c. of this oxalic acid solution were required to precipitate the gold from its chloride solution completely and within 20 minutes.

Effect of heat on precipitation of gold by oxalic acid.

Heating the solution containing gold + oxalic acid had no appreciable effect in increasing or decreasing the completeness or speed of gold precipitation.

The influence of the quantity of gold present.

Gold in different quantities from 0.10 mg. to 2.0 mg. in 50 c.c. could be thrown down with satisfaction and without affecting the quantity of platinum present; nor did varying the amount of platinum present in solution from 0.1 mg. to 1.0 mg. in 50 c.c. give rise to any noticeable loss.

Care must be taken, however, in filtering. Furthermore, all solutions from which gold is being precipitated should be allowed to stand for at least 30 minutes, to ensure complete coagulation of the finely-divided gold, so as to facilitate filtering.

With these precautions the oxalic acid precipitation method will be found to be eminently satisfactory.

METHOD ADVISED.—Add 5 c.c. of saturated oxalic acid solution to 50 c.c. of the solution containing not more than 1 mg. of gold. Allow to stand twenty to thirty minutes at room temperature.

(b) PRECIPITATION OF GOLD BY FERROUS SULPHATE.—The conditions to be taken into account when throwing down gold with ferrous sulphate,

in order to obtain the best results, and to avoid excessive losses of platinum and gold, are:—

1. Freedom of the ferrous sulphate from ferric iron.
2. Quantity of the ferrous sulphate used.
3. Temperature at which precipitation is effected.
4. Time—i.e., duration of precipitation. (The time required depends largely on the temperature and the other conditions).

In order to have an ample supply of ferrous sulphate for use as a precipitating agent, the ferrous content of which must remain constant, a large bottle, provided with a two-holed rubber stopper, was nearly filled with ferrous sulphate solution made from recrystallized green vitriol; 5 grams of concentrated H_2SO_4 were then added for each 100 grams of $\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$ used. The strength was determined by titration, with 0.1 NKMnO_4 , and the calculated quantity of water added to make the solution contain 0.1 gram FeSO_4 per c.c. Through one hole of the stopper the tube from a hydrogen-Kipp was inserted, and through the other a siphon for drawing off the solution when required.

If oxidation of the ferrous sulphate is allowed to proceed by exposure to air, not only is the precipitating power of the solution reduced, but, what is even more important, the yellow colour of the accumulating ferric salt affects materially the colour readings.

When precipitating gold with FeSO_4 , a quantity just sufficient or slightly in excess of the amount required, should be used. If a very large excess of FeSO_4 is used, its green colour disguises the rose-pink colour of the platinum in the concluding step of the analysis, or, if the solution containing this large excess of FeSO_4 is allowed to stand, the FeSO_4 becomes oxidized, the whole solution grows murky, and tintometer readings are impossible to make.

The limiting excess of FeSO_4 solution was determined by making up two sets of tests in 50 c.c. Nessler tubes, each tube of which contained 0.2 mg. of platinum and 1 c.c. of KI. To Set No. 1 was added 0.5 c.c. of normal HCl ; to Set No. 2, 2.0 c.c. of normal HCl , after the FeSO_4 had been added. (See Table No. 7).

The above table shows that ferrous sulphate, if present in large excess, has a very detrimental effect upon the colour, and that an excess of 50 mg. is the maximum amount permissible in 50 c.c. of solution, i.e., 1 gram of FeSO_4 per litre more than the amount required to throw down the gold in solution will not prevent the platinum colour from developing to full value.

The amounts of ferrous sulphate required to precipitate known quantities of gold were determined as follows:—

Several solutions, each containing 1.0 mg. of gold, were made up. To

each solution a different quantity of ferrous sulphate was added. (See Table No. 8).

For cold precipitations the proportion of ferrous sulphate to gold used in Solution 3 should, therefore, be fairly closely adhered to, viz., 0.2 g. FeSO_4 in 50 c.c. of test solution.

TABLE NO. 7
EFFECT OF EXCESS FERROUS SULPHATE UPON COLOUR

SET No. 1			SET No. 2		
Tube	Fe SO_4	Observations	Tube	Fe SO_4	Observations
1	5 mg.	Colour good.	1	5 mg.	Colour good.
2	20 mg.	Colour good.	2	20 mg.	Colour good.
3	50 mg.	Colour good, but slow to develop.	3	50 mg.	Colour good, slightly better than No. 3, Set 1.
4	100 mg.	Colour noticeably weaker than No. 1 above.	4	100 mg.	Colour weak.
5	0.5 g.	Colour poor, even after several hours.	5	0.5 g.	Colour weaker than No. 4 above.
6	1.0 g.	Colour weak and greenish due to FeSO_4 .	6	1.0 g.	Colour weak and greenish-yellow.
7	2.0 g.	Colour very weak and greenish.	7	2.0 g.	Colour very weak and greenish-yellow.
8	5.0 g.	Colour very weak and greenish.	8	5.0 g.	Colour very weak and greenish-yellow.

TABLE NO. 8

No. 1	No. 2	No. 3	No. 4
1.0 mg. Au 1.0 mg. Pt. .5 c.c. FeSO_4 = .05 g. FeSO_4 Solution made up to 50 c.c.	1.0 mg. Au 1.0 mg. Pt. 1.0 c.c. FeSO_4 = .1 g. FeSO_4 Solution made up to 50 c.c.	1.0 mg. Au 1.0 mg. Pt. 0.2 g. FeSO_4 Solution made up to 50 c.c.	1.0 mg. Au. 1.0 mg. Pt. 1.0 g. FeSO_4 Solution made up to 50 c.c.
Stood cold 4 hrs. Gold not all down blue colour still distinct.	Stood cold 4 hrs. Slight trace of gold left (blue colour).	Stood cold 4 hrs. Gold down.	Stood cold 4 hrs. Gold down.

Filtered to remove precipitate of gold, took 10 c.c. and made up to 50 c.c.
Developed colour in filtrate with KI and HCl.

Gold interfered.	Gold not enough to interfere greatly.	Colour satisfac- tory; no trace of gold.	Colour poor, yel- lowish, probably due to $\text{Fe}_2(\text{SO}_4)_3$.
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Hot precipitation is more advantageous than cold for three reasons, namely:—

1. Heating coagulates the precipitate, and hence the danger of loss of gold is lessened.
2. The time required for complete precipitation is greatly reduced.
3. There appears to be less platinum carried down by the coarse precipitate from a hot solution than by the fine precipitate from a cold one, and, in the latter case, the loss is not always negligible.

Excess FeSO_4 is rapidly oxidized, when heated, especially if heated nearly to boiling.

The platinum salt in solution is altered in some way by long heating, with the result that KI does not bring up the colour, as desired for the tintometer readings.

The following experiments (Tables No. 9 and No. 10) reveal the effect of heat and time upon the precipitation of gold.

The samples were the same in every respect. In Table No. 9 the only variable in the procedure was the temperature—each sample being heated in a water bath at a different temperature, but for the same period of time. In Table No. 10, each sample was kept at the same temperature in a water bath, but for a different period of time.

Thus the time and temperature conditions for the optimum results were established.

From 9 and 10 tables it is obvious that, if a platinum solution containing gold has sufficient ferrous sulphate added to it, and is then heated at 40°C ., for 45 minutes, all the gold will be thrown down, leaving the platinum salt in the best condition for colour development.

The conditions to be observed in precipitating the gold with FeSO_4 , when platinum is to be determined colorimetrically in the filtrate, are:—

1. The FeSO_4 should be as free as possible from ferric iron.
2. The FeSO_4 added must not be greater than 0.2 grams for 1 mg. of gold in 50 c.c. of solution. If 10 c.c. of the filtrate are taken, the FeSO_4 present will not be too great for colour development.
3. Sufficient time must be allowed for the gold to separate.
4. The solution must not be heated above 40°C .

METHOD ADVISED.—Add not more than 2 c.c. of pure ferrous sulphate solution (0.10 gram FeSO_4 per c.c.) in excess. Warm the solution to 40°C . and maintain at this temperature for forty-five minutes, or allow to stand at room temperature for at least three to four hours.

Gold that had been thrown down at room temperature was analyzed by the same method as in Table No. 10. In each case a slight trace of platinum was found. The error introduced is, however, always very small.

TABLE NO. 9
EFFECT OF TEMPERATURE

1.0 mg. Pt. 1.0 mg. Au. 0.2g. FeSO ₄ in 50 c.c.	1.0 mg. Pt. 1.0 mg. Au. 0.2g. FeSO ₄ in 50 c.c.	1.0 mg. Pt. 1.0 mg. Au. 0.2g. FeSO ₄ in 50 c.c.	1.0 mg. Pt. 1.0 mg. Au. 0.2g. FeSO ₄ in 50 c.c.	1.0 mg. Pt. 1.0 mg. Au. 0.2g. FeSO ₄ in 50 c.c.
At room tem- perature for 30 min.	At 30° C. 30 min.	At 40° C. 30 min.	At 50° C. 30 min.	At 60° C. 30 min.
Filtered	Filtered	Filtered	Filtered	Filtered

Took 10 c.c. of each filtrate, made up to 50 c.c., added KI and where possible read colours.

Gold only partly down, colour very yellow.	Gold still ap- parent, solution very yellow.	Colour of platinum very good, R – 14.75 Y – 9.5 (high.)	Colour fairly good, but weak, no gold.	Colour weak but no trace of gold, yel- lowish, pro- bably due to Fe ₂ (SO ₄) ₃ .
Precipitate not analyzed as gold not all down.		Dissolved filter ash in aqua regia. Boiled off all nitric fumes. Threw down gold with oxalic acid. Filtered and added KI to each filtrate.		
		Very slight trace of platinum.	Slight trace of platinum	Quite distinct colour of platinum.

When metals such as silver, copper, nickel, iron, etc., are present in solution with the gold and platinum, separation is most easily brought about by warming the solution with 20-mesh zinc and some dilute HCl. The precipitated metals, containing all the gold and platinum, can then be filtered off, washed and attacked with cold dilute HNO₃, which leaves the platinum and gold undissolved.

Copper, when present in a solution containing platinum, produces a yellowish colour when KI is added. Its removal is, therefore, a matter of importance, since copper is always present in such a material as “jewellers’ filings”.

To establish the completeness of the precipitation of platinum by zinc in hydrochloric acid solution, exactly 1 mg. of platinum (in 100 c.c. of solution) was treated with 20-mesh zinc and dilute HCl. When all the zinc had been dissolved, the precipitated platinum was filtered off and washed from the filter paper into a clean beaker. The water was then boiled off, the platinum dissolved in aqua regia, the solution carefully evaporated to drive off the nitric fumes, and evaporated again very care-

TABLE NO. 10
EFFECT OF TIME UPON PRECIPITATION

Sample No. 1	Sample No. 2	Sample No. 3	Sample No. 4	Sample No. 5
Each sample contained 1.0 mg. Pt., 1.0 mg. Au., 0.2 g. FeSO ₄ in 50 c.c.				
At 40° C. for 10 min.	At 40° C. 20 min.	At 40° C. 30 min.	At 40° C. 40 min.	At 40° C. 60 min.
Filtered, took 10 c.c. of each filtrate and made up to 50 c.c. Added KI and noted colour.				
Very yellow, therefore gold still present.	Yellowish, therefore some gold present.	Colour good, very little or no gold, R—14.6 Y— 9.7 (high).	Colour good, no gold R—14.75 Y— 8.8.	Colour good, but a little weak, most likely due to too long heating.
		Analyzed filter ash as before. Only very slight trace of platinum in each case, therefore weakness in No. 5 not due to excessive loss of platinum.		

fully with a few drops of concentrated HCl. The almost-dry residue was taken up with distilled water and made up to 50 c.c. This solution was then tested for the amount of platinum present. After standing with the iodide for from 4 to 5 hours, a 0.2 mg. aliquot gave the correct reading. In the filtrate from the platinum, we could not detect even a trace of this metal.

This experiment proves that zinc does throw down all the platinum from a solution containing dilute HCl.

A series of experiments to determine the minimum amount of platinum recoverable was now made. The results of this investigation seemed to show that any quantity of platinum less than 0.1 mg. in 50 c.c. could not be recovered completely. At any rate, the tintometer reading was lower than it should have been.

Having established the limits with pure platinum solution, the next step was to try separating copper, since this is the interfering metal usually present in greatest quantity.

About 0.25 gram of copper (as CuCl₂) was added to a solution of platinum (1 mg. in 100 c.c.). Then a small excess of zinc (20-mesh) was dropped in and some dilute HCl added. By the time the action had ceased, the blue colour due to the CuCl₂ had disappeared and the copper lay on the bottom of the beaker as a beautiful red spongy mass. After filtering carefully, the precipitate was washed back into a beaker, and a

small amount of dilute HNO_3 added to dissolve the copper. Cold dilute nitric acid will not attack platinum, even in this finely-divided state. When all action had ceased, the platinum was carefully filtered and washed free of copper salt. The platinum was then taken up in aqua regia, and an effort made to bring up the platinum colour as before. Somewhat low results were at first obtained; but, by taking every precaution to avoid mechanical loss of the finely-divided platinum, we found that the platinum could be completely recovered. Great care must be taken to remove *all* the copper nitrate, when washing the deposit of black platinum; otherwise it will greatly interfere with the development of the pink colour of the platinum.

SUMMARY

QUANTITY OF PLATINUM.—The optimum amount of platinum is in the neighbourhood of 0.2 mg. in 50 c.c. of solution.

TIME FOR DEVELOPMENT.—The colour intensity developed by one hour's standing at room temperature approximates 90% of the maximum. The rate of increase in colour intensity on longer standing is very slow. If standard platinum solutions are used as comparates, correct results will be obtained, if standard and assay have both stood one hour, or even thirty minutes.

In the case of freshly-made solutions, however, the rose-pink colour does not come up so rapidly, as in older ones that have "ripened" or "aged". We look upon this as a possible source of grave error, if the colour is given only a few minutes to develop before matching, and we consider it to be well worth investigating. We found that a solution that had stood about two weeks developed a pink much more rapidly than one a day or two old. The maximum in each case, however, was the same.

COLOUR COMPARISONS.—In making quantitative colour comparisons for minute amounts of platinum, the tintometer method is by far the most precise method; but, for ordinary work, matching in 50 c.c. Nessler tubes (wide form) is quite satisfactory.

HEATING.—Warming gently and carefully at the proper time during the analysis hastens the colour development. The results, however, are likely to be low; hence, we advise bringing up the colour at room temperature.

AMOUNT OF POTASSIUM IODIDE REQUIRED.—In developing the colour, the amount of KI used has very little effect, so long as sufficient to cause full colour development is added. We used from five to twenty drops of the ordinary shelf reagent (20 g. KI in 1000 c.c. of solution) in 50 c.c.

ACIDS.—Acids have a widely-varying influence on the shade and intensity of the colour, as well as upon the rate of its development. All acids, with the exception of hydrochloric, are detrimental in the final stages of the analysis. The most satisfactory results were obtained by adding between 0.5 c.c. and 1 c.c. of *N* HCl. to a 50 c.c. test.

METALS (such as gold, silver, copper, nickel, etc.).—The heavy metals likely to be present interfere with the platinum colour; hence it is advisable to proceed as follows:—

1. Throw down platinum, gold, silver and copper, with 20-mesh zinc in hydrochloric acid solution.
2. Filter, wash free from chlorides, dissolve silver, copper, etc. in dilute nitric acid without heating. If the ZnCl_2 is hot and too concentrated, cool, dilute and use a Büchner funnel.
3. Filter and wash till completely free from copper.
4. Dissolve the gold and platinum in aqua regia, and evaporate carefully.
5. Precipitate the gold with oxalic acid or ferrous sulphate, exactly as advised above. Make up to the mark in a graduated flask, mix well and allow to stand till the gold has all settled.
6. Take several different aliquots of the clear solution, transfer to 50 c.c. Nessler tubes, (wide form), add the reagents (*N* HCl+KI) as described, make up to the mark, mix and allow to stand before reading.
7. Develop the rose-pink in 50 c.c. of solution, containing 0.2 mg. of platinum. Use this tube as comparison standard. Select the aliquot giving a colour slightly more intense than the standard and remove measured portions until an exact match is obtained. Place the tubes on white paper in a north window for matching.
8. If the tintometer is used, it is of course unnecessary to prepare and use the standard tube referred to in 7. In the 4" cell, a 50 c.c. solution containing 0.2 mg. of platinum will give a reading of 12.6 red, 7.1 yellow in ten minutes; and 13.15 red, 7.3 yellow in one hour.

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A SIMPLE, RAPID AND ECONOMICAL METHOD OF SEPARATING NICKEL AND COPPER FROM IRON

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For separating nickel from iron in ores such as those mined in the Sudbury, Ontario, district numerous methods have been recommended. Of these the best known are the following:

- (1) The Ammonium Carbonate Method.¹
- (2) The Basic Acetate Method.¹⁵
- (3) The Phosphate Method.³
- (4) The Ether Method.⁵
- (5) The Lead Oxide Method.⁶
- (6) The Succinate Method.¹¹
- (7) The Dimethylglyoxime Method, using a citrate or tartrate solution.¹⁷

Objections can be raised to every one of these methods. For example: (1) is long and tedious, and furthermore does not give a complete separation,² (2) requires too great dilution; the precipitate is difficult to wash and often retains Ni,³ (3) requires a double precipitation,⁴ (4) is inconvenient, tedious and in addition requires special apparatus, (5) does not give complete separation.⁴

One can scarcely imagine a simpler and more convenient way of separating Cu and Ni from Fe than the old ammonia+ammonium chloride method used in qualitative analysis. If this method can be modified so that Ni (and Cu also) can be completely separated from Fe by one precipitation without at the same time requiring abnormal quantities of ammonia and ammonium chloride or sulphate, the time consumed on much routine work, as well as the cost of chemicals required for the same, will be materially reduced. In their efforts to this end the authors have been completely successful.

SEPARATION OF NICKEL FROM IRON.

Standard Solutions Containing Nickel and Iron.

37.0 grams of $\text{Ni}(\text{NH}_4)_2(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$ were dissolved and made up to two litres. Determination of the Ni in this solution by dimethylglyoxime gave (1) 0.0258 and (2) 0.0260 grams Ni in 10 c.c. Electrolytic

deposition gave (1) 0.0258 and (2) 0.0259 grams. Mean value 0.0259 grams Ni in 10 cc.

Enough $\text{Fe}_2(\text{SO}_4)_3$ was dissolved in two litres of warm water slightly acidified with H_2SO_4 , to give a solution containing very close to 0.1 gram Fe in 10 cc.

Determination of Nickel Separated from the Iron

To determine the Ni in the experiments that follow, the cyanide method was selected because of its rapidity. 10 cc. of the Ni solution were diluted to 150 cc., 5 drops of conc. NH_4OH added and then 5 cc. of 0.1% AgNO_3 and 5 cc. of 2% KI. The NaCN solution (50 grams diluted to 2700 cc.) was run in with constant stirring until the AgI opalescence cleared. Titration was carried out at room temperature. The blank for the opalescence alone was 0.2 cc. NaCN.

10 cc. Ni solution (0.0259 grams Ni) required 5.2 cc. NaCN net.

To determine the effect of excess NH_4OH , the following titrations were carried out with 10 cc. Ni solution diluted to 200 cc.:

c.c. Ni Solution Used	c.c. NH_4OH (d. 0.90) Added	c.c. NaCN Required
10	0.1	5.4
10	1.0	5.4
10	3.0	5.4
10	5.0	5.4
10	6.5	5.3
10	10.0	3.0

It is evident that as much as 5 cc. of conc. NH_4OH may be present in 200 cc. without affecting the titration.

INFLUENCE OF VARYING CONCENTRATIONS OF NH_4OH and NH_4Cl OR $(\text{NH}_4)_2\text{SO}_4$, WORKING WITH A 200 C.C. SOLUTION.

Ibbotson⁹ and Brearley have shown that a precipitate of $\text{Fe}(\text{OH})_3$ will absorb Ni just as readily if the Ni solution is added after the $\text{Fe}(\text{OH})_3$ has been thrown down as when the Ni is present in solution during the actual precipitation of the iron. In the same article it is shown that if very large quantities of NH_4Cl (100 grams) and of conc. NH_4OH (50 cc. in excess) are added, a very complete separation can be obtained. Such extravagant quantities of these reagents are, of course, out of the question for routine work.

Series A

The use of Ammonium Sulphate and Ordinary Reagent
Ammonia (1 conc. NH_4OH to 1.5 H_2O).

In this series 10 cc. of Ni solution and 10 cc. Fe solution were taken,

diluted to 200 cc. and heated before precipitation. The precipitated $\text{Fe}(\text{OH})_3$ was washed with water.

	Grams $(\text{NH}_4)_2\text{SO}_4$ Added	c.c. Reagent NH_4OH Present in Excess	Ni in Filtrate in Terms of % of Amount Added
(1)	none	Just enough to give faint smell	2
(2)	5	5	59
(3)	5	5	(a) 49 (b) 51
(4)	10	5	71
(5)	15	5	73
(6)	20	5	84.5
(7)	25	5	83
(8)	none	5	11.5

(2) NH_4OH added before the $(\text{NH}_4)_2\text{SO}_4$. In all the other trials the $(\text{NH}_4)_2\text{SO}_4$ was added first.

It can be seen that very large quantities of $(\text{NH}_4)_2\text{SO}_4$ and NH_4OH would be necessary to give complete separation.

Series B

The Use of Ammonium Sulphate and Conc. Ammonia (d. 0.90)

In this series the 200 cc. of solution were heated to boiling before the reagents were added, filtration was performed while the solution was hot, and the precipitate was washed with hot water.

	Grams $(\text{NH}_4)_2\text{SO}_4$ Added	c.c. Conc. NH_4OH Added	Ni in Filtrate in Terms of % of Amount Added
(1)	5	10	75
(2)	10	10	81
(3)	15	10	84.5
(4)	15	10	86.5
(5)	5	25	92.5
(6)	10	25	92.5
(7)	15	25	92.5

(3) Stood over-night, was filtered cold and washed with cold water.

It is again clearly to be seen that very large quantities of the reagents must be present to bring about complete separation.

Series C

In this series the mixed Ni-Fe solution was evaporated to about 2 cc. and then NH_4Cl was stirred in and just sufficient water added to dissolve the NH_4Cl . The conc. NH_4OH was then added. The precipitate was washed three or four times with water.

One very noteworthy effect of this procedure proved to be the complete transformation of the usual bulky, flocculent $\text{Fe}(\text{OH})_3 \cdot x\text{H}_2\text{O}$ precipitate to a dense form that takes up comparatively little space in the filter, and that can be washed quickly with gentle suction. Low²⁴ has also observed this phenomenon, and refers to it in his chapter on the determination of zinc.

	Grams NH_4Cl Added	c.c. Conc. NH_4OH Added	Ni in Filtrate in Terms of % of Amount Added
(1)	5	25	84.7
(2)	10	20	(a) 96 (b) 94.5
(3)	10	25	(a) 94.5 (b) 96
(4)	15	25	96

The $\text{Fe}(\text{OH})_3$ precipitates from trials (3) and (4) were examined qualitatively with dimethylglyoxime. Ni was present in small amount.

This series shows that a somewhat better separation of Ni can be made by working with a small bulk of solution, but that the $\text{Fe}(\text{OH})_3$ precipitate still retains a little Ni. The precipitates were easier to wash than those obtained in series A and B.

Series D

The procedure followed in this series was exactly the same as in Series C except that the precipitate of $\text{Fe}(\text{OH})_3$ was washed with a solution containing NH_4Cl [or $(\text{NH}_4)_2\text{SO}_4$] and NH_4OH instead of with water.

	Grams NH_4Cl Added	c.c. Conc. NH_4OH Added	Grams Fe Taken	Grams Ni Taken	Ni in Filtrate in Terms of % of Ni Taken
(1)	10	20	0.1	0.0259	100
(2)	10	20	0.1	0.0259	100
(3)	10	20	0.5	0.0259	100
(4)	5	10	0.5	0.0518	100
(5)	5	10	0.5	0.0518	100
	Grams $(\text{NH}_4)_2\text{SO}_4$ Added				
(6)	10	20	0.5	0.0518	100

In each case the washed $\text{Fe}(\text{OH})_3$ was dissolved in a little conc. HCl , conc. NH_4OH added in excess, the precipitate filtered and the filtrate tested for Ni with dimethylglyoxime. In trials (1) and (2) Ni could not be detected. In the remainder a trace only of Ni was found; in (4) about 0.1 mg, in (5) much less than 0.05 mg. and in (6) about 0.05 mg. Ni.

The ore from the Creighton mine assays about 4.5% Ni and 45% Fe, hence in trials (4), (5) and (6) we used about the same quantity of Ni and Fe as a one gram sample of this ore would contain.

The wash solution, which was used cold, contained 10 grams NH_4Cl and 10 cc. conc. NH_4OH in 100 cc. In trial (6) 10 grams of $(\text{NH}_4)_2\text{SO}_4$ were used in place of the NH_4Cl .

The most satisfactory technique was found to be the following, when such a large quantity as 0.5 gram Fe was present.

To the Ni-Fe solution in a beaker or casserole add a few drops of conc. HCl and evaporate to about 5 cc. taking care to avoid local baking. Then take the vessel in the hand and swirl contents while continuing the evaporation over a small flame, until the volume is reduced to about 2 or 3 cc. While still warm add 5 grams NH_4Cl and work up the mass with a stirring rod. Add 10 cc. conc. NH_4OH , break up lumps, add about 25 cc. cold water and filter on a 7 cm. paper in an 8 cm. Büchner funnel. Wash back the bulk of the precipitate into the beaker or casserole, using the ammoniacal wash and transfer again to the filter. Wash six to eight times on the filter, consuming about 100 cc. of the ammoniacal wash-liquid in all. A powerful suction packs the precipitate on the paper too tightly and hence should be avoided. A suction equivalent to about three inches of mercury works very satisfactorily, and with it the whole operation of filtering and washing should not require more than about five minutes. When handling smaller quantities of Fe, say 0.1 to 0.2 gram, an ordinary funnel is quite satisfactory, but with larger quantities the precipitate packs in the apex of the paper.

SEPARATION OF COPPER FROM IRON BY AMMONIA AND AMMONIUM CHLORIDE OR SULPHATE

Fresenius (Quant. Chem. Analysis) states that to effect complete separation of Cu from Fe(ic) by NH_4OH , the precipitation must be repeated two or even three times. De Koninck. (*Qual. u. quant. chemische Analyse*, Vol. II., section 1050, p. 121) lays stress on the uselessness for practical purposes of attempting to obtain a complete separation by means of NH_4OH and ammonium salts.

In the following series of experiments the Cu in the filtrate was determined by the Cyanide Method. This method is rapid, and though not very reliable, even when performed under the optimum conditions, it is nevertheless sufficiently accurate to serve as a guide, so that we could tell if our separations were improving or not as we changed the conditions of precipitation. We did not rely on our figures for Cu as we approached 100% separation, but examined the precipitate to determine whether or not it still retained any Cu.

The Cu solution contained 39.35 grams of $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ per litre. A few drops of H_2SO_4 were also added to prevent hydrolysis. 10 cc. of this solution, containing 0.1 gram Cu and 10 cc. of the Fe(ic) solution, containing 0.1 gram Fe, were used for each experiment. Only those conditions that gave the best results with Ni were considered worthy of trial.

Series A

The precipitation was carried out in a 20 cc. solution and the precipitate was washed three times with water—

	Grams (NH_4) $_2$ SO $_4$ Added	c.c. Conc. NH $_4$ OH Added	Cu in Filtrate in Terms of % of Cu Added	
(1)	10	20	(a) 95.5	(b) 96.5
(2)	15	20	(a) 96.5	(b) 96.5

In trial (2) the precipitate was washed six times.

The precipitates are dense and easy to wash, but evidently water alone does not remove the last traces of Cu.

Series B

The method was identical with Series A, but the precipitate was washed with a solution containing 10 grams of $(\text{NH}_4)_2\text{SO}_4$ and 10 cc. conc. NH_4OH in 100 cc.

	Grams (NH_4) $_2$ SO $_4$	c.c. Conc. NH $_4$ OH	Cu in Filtrate in Terms of % of Cu Added	
(1)	10	20	(a) 99.5	(b) 100
(2)	10	20	100	
(3)	10	20	98.5	
(4)	10	20	100	
(5)	10 grams NH $_4$ Cl	20	100	
(6)	10 grams NH $_4$ Cl	20	100	

(1) was washed three times. The precipitate when tested for Cu with $\text{K}_4\text{Fe}(\text{CN})_6$ was found to contain about 1 mg.

(2) was washed six times. No Cu in the $\text{Fe}(\text{OH})_3$ precipitate.

(3) used 40 cc. Fe(ic) solution in place of 10 cc. Evaporated to

about 3cc. before adding $(\text{NH}_4)_2\text{SO}_4$ and NH_4OH . Washed three times. The precipitate contained about 1.5 mg. Cu.

(4) Same as (3), but washed six times. No Cu in the $\text{Fe}(\text{OH})_3$ precipitate.

(5) and (6) same as (2), but NH_4Cl used in precipitating and washing in place of $(\text{NH}_4)_2\text{SO}_4$.

Evidently 0.1 g. Cu can be completely separated from 0.4 g. of Fe by this method. This is approximately as much Fe as a 1 gram sample of Creighton ore or a 2 gram sample of Crean Hill ore would contain.

Application of the Method to Crean Hill Ore and Creighton Ore

The two samples of ore were ground to such fineness that approximately 90% of each would pass an 80 mesh screen. The ores on analysis gave the following figures:

Crean Hill ore	Ni 1.59%	Cu 3.30%	Fe 21.72%	S 11.88%
Creighton ore	Ni 4.41%	Cu 1.62%	Fe 43.50%	S 27.37%

One gram of the ore was evaporated to dryness with 10 cc. conc. HCl and 5 cc. conc. HNO_3 . After addition of 10 cc. conc. HCl to the residue it was evaporated to about 3 cc., 10 grams NH_4Cl and 20 cc. conc. NH_4OH were now added, the precipitate filtered off and washed three times with wash solution containing 10 grams NH_4Cl and 10 cc. conc. NH_4OH in 100 cc.

The $\text{Fe}(\text{OH})_3$ precipitate on examination was found to contain 1.5 mg. of Cu, but to be free from Ni. Repeating the experiment we found the $\text{Fe}(\text{OH})_3$ to contain 1.0 mg. Cu, but to be free from Ni. Repeating the experiment a third time, in which we washed the $\text{Fe}(\text{OH})_3$ six times with the special solution, we found only 0.1 mg. Cu in the precipitate.

The Creighton ore was treated in exactly the same way.

The $\text{Fe}(\text{OH})_3$ precipitate which had been washed four times was found to be free from both Ni and Cu. Repetition of the experiment gave the same result.

The siliceous residues from both ores were tested for Ni and Cu, and in each case both metals were absent.

Determination of the Ni and Cu in the filtrates from the $\text{Fe}(\text{OH})_3$ + gangue in the above samples was carried out by precipitating the Cu as CuS and then determining the Cu and Ni by titration with standard NaCN . This method is, of course, not to be recommended for precise work, especially for Cu, but it served as a means of confirming our qualitative tests already given.

Crean Hill sample	(1)	Ni 1.61%	Cu 3.10%
	(2)	Ni 1.58%	Cu 3.10%
Creighton sample	(1)	Ni 4.39%	Cu 1.59%
	(2)	Ni 4.35%	Cu 1.71%

We are of the opinion that excellent separations can be made with much smaller amounts of NH_4Cl [or $(\text{NH}_4)_2\text{SO}_4$] and NH_4OH than those employed in the foregoing experiments. We did not, however, attempt to determine the minimum quantities it is possible to use successfully, but have left that for others to work out.

SUMMARY

- (1) A simple, rapid and inexpensive method for separating nickel and copper from iron has been worked out.
- (2) The method has been shown to give excellent results on typical nickel ores from the Sudbury District.
- (3) A method by which ferric hydroxide can be precipitated in a dense form, easy to filter and wash, has been indicated.

REFERENCES

The following list of references is intended principally to serve as a guide to the history of the methods suggested for separating Ni and Cu from Fe. There are also included some of the original articles on the determination and detection of Ni and Cu, and two or three standard works that contain useful information on this subject.

I. Methods of separating nickel from iron.

1. Schwarzenberg: Liebig's Annalen **97** (1856), p. 216.
2. Laby: Chem. News **89** (1904), p. 280.
3. Cheney and Richards: Chem. News **36** (1877), p. 161.
4. Moore: Chem. News **54** (1886), p. 300.
5. Rothe: Chem. News **66** (1892), p. 182.
6. Field: Chem. News **1** (1859), p. 4.
7. Thomas: Chem. News **35** (1877), p. 187.
8. Hassreidter: Z. angew. Chem. **22** (1909), p. 1492.
9. Ibbotson & Brearley: Chem. News **81** (1900), p. 193.
10. Thompson: J. Ind. Eng. Chem. **3** (1911), p. 950.
11. Fresenius: Quant. Chem. Analysis.
12. Mellor: Quant. Inorg. Analysis.

II. Methods of separating copper from iron.

13. Rose: Poggendorff's Annalen **110** (1860), p. 424.
14. Fresenius: Quant. Chem. Analysis.

III. Determination of Nickel.

15. Gibbs: Z. anal. Chem. **3** (1864), p. 336. (Earliest description of electrolytic method.)
16. Campbell and Andrews: J. Am. Chem. Soc. **17** (1895), p. 127. (Earliest description of cyanide method.)
17. Brunck: Z. angew. Chem. **20** (1907), p. 1844. (Earliest description of quant. detn. by dimethylglyoxime.)

IV. Determination of Copper.

18. Parkes: Mining Journal, 1851. (Earliest description of cyanide method.)
19. Steinbeck: Z. anal. Chem. **8** (1869), p. 12. (Cyanide.)
20. Dulin: J. Am. Chem. Soc. **17** (1895), p. 346. (Cyanide.)
21. Gibbs: Z. anal. Chem. **3** (1864), p. 334. (Earliest description of electrolytic method.)
22. Low: Technical Methods of Ore Analysis.

